Curriculum Vitae

Piotr M. Hajac

30 September 2017

1. Education

(1) 3.8.1988, M.Sc. summa cum laude conferred by Wrocław University. Thesis title: *Applications of holonomy maps in Yang-Mills theory.* Supervisor: Arkadiusz Zbigniew Jadczyk.
(2) 1989/90, Visiting Graduate Student at the Mathematical Institute of Oxford University, G. Britain.
(3) 1990–1994, Ph.D. student in the Department of Mathematics of the University of California at Berkeley, USA.
(4) 21.5.1994, Ph.D. conferred by the University of California at Berkeley. Thesis title: *Strong connections and $U_q(2)$-Yang-Mills theory on quantum principal bundles.* Supervisor: Marc A. Rieffel.
(5) 14.6.2002, habilitation degree conferred by the Institute of Mathematics of the Polish Academy of Sciences (IMPAN). Dissertation title: *Noncommutative geometry of algebraic bundles.* Referees: Alain Connes, Piotr Podleś, Piotr Pragacz, Hans-Jürgen Schneider.
(6) 26.6.2014, the academic title of professor awarded by the President of Poland.

2. Postdoctoral positions

(1) Mathematics Section of the International Centre for Theoretical Physics, Trieste, Italy: 6–12.1995, 10–12.1996, 12.1998, 6–8.1999.
(2) 1.4.1997–31.3.1998, postdoctoral fellow in the Department of Applied Mathematics and Theoretical Physics, Faculty of Mathematics, University of Cambridge, G. Britain.
(3) International School for Advanced Studies (SISSA), Trieste, Italy: 2.1997, 4–9.1998, 4.1999 and 12.1999.
(4) 1.3.2001–28.2.2003, Marie Curie postdoctoral fellow at the Mathematical Institute of Munich University, Germany.

3. Employment

(1) 1.10.1994–30.9.2014, a tenured Assistant Professor in the Department of Mathematical Methods in Physics, Warsaw University.
(2) 1.10.1999–30.9.2003, an Assistant Professor in IMPAN
(3) 1.10.2003–30.9.2014, an Associate Professor in IMPAN
(4) 1.7.2012–30.6.2016, an Adjunct Professor at the University of New Brunswick, Fredericton, Canada.
(5) 1.9.2014-30.9.2014, a Visiting Full Professor at the University of Bonn, Germany.
(6) 1.10.2014, a tenured Full Professor in IMPAN, chair of Noncommutative Geometry.

4. Teaching activities

4.1. **Lecture courses.**

4.1.1. *Department of Mathematics, University of California at Berkeley.* In the period 20.6.1994–12.8.1994, I was employed as a lecturer, and taught a lecture course on real analysis.

4.1.2. *Warsaw University.*

(1) *An introduction to contemporary mathematics*, a year-long special topic lecture course in Polish (2010).
(2) *From classical to quantum spaces*, a semester-long special topic lecture course in Polish (2011).
(3) *From Hilbert spaces to quantum geometry*, a semester-long special topic lecture course (2012).
(4) *Algebra z with geometry*, a year-long regular lecture course in Polish (2012/13).

4.2. **Seminars.**

(1) *Noncommutative Geometry Seminar:* This is a weekly seminar at IMPAN run jointly with T. Maszczyk. The seminar was started in November 1999. After a year, there was a 2-year gap due to my fellowship in Munich. The seminar was resumed in October 2003. However, only on 6 October 2004, with the inaugural lecture of Alain Connes, the undisputed father of noncommutative geometry and a Fields medalist, the seminar started working full steam. Since then the seminar gained its international status with all talks in English and the majority of talks given by guests from all over the world. Thus far the peak of attendance was reached on 22 January 2007 when the lecture of Maxim Kontsevich (also a Fields medalist) was attended by at least 82 people. Since October 2009 seminar talks are video-recorded and the recordings are available online. The seminar announcements are e-mailed to about 200 mathematicians in the world. Out of 115 seminar talks in the years 2010–2013, as many as 74 talks were delivered by international guests. This is probably the most international mathematical seminar in Poland.
(2) *Hopf-Cyclic Cohomology:* This was a weekly seminar run by me in the winter semester of 2002/03 in the Mathematical Institute of the University of Munich.
(3) *Mathematical Colloquium:* Together with B. Jakubczyk and P. Strzelecki, in the years 2008–2010, I ran a monthly mathematical colloquium of the Polish Mathematical Society and the Banach Center.
(4) *Seminar of the Department of Mathematical Methods in Physics, Warsaw University:* This is a weekly seminar which, in the academic year 2009/10, I ran jointly with P. Urbański.

