European Mathematical Society Conference

ANALYSIS ON METRIC MEASURE SPACES

Mathematical Research and Conference Center, Będlewo
July 15 - 23, 2004

Stefan Banach International Mathematical Center

8 Sniadeckich Str., 00-956 Warsaw, Poland

Phone: (+48 22) 628 01 92 Fax: (+48 22) 622 39 97

E-mail: banach@impan.gov.pl WWW: http://www.impan.gov.pl/BC

Institute of Mathematics Polish Academy of Sciences

and

European Mathematical Society

Conference
ANALYSIS ON METRIC MEASURE SPACES

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Anna Zatorska-Goldstein (Warsaw University)
Very weak solutions of nonlinear subelliptic equations

ABSTRACTS

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Zoltan M. Balogh (University of Bern)
Stepanov's differentiability theorem in metric measure spaces

We extend Cheeger's theorem on differentiability of Lipschitz functions in metric measure spaces to the class of functions satisfying Stepanov's condition of boundedness of the upper-scaled oscillation. As consequence, we obtain the analogue of Calderon's differentiability theorem of Sobolev functions in metric measure spaces satisfying a Poincaré inequality. We show by examples that a Stepanov type differentiability theorem does not hold under the boundedness assumption of the lower scaled-oscillation only and give additional conditions in terms of the lower scaled-oscillation for a function to belong to the Sobolev or Lipschitz class.

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Paweł Bechler (Polish Academy of Sciences)
Greedy wavelet projections and the spaces $BV(R^d)$

In the space BV of functions of bounded variation on a d-dimensional euclidean space we consider a compactly supported wavelet basis normalized in BV. Each function from BV has a unique expansion with respect to this wavelet basis. An N-term (nonlinear) greedy projection is obtained by taking the first N largest terms of that expansion. It is shown that these projections are uniformly bounded in the BV norm.

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Jana Björn (Linköping University)
Boundary regularity for p-harmonic functions on metric spaces

In 1924, Wiener proved his famous criterion characterizing regular boundary points, i.e. those points $x_0 \in \partial\Omega \subset \mathbf{R}^3$, such that for every continuous $f : \partial\Omega \to \mathbf{R}$, the solution u of the Dirichlet problem for the Laplace equation in Ω with the boundary data f satisfies

$$\lim_{\Omega \ni x \to x_0} u(x) = f(x_0).$$

The same year, Lebesgue characterized regular boundary points for the Laplace equation using bariers.

Similar results were later obtained for the p-Laplace equation

$$\operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0$$

and other generalizations of harmonic functions.

During the last decade, p-harmonic functions have been studied in a general setting of metric spaces equipped with a Borel regular doubling measure satisfying a Poincaré inequality. In this setting, it is possible to define a notion of upper gradients, which generalizes the usual gradient. This makes it possible to define p-harmonic functions as minimizers of the p-energy integral involving the upper gradient and to solve the Dirichlet problem for them.

More generally, we can consider a one-sided obstacle problem for p-energy minimizers on bounded open sets in a metric space satisfying the above assumptions, and study the regularity of boundary points with respect to the solutions of the obstacle problem with continuous boundary data. It turns out that a boundary point is regular for the obstacle problem with continuous boundary data and certain obstacles if and only if it is regular for p-harmonic functions with continuous boundary data. This can be used to construct continuous bariers at regular boundary points and to characterize regular boundary points using bariers, thus extending Lebesgues's results to the metric space setting. These results are a joint work with A. Björn.

As for the Wiener criterion, its sufficiency for the regularity of a boundary point for p-harmonic functions on metric spaces follows from results by MacManus, Shanmugalingam and the speaker. Recently, a similar condition has been shown to be necessary for the regularity of a boundary point.

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Steve Buckley (NUI Maynooth)
Sphericalization and flattening

We discuss a certain family of conformal dilations that take unbounded length spaces to bounded incomplete length spaces (sphericalization) and vice versa (flattening). In particular, we study what happens to the associated quasihyperbolic metric under sphericalization or flattening, and give conditions under which sphericalization and flattening are inverse processes. We also discuss the relationship between the boundary at infinity under sphericalization and the ideal boundary. This talk is based on work with Zoltan Balogh and with Simon Kokkendorff.

