TALK: A MARSTRAND THEOREM FOR MEASURES WITH POLYTOPE DENSITY

Abstract: We will give a sketch of some methods used to prove some classic theorems in Geometric Measure Theorey related to (and precursors of) Preiss rectifiability theorem. In particular we will focus on Marstrand's Theorem that a measure in Euclidean space whose s-density exists and is positive finite everywhere has to be such that s is and integer and the measure has a weak s tangents almost everywhere. Formally

Theorem 1 (Marstrand, 1964)**.** *Let μ be a Radon measure on $\mathbb{R}^n$ with the property*

$$0 < \lim_{r \to 0} \frac{\mu(B_r(x))}{r^s} < \infty$$

for μ a.a. points $x \in \mathrm{Spt}\mu$, then

- *s is an integer,*
- *for μ almost all points $x \in \mathrm{Spt}\mu$ we have that there exists an s-plane V going through x such that*

$$\liminf_{r \to 0} \frac{\mu(B_r(x) \setminus N_{\epsilon r}(V))}{r^s} = 0.$$

We will state a recent theorem which generalises the first part of Marstrands theorem for a wide class of finite dimensional normed vector spaces for the range $s \in (0, 2]$

Theorem 2 (Lorent, 2005)**.** *Let $s \in (0, 2]$. Let Θ denote a centrally symmetric convex polytope centred on 0. Let $\Theta_r(z) := r\Theta + x$.*

Suppose μ is a Radon measure with the property

$$0 < \lim_{r \to 0} \frac{\mu(\Theta_r(x))}{r^s} < \infty \text{ for } \mu \text{ a.e. } x \in \mathrm{Spt}\mu$$

then s is an integer.