4.3. **Supervision.**

4.3.1. *Postdoctoral Fellows.*

(1) *Ulrich Krähmer:* In the period 1.9.2005–31.8.2007, I hosted him as a Marie Curie Fellow in IMPAN.
(2) *Emily Burgunder:* In the period 1.10.2008–30.9.2009, I hosted her at IMPAN as a European Postdoctoral Institute (EPDI) Fellow.
(3) *Adam Skalski:* From 1.10.2010 till 30.9.2012, I hosted him as a Marie Curie Fellow in IMPAN.
(4) *Karen Strung:* 1.10.2014–31.3.2016, IMPAN's postdoctoral fellow till 30.9.2015, then a recipient of an IMPACT international fellowship.
(5) *Réamonn Ó Buachalla:* IMPAN's postdoctoral fellow within the IMPACT international fellowship programme for the period 1.1.2015–31.12.2016.
(6) *Safoura Zadeh:* IMPAN's postdoctoral fellow for the period 1.9.2017–31.8.2018.

4.3.2. *Ph.D. students.*

(1) *Dorota Marciniak:* I was her supervisor in 2004–2007. She graduated from IMPAN without obtaining a Ph.D.
(2) *Marcin Szamotulski:* I was his supervisor in 2004–2007. He graduated from IMPAN without obtaining a Ph.D.
(3) *Jan A. Rudnik:* I was his supervisor in 2008–2013. His Ph.D. in Mathematics was awarded by a unanimous decision of the Scientific Council of IMPAN on 24.1.2013. His thesis title was: *The noncommutative topology of triple-pullback C*-algebras.* Referees: L. Dąbrowski and P. M. Sołtan.
(4) *Mariusz Tobolski:* A Ph.D. student in IMPAN since 2016.

5. Organizational activities

5.1. **Directing grant projects.** Since 2003, I was/am the director of 13 Polish grants with the total value ca. 1 284 693 EUR, 7 EU grants with the total value ca. 1 172 730 EUR, 3 US grants of ca. 72 960 EUR, and 2 Canadian grants of ca. 66 539 EUR. In particular, these funds financed the employment of dozens of mathematicians at Warsaw University and IMPAN. During 14 years the total value of my grants reached about 2 596 922 EUR. This is several times more than IMPAN and Warsaw University combined spent on my salaries. In 2015 alone, I received 8 grants from funding agencies in the EU (REA, Polish Government, Banach Center), the US (NSF, Simons Foundation, Penn State) and Canada (the Fields Institute) adding up to about 461 040 EUR. Details of my grants are described below. I also participated in 3 other grant projects: *Geometric Analysis* (EU grant), *Quantum Information and Engineering* (Polish grant), and *Geometry in Mathematical Physics* (Polish matching grant).