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Luigi D'Onofrio (Universitá degli Studi di Napoli)
The p-harmonic transform beyond its natural domain of definition, interpolation and continuity
(joint work with Tadeusz Iwaniec)

The p-harmonic transforms are the most natural nonlinear counterparts of the Riesz transforms in $\mathbb{R}^n$. They originate from the study of the p-harmonic type equation $\mathrm{div}|\nabla \mathrm{u}|^{\mathrm{p}-2}\nabla \mathrm{u} = \mathrm{div}\ f$, where $f : \Omega \longrightarrow \mathbb{R}^n$ is a given vector field in $L^q(\Omega, \mathbb{R}^n)$ and u is an unknown function of Sobolev class $W_0^{1,p}(\Omega, \mathbb{R}^n)$, $p + q = p\,q$. The p-harmonic transform $H_p : L^q(\Omega, \mathbb{R}^n) \longrightarrow L^p(\Omega, \mathbb{R}^n)$ assigns to f the gradient of the solution:$H_p f = \nabla u \in L^p(\Omega, \mathbb{R}^n)$. More general PDE's and the corresponding nonlinear operators are also considered. We investigate the extension and continuity properties of the p-harmonic transform beyond its natural domain of definition. In particular, we identify the exponents $\lambda > 1$ for which the operator $H_p : L^{\lambda q}(\Omega, \mathbb{R}^n) \longrightarrow L^{\lambda p}(\Omega, \mathbb{R}^n)$ is well defined and remains continuous. Rather surprisingly, the uniqueness of the solution $\nabla u \in L^{\lambda p}(\Omega, \mathbb{R}^n)$ fails when λ exceeds certain critical value. In case $p = n = \dim \Omega$ there is no uniqueness in $W^{1,\lambda n}(\mathbb{R}^n)$ for any $\lambda > 1$.

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Alexander Egorov (Sobolev Institute of Mathematics)
Stability of classes of mappings and quasiconvex functions

This research was partially supported by the Russian Foundation for Basic Research (Grant 02–01–01009), by the Ministry of Education of the Russian Federation (Grant PD02–1.1–455), by the State Program of Support for Leading Scientific Schools (Grant NSh–311.2003.1), and by the Russian Science Support Foundation.

We consider classes of solutions $u : U \to \mathbb{R}^m$ to partial differential relations $F(u'(x)) = G(u'(x))$ a.e. in $U \subset \mathbb{R}^n$ with quasiconvex functions $F : R^{m\times n} \to \mathbb{R}$ and null Lagrangians $G : \mathbb{R}^{m\times n} \to \mathbb{R}$. We study stability problems for these classes. The theorems presented in the talk generalize some well-known results on stability of classes of holomorphic functions, conformal transformations, and homotheties. The case when F is convex was considered in [1].

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Uta Freiberg (University Jena)
Analysis on fractals

The physical relevance of fractal sets lies for example in the practical importance of porous media. In particular, one is interested in continuous transport on fractals which can be described by means of a "Laplacian". There are several approaches how to define such a differential operator on finitely ramified self-similar fractals - which are classical examples for non differentiable structures. After presenting the main ideas in this case, we explain how to overcome the lack of self-similarity by using a Lagrangian approach.

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NICOLA GAROFALO (Purdue University)
Minimal surfaces in Carnot groups (3 lectures)

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PETER HÄSTÖ (University of Helsinki)
Arc-wise hyperbolic metrics

Let G be a domain in $\mathbb{R}^n$. A metric is said to be arc-wise hyperbolic if the following condition holds: if B is a ball in G which touches the boundary at two points, and C is the arc of a circle perpendicular to B at these points that joins them in B, then the metric restricted to C equals the hyperbolic metric. Examples of metrics with this property are the Apollonian metric, Ferrand's metric, the Kulkarni-Pinkal metric and Seittenranta's metric. In this talk I will discuss the problem of determining the isometries of such metrics, with a focus on the Apollonian metric.

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DARIA ISANGULOVA (Sobolev Inst. Math. SB RAS)
Differentiability of mappings between Carnot - Carathéodory spaces in BV-topology

We prove *cc*-differentiability of mappings on Carnot - Carathéodory spaces belonging to BV-class in BV-topology. As a consequence we obtain analogue of Calderon-Zygmund theorem for mappings of Carnot - Carathéodory spaces.

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IYA IVANISHKO (Belarussian State University)
Sobolev type spaces on spaces of homogeneous type: different descriptions and embedding theorems

We consider the scale of Sobolev type spaces W^p_η on metric spaces, that depends on the modulus of continuity η and $p > 1$. We give different descriptions for these spaces. In case $\eta(t) = t^s$ common descriptions were given by D. Yang. Also we obtain for these spaces embedding theorem of Sobolev type, which extends the result of V. Kolyada onto spaces of homogeneous type.

