

**Peter Simon**

INSTITUTE OF MATHEMATICS, EOTVOS LORAND UNIVERSITY, BUDAPEST  
e-mail: [simonp@cs.elte.hu](mailto:simonp@cs.elte.hu)

## **Exact and approximate epidemic models on networks**

The rigorous linking of exact stochastic models to mean-field pair and triple approximations is studied. Using a continuous time Markov Chain, we start from the exact formulation of a simple epidemic model on a completely connected network and rigorously derive the well-known mean-field pair approximation that is usually justified under the hypothesis that infected nodes are distributed randomly.

In addition, we propose a new approach that is based on deriving a countable system of ordinary differential equations for the moments of the distribution of the number of infected nodes. We show how the usual mean-field pair approximation can be derived from this countable system, and prove that this converges to the exact solution given by the Kolmogorov equations as order  $1/N$ . We discuss how our new approach relates to the generally cited results by Kurtz.

Finally, the performance of the triple closure approximation is investigated numerically. It will be shown that the usual triple closure yields a solution that also converges as order  $1/N$  to the exact solution, and we propose a novel triple closure where the rate of convergence is of order  $1/N^2$ .