(1) IMPAN (19.3.2003–18.3.2004): Personal Polish grant *Index theory of noncommutative fibrations*, ca. 10 654 EUR.
(2) IMPAN (1.10.2004–30.9.2005): European Reintegration Grant (ERG) *Quantum fibrations*, 40 000 EUR. Among other things, this grant sponsored seminar guests at IMPAN, a Banach Center *Spring School on Noncommutative Geometry and Quantum Groups*, and 4 stipends for Ph.D. students: P. Ł. Kasprzak (Warsaw University), D. Marciniak (IMPAN), M. Szamotulski (IMPAN), P. Witkowski (Warsaw University).
(3) IMPAN (12.5.2004–11.5.2007): *Hopf-cyclic cohomology and noncommutative index theory*, a Polish grant of ca. 41 693 EUR. Within this project, I directed a 5-member international research team.
(4) IMPAN (1.9.2005–31.8.2007): European Individual Fellowship (EIF) *Noncommutative geometry of quantum homogeneous spaces*, ca. 115 810 EUR. This was the Marie Curie fellowship of U. Krähmer, directed by me.
(5) Faculty of Physics, Warsaw University (1.10.2004–30.9.2008): Transfer of Knowledge (TOK) *Noncommutative geometry and quantum groups*, ca. 500 000 EUR. Within this project, I directed a 20-member international team of lecturers and research collaborators.
(6) Faculty of Physics, Warsaw University (1.1.2005–30.9.2008): a Polish matching grant for the aforementioned EU-grant *Noncommutative geometry and quantum groups*. The budget was ca. 258 720 EUR. Within this project, I directed an 11-member Polish research team.
(7) IMPAN (30.11.2007–29.11.2010): *K-theory and spectral geometry for Galois-type extensions of noncommutative algebras*. A Polish grant of ca. 121 598 EUR. Within this project, I directed an 11-member international research team.
(8) IMPAN (1.1.2009–31.12.2012): International Research Staff Exchange Scheme (IRSES), *Geometry and symmetry of quantum spaces*. An EU-grant of 50 400 EUR for co-financing transcontinental collaboration. Within this project, I directed the collaboration of 7 European nodes and 5 nodes in North America and Australia.
(9) IMPAN (1.1.2009–31.12.2012): a Polish matching grant for the aforementioned EU-grant *Geometry and symmetry of quantum spaces*. The budget was ca. 144 547 EUR. Within this project, I directed a 4-member Polish research team.
(10) IMPAN (1.10.2010–30.9.2012): *Probability and quantum groups*, an EU-grant no. PIEF-GA-2009-252212 of ca. 111 180 EUR. This was the Marie Curie fellowship of Adam Skalski, directed by me.
(11) IMPAN (21.12.2011–20.12.2014): *Coefficients in cyclic homology and the noncommutative geometry of C*-algebras and Dirac operators*. This is a Polish National Research Centre grant of ca. 99 369 EUR. Within this project, I direct a 7-member international research team.
(12) The Fields Institute, Toronto, Canada (June 2013): *Focus Program on Noncommutative Geometry and Quantum Groups*. On 23.1.2012 the Fields Institute awarded me and my collaborators a grant of ca. 55 194 EUR to run a month-long scientific programme.

(13) IMPAN (1.3.2013–28.2.2018): *IMPAN International Fellowship Programme (IMPACT)*, a COFUND grant of 108 284 EUR. This is an EU matching grant for co-funding international postdoctoral fellowships (10 person years) at IMPAN.
(14) IMPAN (1.1.2014–28.2.2018): A Polish Government grant of ca. 171 540 EUR for the IMPACT programme.
(15) IMPAN (11.4.2013–10.4.2016): *Deformation and non-deformation quantization: from Poisson brackets to universal symmetries.* This is a Polish National Research Centre grant of ca. 144 541 EUR designed for international collaboration. Within this project, I direct a 19-member international research team.
(16) IMPAN (1.1.2016–31.12.2019): *New geometry of quantum dynamics.* This a Research and Innovation Staff Exchange (RISE) EU-network of 20 research institutions from 3 continents. The EU-contribution to this project is 288 000 EUR.
(17) IMPAN (1.1.2016–31.12.2019): *New geometry of quantum dynamics.* A Polish Government contribution of ca. 2 500 EUR as the reimbursement of expenses incurred while preparing the project.
(18) IMPAN (1.1.2016–31.12.2019): *New geometry of quantum dynamics.* A Polish Government matching grant of ca. 167 230 EUR.
(19) IMPAN (1.1.2016–31.12.2019): *New geometry of quantum dynamics.* A Polish Government award of ca. 33 750 EUR as additional remuneration for carrying out the project.
(20) The Fields Institute, Toronto, Canada (18.7.2016–22.7.2016): *Conference on Geometry, representation theory and the Baum-Connes conjecture.* Sponsored by the Fields Institute (ca. 11 314 EUR), the National Science Foundation, USA (ca. 5 657 EUR), the Pennsylvania State University, USA (ca. 10 608 EUR).
(21) IMPAN (1.9.2016–30.11.2016): *Noncommutative geometry the next generation.* This is a Simons semester at the Banach Center sponsored by Simons Foundation (ca. 56 695 EUR), the Polish Government (ca. 70 229 EUR), and the Banach Center (ca. 16 019 EUR).
(22) IMPAN (14.1.2018–20.1.2018): *New geometry of quantum dynamics* conference at the Banach Center (ca. 2 351 EUR).