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Tadeusz Iwaniec (Syracuse University)
Weekly differentiable mappings between manifolds

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Aigerim Kalybay (Kazakh National University)

On properties of multiweighted spaces and their applications to singular differential equations

Let $\overline{\alpha} = (\alpha_0, \alpha_1, ..., \alpha_n)$, $1 < p < \infty$. For the function $x : (0,1) \to R$ assume $D^0_{\overline{\alpha}} x(t) \equiv t^{\alpha_0} x(t)$; $D^i_{\overline{\alpha}} x(t) = t^{\alpha_i} \frac{d}{dt} D^{i-1}_{\overline{\alpha}} x(t)$, $i = 1, 2, ..., n$, where all derivatives are understood in generalized sense.

Let $W^n_{p,\overline{\alpha}}(0,1)$ be a class of functions $x : (0,1) \to R$ for which the norm $\|x\|_{W^n_{p,\overline{\alpha}}} = \|D^n_{\overline{\alpha}} x\|_p + \sum\limits_{i=0}^{n-1} |D^i_{\overline{\alpha}} x(1)|$ is finite.

We get that if $\gamma_{\max} = \max\limits_{0 \le i \le n-1} \gamma_i < 1 - \frac{1}{p}$, where $\gamma_i = \alpha_n + \sum\limits_{j=i+1}^{n-1} (\alpha_j - 1)$, $i = 0, 1, ..., n-2$, $\gamma_{n-1} = \alpha_n$, then $\forall x \in W^n_{p,\overline{\alpha}}(0,1)$ and $\forall t_0 > 1$ there exist the finite limits

$$\lim_{t \to 0} B^{t_0}_i x(t) = \lim_{t \to 0} \sum_{j=i}^{n-1} K_{i+1,j}(t, t_0) D^j_{\overline{\alpha}} x(t) = a_i, \quad i = 0, 1, ..., n-1, \tag{1}$$

where $K_{i,j}(s,t) = \int\limits_s^t t_i^{-\alpha_i} \int\limits_s^{t_i} t_{i+1}^{-\alpha_{i+1}} ... \int\limits_s^{t_{j-1}} t_j^{-\alpha_j} dt_j dt_{j-1} ... dt_i$, $i \le j$, $0 < s \le t < \infty$, and in reverse for any set $a_i \in R$, $i = 0, 1, ..., n-1$ there exists $x \in W^n_{p,\overline{\alpha}}(0,1)$ for which (1) takes place.

This property of $W^n_{p,\overline{\alpha}}$ we apply to the problem of initial conditions at singular points for ordinary differential equation. One of the ways to solve such problem was proposed by L.D. Kudryavtsev [see $1-3$], where he considered the problem of existence only one solution and its stability of the equation

$$x^{(n)}(t) = f(t, x, x', ..., x^{(n-1)}), t \ge t_0 \tag{2}$$

and the initial conditions

$$\lim_{t \to +\infty} y_{x,i}(t) = a_i, \ i = 0, 1, ..., n-1, \tag{3}$$

where $y_{x,i}(\cdot) = a_i$, $i = 0, 1, ..., n-1$ is the coefficients of Lagrange expansion of required solution by fundamental solution system of the equation $x^{(n)}(t) = 0$.

Using the connection between $x^{(k)}(t)$ and $D^k_{\overline{\alpha}} x(t)$, $k = 0, 1, ..., n$, due to (1) we pass from the problem (2) and (3) to

$$D^n_{\overline{\alpha}} x(t) = f(t, D^0_{\overline{\alpha}} x, D^1_{\overline{\alpha}} x, ..., D^{n-1}_{\overline{\alpha}} x), t \in (0, t_0], t_0 > 0,$$

$$B^{t_0}_k f(0) = a_k, k = 0, 1, ..., n-1,$$

where $f : (0, t_0] \times R^n \to R$ is a continuous function and $a_k \in R$, $k = 0, 1, ..., n-1$ is given numbers.

