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On dynamics of quadratic stochastic processes and their applications in biology

A *quadratic stochastic operator* $\mathbf{Q} : \mathfrak{X} \to \mathfrak{X}$ is defined by a cubic (finite or infinite) array of nonnegative real numbers $[q_{ij,k}]_{i,j,k\geq 1}$ which satisfy

(1) $0 \leq q_{ij,k} = q_{ji,k} \leq 1$ for all $i, j, k \geq 1$,

(2) $\sum_{k=1} q_{ij,k} = 1$ for any pair (i, j),

where $\mathfrak{X}$ is ℓ^1 or ℓ^1_{d} equipped with a standard norm. The family of all quadratic stochastic operators is denoted by $\mathfrak{Q}$. Any quadratic stochastic operator (process) $\mathbf{Q}$ may be viewed as a bilinear mapping $\mathbf{Q} : \mathfrak{X} \times \mathfrak{X} \to \mathfrak{X}$ if we set $\mathbf{Q}(\underline{x}, \underline{y})(k) = \sum\limits_{i=1,j=1} x_i y_j q_{ij,k}$. Clearly $\mathbf{Q}$ is monotone (i.e. $\mathbf{Q}(\underline{x}, \underline{y}) \geq \mathbf{Q}(\underline{u}, \underline{w})$ whenever $\underline{x} \geq \underline{u} \geq 0$ and $\underline{y} \geq \underline{w} \geq 0$) and is bounded as $\sup_{\|\underline{x}\|_1, \|\underline{y}\|_1 \leq 1} \|\mathbf{Q}(\underline{x}, \underline{y})\|_1 = 1$. It follows that $\mathbf{Q}$ may also be viewed as a mapping $\mathbf{Q} : \mathcal{D} \times \mathcal{D} \to \mathcal{D}$, where $\mathcal{D}$ stands for the simplex of probability vectors. In population genetics a special attention is paid to a nonlinear mapping $\mathcal{D} \ni \underline{p} \to \mathbb{Q}(\underline{p}) = \mathbf{Q}(\underline{p}, \underline{p})$. Here $\mathbb{Q} : \mathcal{D} \to \mathcal{D}$. Roughly speaking $\mathbb{Q}(\underline{p})$ represents a distribution of genes in the next generation if parent's gens have a distribution $\underline{p}$. In this simplified model the iterates $\mathbb{Q}^k(\underline{p})$, where $k = 0, 1, \ldots$, describe the evolution of a genom. Given an initial distribution $\underline{p} \in \mathcal{D}$ one may ask about asymptotic behaviour of the trajectory (i.e. the sequence $(\mathbb{Q}^n(\underline{p}))_{n\geq 0}$). Because of nonlinearity, the trajectories enjoy several unexpected features (as it was conjectured by S. Ulam). In this talk we discuss some generic properties in the set $\mathfrak{Q}$. We also present conditions for asymptotic stability of $\mathbf{Q} \in \mathfrak{Q}$.