5.2. **Organizing conferences and other meetings.**

2001 *Noncommutative Geometry and Quantum Groups* (17–29.9.2001). Stefan Banach International Mathematical Center School/Conference, Warsaw, Poland. Organizers: Piotr M. Hajac, Wiesław Pusz and Stanisław L. Woronowicz (director). This meeting attracted 82 participants from 20 countries and was co-financed by a special EU-grant.

2003 *A Midsummer Day's Workshop on Noncommutative Geometry and Quantum Groups* (20.8.2003) IMPAN, Warszawa. Organizers: Piotr M. Hajac and Rainer Matthes.

2007 *Noncommutative Geometry and Quantum Groups 2007* (30.7–3.8.2007). A meeting in honour of Paul F. Baum on the occasion of his 70th birthday, Stefan Banach International Mathematical Center. Organizers: Alain Connes, Piotr M. Hajac and Ryszard Nest.

2010 *Noncommutative Geometry Mini-School* (27.9–1.10.2010). Organizers: Piotr M. Hajac and Andrzej Sitarz. Also, I was a member of the Organizing Committee of the Warsaw part of the meeting *Road to Reality with Roger Penrose: Mathematics, Physics and Philosophy.*

2011 In the Banach Center, I organized 3 research meetings: 3–8 April, Noncommutative geometry of line bundles over quantum spaces; 21–24 May, New results in Hopf-cyclic cohomology; 27 October – 8 November, New results in noncommutative topology. Also, I was a member of the Organizing Committee of the Banach Center conference *Operator Algebras and Quantum Groups*, 19–23 September, on the occasion of the 70th birthday of Stanisław L. Woronowicz.

2012 In the Banach Center, I organized 4 research meetings: 16 January, The K-theory and symmetry of geometry given C*-algebras; 5–9 March, The foundations of Peter-Weyl-Galois theory; 7–10 May, Noncommutative topology and geometry of quantum fibrations; 17–19 December, Noncommutative geometry of quantum SU(2) revisited.

2013 In the Banach Center, I organized 3 research meetings: 18–25 February, Noncommutative deformations; 11–15 March, Multi-purpose role of modules in noncommutative geometry; 20–24 May, Classical and quantum groups in action (together with Stanisław L. Woronowicz).

2014 In the Banach Center, I organized 3 research meetings: 7–14 January, Homological algebra of quantisations; 27 March – 17 April, Braided noncommutative join construction; 12–19 May, From quantum groups to quantum groupoids; and a conference *From Poisson brackets to universal quantum symmetries* (18–22 August). At the University of New Brunswick, in Fredericton, Canada, on 27 June, together with Bahram Rangipour, I organized a day-long meeting *Hopf algebras in (co)action*.

2015 In SISSA, Trieste, Italy, together with Giovanni Landi and Gherardo Piacitelli, I organized a conference *Quantum groups in noncommutative geometry* (15–16 July).

2016 In the Fields Institute, Toronto, Canada, together with Alan Carey, George Elliott and Ryszard Nest, I am organizing a conference on *Geometry, representation theory and the Baum-Connes conjecture* (18–22 July). In the Banach Center, together with Paul F. Baum, Alan Carey and Tomasz Maszczyk, I organize a Simons semester *Noncommutative geometry the next generation* (1 Sep.–30 Nov.).