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Maria Karmanova (Novosibirsk State University)

Area & coarea formulas on rectifiable sets in metric spaces and Sobolev mappings

We study the metric differentiability of mappings defined on measurable subsets of $\mathbb{R}^n$ with values in an arbitrary complete metric space. Using results for Lipschitz mappings we investigate the metric differentiability of more general mappings including Sobolev ones. We prove also Stepanov type theorem on metric differentiability of mappings. We give suitable definitions of coarea factor both for mappings defined on n-rectifiable metric spaces with the values in a k-rectifiable metric space, $k \leq n$, and of Jacobian of mappings defined on n-rectifiable metric space with the range in an arbitrary complete metric space. We give a direct proof of the area and coarea formulas for mappings under consideration without an isometric embedding in a Banach space. We generalize the area and coarea formulas for Sobolev mappings defined on Riemannian manifolds with values in complete metric spaces.

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Stephen Keith (Australian National University)

Differentiable structures for metric measure spaces (3 lectures)

My talk will be structured around my thesis, the main result being the provision of conditions under which a metric measure space admits a differentiable structure. This differentiable structure gives rise to a finite dimensional L_∞ cotangent bundle over the given metric measure space and then to a Sobolev space $H^{1,p}$ over the given metric measure space, the latter which is reflexive for $p > 1$: This extends results of Cheeger (Geom. Funct. Anal. 9 (1999) (3) 428) to a wider collection of metric measure spaces.

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Juha Kinnunen (University of Oulu)
Pointwise behaviour of Sobolev functions on metric spaces

We study Lebesgue points for Sobolev functions on metric measure spaces over collections of sets called differentiation bases. Our main result gives several conditions for a differentiation basis which characterize the existence of Lebesgue points outside a set of capacity zero. An interesting feature is that a qualitative result, as the existence of Lebesgue points, is implies quantitative estimates.

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Mikhail Korobkov (Sobolev Institute of Mathematics)
Stability of a class of nonsmooth Lipschitz functions

In our talk we continue with study stability in the C-norm of classes of mappings. This theory originated from the well-known results of Yu.G. Reshetnyak, F.John and others on stability of classes of conformal and isometric mappings (see [1]). In essence, stability of a class G means that local proximity of a mapping $f : \Delta \subset \mathbb{R}^n \to \mathbb{R}^m$ to the mappings of the class G implies global proximity of f to them in the C-norm.

Recall that a class G of Lipschitz functions is called *ω-stable* [2] if there exists a function $\sigma : [0, +\infty) \to [0, +\infty)$ such that (1) $\sigma(\varepsilon) \to \sigma(0) = 0$ as $\varepsilon \to 0$; (2) the inequality $\omega(f, G) \le \sigma(\varepsilon)$ holds for every function $f : \Delta \to \mathbb{R}$ of a domain $\Delta \subset \mathbb{R}^n$ such that $\Omega(f, G) \le \varepsilon$.

Here $\omega(f, G) = \sup_{B \subset \Delta} \omega_B(f, G)$, $\Omega(f, G) = \sup_{x \in \Delta}\{\limsup_{r \to 0} \omega_{B(x,r)}(f, G)\}$, where B=$B(x, r)$ is a ball in Δ and

$$\omega_B(f, G) = \inf_{g:B \to \mathbb{R},\ g \in G} \{r^{-1} \sup_{y \in B} |f(y) - g(y)|\}.$$

The functionals $\omega(\cdot, G)$ and $\Omega(\cdot, G)$ are referred to as the functionals of *global* and *local proximity* to the class G.

One of our result is as follows. Let $G \subset \mathbb{R}^n$ be a compact set and $\delta : \mathbb{R}^n \setminus G \to \mathbb{R}$ be a positive function such that $0 < \delta(x) \le \operatorname{dist}(x, G)$. Let $Z^+_\delta(G)$ be the class of all Lipschitz functions $g : \Delta \to \mathbb{R}$ defined on domains $\Delta \subset \mathbb{R}^n$ such that

$$g'(x) \in G \text{a.e.}$$

and

$$\forall a \in \mathbb{R}^n \setminus G\ \forall B(x, r) \subset \Delta \sup_{y \in B(x,r)} \big(g(y) - g(x) - a(y - x)\big) \ge \delta(a)r.$$

Theorem. The class $Z^+_\delta(G)$ is ω-stable.

Examples. If $n = 1$ and $G \subset \mathbb{R}$ is a totally disconnected set then the class $Z^+_\delta(G)$ coincides with the class of convex functions satisfying inclusion (1) (see [3]). (In this case $Z^+_\delta(G)$ does't depend on the function δ.) In general case the class $Z^+_\delta(G)$ always contains the class of convex functions satisfying (1).

