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Structured and unstructured continuous models for *Wolbachia* infections

Wolbachia is a maternally transmitted bacterium that lives in symbiosis with many arthropod species. We introduce and investigate a series of models for an infection of a diploid host species by *Wolbachia*. The continuous models are characterized by partial vertical transmission, cytoplasmic incompatibility and fitness costs associated with the infection. A particular aspect of interest is competitions between mutually incompatible strains. We further introduce an age-structured model that takes into account different fertility and mortality rates at different stages of the life cycle of the individuals. With only a few parameters, the ordinary differential equation models exhibit already interesting dynamics and can be used to predict criteria under which a strain of bacteria is able to invade a population. Interestingly, but not surprisingly, the age-structured model shows significant differences concerning the existence and stability of equilibrium solutions compared to the unstructured model.

Keywords: *Wolbachia*, endosymbiosis, cytoplasmic incompatibility