2018 The Banach Center, Warsaw, Poland, a conference *New geometry of quantum dynamics* organized together with Tomasz Maszczyk (14.1.2018–20.1.2018).

5.3. **Other activities.** I am the director of the Noncommutative Geometry Research Group at IMPAN, and a member of IMPAN's Scientific Council. In 2015, my research group consisted of 13 mathematicians from 8 countries: Poland (6), Canada (1), China (1) France (1), India (1), Iran (1), Ireland (1), USA (1). Since 2010, I supervise all EU programmes at IMPAN, and since 2014, I am a Legal Entity Appointed Representative for IMPAN.

6. Talks and mini courses

Not counting talks at the place of employment, within about 23 years since receiving my Ph.D. (1994–2017), I delivered the total of 205 (including 64 at conferences) talks or series of lectures in 25 countries on 5 continents: Canada 29, USA 29, Italy 26, Germany 23, Poland 21, Australia 13, France 13, G. Britain 10, Belgium 8, Luxembourg 4, Denmark 4, Japan 3, Argentina 3, Ireland 3, Finland 3, Austria 2, Czeck Republic 2, Thailand 2, Mexico 1, New Zealand 1, Norway 1, Spain 1, Sweden 1, Netherlands 1, S. Korea 1. Thus, on average, more less 9 times per year I was invited to present my mathematical results or deliver a mini lecture course. In 2015 alone, I delivered 26 talks/lectures (including 6 at conferences) on 3 continents.

7. Publication list

According to the official data published by the American Mathematical Society (www.ams.org/mathscinet), in September 2017 my papers scored over 400 citations, and my Hirsch index (equal to the number of papers h cited at least h times) was 12. The following list of my 35 published papers does not include chapters written by me of the book *Quantum symmetry in noncommutative geometry*, accepted for publication.

(1) Hajac, Piotr M.: Axiomatic holonomy maps and generalized Yang-Mills moduli space. Lett. Math. Phys. 27 (1993), no. 4, 301–309.

(2) Hajac, Piotr M.: The Einstein action for algebras of matrix valued functions — toy models. J. Math. Phys. 37 (1996), no. 9, 4549–4556.

(3) Dąbrowski, Ludwik; Hajac, Piotr M.; Landi, Giovanni; Siniscalco, Pasquale: Metrics and pairs of left and right connections on bimodules. J. Math. Phys. 37 (1996), no. 9, 4635–4646.

(4) Hajac, Piotr M.: Strong connections on quantum principal bundles. Comm. Math. Phys. 182 (1996), no. 3, 579–617.

(5) Hajac, Piotr M.: A note on first order differential calculus on quantum principal bundles. Quantum groups and integrable systems, Part I (Prague, 1997). Czechoslovak J. Phys. 47 (1997), no. 11, 1139–1143.

(6) Hajac, Piotr M.; Masuda, Tetsuya: Quantum double-torus. C. R. Acad. Sci. Paris Sér. I Math. 327 (1998), no. 6, 553–558.

(7) Hajac, Piotr M.; Matthes, Rainer: Frame, cotangent and tangent bundles of the quantum plane. Proceedings of the workshop "Lie Theory and its Applications in Physics II", H.-D. Doebner, J. Hilgert, V. K. Dobrev (eds.), World Scientific Publ., pp.377–385, 1998

(8) Brzeziński, Tomasz; Hajac, Piotr M.: Coalgebra extensions and algebra coextensions of Galois type. Comm. Algebra 27 (1999), no. 3, 1347–1367.

(9) Dąbrowski, Ludwik; Grosse, Harald; Hajac, Piotr M.; Siniscalco, Pasquale: Explicit Hopf-Galois description of $SL_{e^{\frac{2i\pi}{3}}}(2)$-induced Frobenius homomorphisms. "Enlarged Proceedings of the ISI GUCCIA Workshop on quantum groups, non commutative geometry and fundamental physical interactions", D. Kastler, M. Rosso, T. Schucker (eds.), Commack – New York, Nova Science Pub, Inc., pp.279–298, 1999

(10) Hajac, Piotr M.; Majid, Shahn: Projective module description of the q-monopole. Comm. Math. Phys. 206 (1999), no. 2, 247–264.

(11) Hajac, Piotr M.: Bundles over quantum sphere and noncommutative index theorem. K-Theory 21 (2000), no. 2, 141–150.

(12) Dąbrowski, Ludwik; Grosse, Harald; Hajac, Piotr M.: Strong connections and Chern-Connes pairing in the Hopf-Galois theory. Comm. Math. Phys. 220 (2001), no. 2, 301–331.

(13) Hajac, Piotr M.; Matthes, Rainer; Szymański, Wojciech: Quantum real projective space, disc and spheres. Algebr. Represent. Theory 6 (2003), no. 2, 169–192.

(14) Hajac, Piotr M.; Matthes, Rainer; Szymański, Wojciech: Chern numbers for two families of noncommutative Hopf fibrations. C. R. Math. Acad. Sci. Paris 336 (2003), no. 11, 925–930.

(15) Hajac, Piotr M.; Matthes, Rainer; Szymański, Wojciech: Graph C^*-algebras and $\mathbb{Z}_2$-quotients of quantum spheres. Proceedings of the XXXIV Symposium on Mathematical Physics (Toruń, 2002). Rep. Math. Phys. 51 (2003), no. 2-3, 215–224.

(16) Brzeziński, Tomasz; Hajac, Piotr M.: The Chern-Galois character. C. R. Math. Acad. Sci. Paris 338 (2004), no. 2, 113–116.

(17) Hajac, Piotr M.; Khalkhali, Masoud; Rangipour, Bahram; Sommerhäuser, Yorck: Stable anti-Yetter-Drinfeld modules. C. R. Math. Acad. Sci. Paris 338 (2004), no. 8, 587–590.

(18) Hajac, Piotr M.; Khalkhali, Masoud; Rangipour, Bahram; Sommerhäuser, Yorck: Hopf-cyclic homology and cohomology with coefficients. C. R. Math. Acad. Sci. Paris 338 (2004), no. 9, 667–672.

(19) Baum, Paul F.; Hajac, Piotr M.; Matthes, Rainer; Szymański, Wojciech: The K-theory of Heegaard-type quantum 3-spheres. K-Theory 35 (2005), no. 1-2, 159–186.

(20) Hajac, Piotr M.; Matthes, Rainer; Szymański, Wojciech: A locally trivial quantum Hopf fibration. Algebr. Represent. Theory 9 (2006), no. 2, 121–146.

(21) Hajac, Piotr M.; Matthes, Rainer; Szymański, Wojciech: Noncommutative index theory for mirror quantum spheres. C. R. Math. Acad. Sci. Paris 343 (2006), no. 11-12, 731–736.

(22) Hajac, Piotr M.; Krähmer, Ulrich; Matthes, Rainer; Zieliński, Bartosz: Piecewise principal comodule algebras. J. Noncommut. Geom. 5 (2011), no. 4, 591–614.

(23) Hajac, Piotr M.; Kaygun, Atabey; Zieliński, Bartosz: Quantum complex projective spaces from Toeplitz cubes. J. Noncommut. Geom. 6 (2012), no. 3, 603–621.

(24) Dąbrowski, Ludwik; Hadfield, Tom; Hajac, Piotr M.; Matthes, Rainer; Wagner, Elmar: Index pairings for pullbacks of C*-algebras; Banach Center Publ., Volume 98, 2012, 67–84.

(25) Hajac, Piotr M.; Kaygun, Atabey; Zieliński, Bartosz: Finite closed coverings of compact quantum spaces, Banach Center Publ., Volume 98, 2012, 215–237.

(26) Hajac, Piotr M.; Zieliński, Bartosz: Cocycle condition for multi-pullbacks of algebras; P. M. Hajac, B. Zieliński; Banach Center Publ., Volume 98, 2012, 239–243.

(27) Hajac, Piotr M.; Rennie, Adam; Zieliński, Bartosz: The K-theory of Heegaard quantum lens spaces; J. Noncommut. Geom., 7 (2013), no. 4, 1185–1216.

(28) Hajac, Piotr M.; Zieliński, Bartosz: Nontrivial deformation of a trivial bundle; SIGMA Symmetry Integrability Geom. Methods Appl. 10 (2014), Paper 031, 7 pp.

(29) Baum, Paul F.; Hajac, Piotr M.: Local proof of algebraic characterization of free actions; SIGMA Symmetry Integrability Geom. Methods Appl. 10 (2014), Paper 060, 7 pp.

(30) Piotr M. Hajac, Elmar Wagner: The pullbacks of principal coactions; Documenta Math. 19 (2014) 1025–1060.

(31) Ludwik Dąbrowski, Tom Hadfield, Piotr M. Hajac: Equivariant join and fusion of noncommutative algebras; SIGMA Symmetry Integrability Geom. Methods Appl. 11 (2015), Paper 082, 7 pp.

(32) Paul F. Baum, Ludwik Dąbrowski, Piotr M. Hajac: Noncommutative Borsuk-Ulam-type conjectures; Banach Center Publ., Volume 106, 2015, 9–18.

(33) Ludwik Dąbrowski, Tom Hadfield, Piotr M. Hajac, Elmar Wagner: Braided join comodule algebras of bi-Galois objects; New York J. Math. 22 (2016) 1085–1109.

(34) Piotr M. Hajac, Jan Rudnik: Noncommutative bundles over the multi-pullback quantum complex projective plane; New York J. Math. 23 (2017) 295–313.

(35) Paul F. Baum, Kenny De Commer, Piotr M. Hajac: Free actions of compact quantum groups on unital C*-algebras; Documenta Math. 22 (2017) 825–849.

8. Research interests and achievements

Since the 2nd year of my Ph.D. studies, I work on noncommutative geometry and quantum groups. Key words of my research are: K-theory of C*-algebras, Hopf algebras, cyclic homology. My results can grouped into two main streams: the noncommutative topology of compact quantum principal bundles and Hopf-cyclic cohomology.

As my most important research achievements I consider the following:

(1) Introducing the concept of a strong connection (Ph.D. thesis), and then developing the theory of strong connections (habilitation dissertation). My key papers on strong connections are: *Strong connections on quantum principal bundles; Projective module description of the q-monopole; Strong connections and Chern-Connes pairing in the Hopf-Galois theory.* Strong connections play the role of connections on principal bundles, and are given in general algebraic terms. They turned out to be pivotal (both conceptually and in calculations) in bringing together representations of compact quantum groups and K-theory invariants.

(2) An application of noncommutative index theory to compute K_0-invariants for all Podleś quantum spheres (habilitation dissertation and its follow up). Here the key papers are: *Bundles over quantum sphere and noncommutative index theorem; Chern numbers for two families of noncommutative Hopf fibrations.* They provide solutions of concrete mathematical problems.

(3) Introducing and applying the Chern-Galois character (posthabilitation work). The relevant articles are: *The Chern-Galois character; Noncommutative index theory for mirror quantum spheres.* The Chern-Galois character is a fundamental point for developing noncommutative Chern-Weil theory.

(4) Working out the theory of coefficients of Hopf-cyclic cohomology (posthabilitation work). Here the relevant articles are: *Stable anti-Yetter-Drinfeld modules; Hopf-cyclic homology and cohomology with coefficients.* Unlike the Hochschild homology, which is born with general coefficients, the intimately related cyclic homology resisted general coefficients for a long time. Discovered by Connes and Moscovici, Hopf-cyclic cohomology suffered the same problem. However, it came equipped with a richer Hopf-algebraic structure that allowed the development of Hopf-cyclic cohomology with general coefficients. This new cohomology theory turned out to be very fruitful and popular. Now it is clear that the introduction of general coefficients opened a door to a new branch of noncommutative geometry. At the same time, Hopf-cyclic homology enjoys deep roots in difficult and concrete problems of the transverse geometry of foliations, and a fascinating general structure approached from an advanced categorical point of view. This is, presumably, one of the reasons for its success. Taking into account the vast influence of this construction on the work of others, I consider this result as my most important mathematical achievement thus far.

(5) Proving the equivalence of the freeness of actions of compact quantum groups on unital C*-algebras and the Galois condition for the associated Peter-Weyl comodule algebra (*Free actions of compact quantum groups on unital C*-algebras*). For a very long time, there was a worrisome conceptual gap between the plentiful algebraic constructions of noncommutative principal (co)actions and the topological concept of a classical compact principal bundle. Following Woronowicz's idea of defining a compact quantum group, one can easily provide an analytical definition of a compact quantum principal bundle (David A. Ellwood) including classical compact principal bundles as a special case. However, it remained unclear how the analytical and algebraic definitions of principal (co)actions relate to each other. Only after extending Woronowicz's Peter-Weyl theory from compact quantum groups to compact quantum principal bundles, and applying sophisticated methods of Hilbert modules, we obtained a desired proof of the equivalence of the analytical and algebraic approaches. This result has its virtue already in the classical setting of topological spaces because it equips compact principal bundles with a new algebraic tool (strong connections) for computing K-theory invariants.

(6) Proving that the pullback of principal coactions is principal. The main theorems are in the articles: *Piecewise principal comodule algebras; The pullbacks of principal coactions.* The classical counterpart of this result is trivial: if an action of a group G is free on G-subspaces covering the space, then it is free on the whole space. However, the situation drastically changes its nature when moved to the noncommutative setting. The problem can be easily formulated in terms of multi-pullbacks of comodule algebras, but its solution is no longer obvious. Only an application of strong connections replaced the lack of classical intuition and afforded a solution of this problem. The solution is technically very complicated. Its linearization yields the celebrated Milnor construction of a connecting homomorphism in the Mayer-Vietoris long exact sequence in algebraic K-theory.

(7) Applying the Milnor construction of a connecting homomorphism in algebraic K-theory to studying the K-theory of Pedersen's multi-pullbacks of C*-algebras (*The K-theory of Heegaard quantum lens spaces*). It turns out that not only can Milnor's construction be transplanted from the algebraic to the C*-algebraic setting, but also it is a better computational tool than the standard C*-algebraic construction of a connecting homomorphism in the Mayer-Vietoris 6-term exact sequence of K-groups of C*-algebras. Matching idempotents obtained from the Milnor construction with idempotents provided by the Chern-Galois theory of strong connections yielded unexpectedly strong results in the K-theory of pullbacks of C*-algebras.

In his letter of recommendation, the creator of noncommutative geometry Alain Connes described my results detailed above in points 2 and 4 in the following way:

"I am very impressed by the recent work of Piotr Hajac. He has made recently two important breakthrough contributions to noncommutative geometry.

The first is his computation of Chern numbers for generic quantum spheres, a long open and important problem in the general development of the theory which passes of necessity through explicit and hopefully general computations. In that respect he combines both an excellent skill in concrete examples as well as a very good sense for what is conceptual.

The second which is probably the most important from the conceptual point of view is on the cyclic cohomology of Hopf algebras which I introduced with Henri Moscovici motivated by explicit computations of Chern characters in the context of foliations. What Piotr Hajac discovered is the proper setting for such theories and their relation with Yetter-Drinfeld modules. He showed that what lies behind the complicated algebra responsible for the periodicity of the analogue of the cyclic operation in the cyclic cohomology of Hopf algebras is due to an underlying structure of Yetter-Drinfeld module. This was highly non-trivial since in fact the latter structure had to be twisted in the process. It obviously opens up totally new territories and is a perfect witness of his expertise (he knows both domains in a rather unique manner) and of his creativity."