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Pekka Koskela (University of Jyväskylä)

Quasiconformal mappings on metric spaces (3 lectures)

Taking the so-called metric definition as the starting point, I will describe how and when this guarantees analytic properties of the mapping. This is mostly based on the results by Heinonen and Koskela from 1998 but I will also discuss the very recent improvements on that work.

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Kazuhiro Kuwae (Yokohama City University)

On Calabi's strong maximum principle via local Dirichlet forms

I will talk about an extension of E. Calabi's strong maximum principle in the framework of strong Feller diffusion processes associated with local regular semi-Dirichlet forms. The proof is quite stochastic and the result can be applicable to the Gromov-Hausdorff limit spaces over compact Riemannian manifolds with uniform bound of diameter and uniform lower bound of Ricci curvature.

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Valentino Magnani (Pisa University)

Blow-up and transverse mass of submanifolds in the Heisenberg group

Aim of this talk is to present an explicit formula for the spherical Hausdorff measure of submanifolds of arbitrary dimension in the Heisenberg group. This formula is obtained by a blow-up procedure with respect to the anisotropic dilations of the group and it suggests a privileged class of left invariant forms in the Heisenberg group. These forms allow us to introduce a reasonable notion of mass for currents in the Heisenberg group, which fits the well known theory in codimension one. A discussion of this notion and some remarks are added.

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JAN MALÝ (Charles University in Prague)
Fine topologies on metric spaces

The density topology, related to the points of Lebesgue density, and the fine topology from the potential theory are usually regarded as two basic examples of fine topologies on Euclidean spaces (or, more generally, on metric spaces). The aim of this talk is to study a unified scale of fine topologies. In this framework, the density topology also comes from a "potential theory".

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OLLI MARTIO (University of Helsinki)
Quasiminimizers in metric measure spaces

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ALEXANDER MESKHI (Scuola Normale Superiore di Pisa)
Fractional integrals on nonhomogeneous spaces

Necessary and sufficient conditions on a measure governing two-weight inequalities for potential operators defined on nonhomogeneous spaces are established, where the weights are of power type. Theorems of Sobolev and Adams type for fractional integrals on measure spaces will be also discussed. The talk is based on the monograph D. E. Edmunds, V. Kokilashvili and A. Meskhi, Bounded and Compact Integral Operators, *Kluwer Academic Publishers, Dordrecht, Boston, London*, 2002, and the paper V. Kokilashvili and A. Meskhi, On some weighted inequalities for fractional integrals on nonhomogeneous spaces (to appear).

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OLEKSANDR METELICHENKO (Odessa University & UCL)
Isodiametric problem in the groups with dilations

We show that the isodiametric inequality (a metric ball maximizes the volume for the given diameter) fails in the Heisenberg group equipped with the Carnot-Carathéodory (geodesic) metric and give estimates for the ratio of the volume of the ball and the maximal volume of a set of the same diameter. We also give some examples of the similar nature in the additive group $\mathbb{R}^n$ with non-isotropic dilations.

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Michele Miranda (University of Lecce)

Special functions of bounded variation in metric measure spaces

In this lecture we present the theory of special functions of bounded variation (characterised by a total variation measure which is the sum of a "volume" energy and of a "surface" energy) to metric measure spaces endowed with a Poincare' inequality. In this framework, which includes all Carnot–Caratheodory spaces, we use and improve previous results in [1], [2] and [5] to show the basic compactness theorem of special functions of bounded variation. In a particular class of "isotropic" spaces, which includes all Carnot groups of step 2 and spaces induced by a continuous and strong A-infinity weight, we are able to show the lower semicontinuity of a Mumford-Shah type functional, extending previous results by Song and Yang [6], Citti, Manfredini and Sarti [4] in the Heisenberg group and by Franchi and Baldi [3] in weighted spaces.

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Gerard Misiołek (University of Notre Dame)

Exponential map on diffeomorphism groups

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Daniele Morbidelli (Universitá di Bologna)

Conformal inversions for the Grushin metric and applications to non-linear PDEs

We use a suitable inversion to study entire positive solutions of a Yamabe type equation for the Grushin operator. We discuss how conformal invariance is related to a kind of hyperbolic symmetry and we give some uniqueness results (up to conformal transformations).

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Jani Onninen (University of Jyväskylä)

Mappings of finite distortion

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Carlos Perez (Universidad de Sevilla)

Self-improving properties of generalized Poincaré type inequalities through rearrangements

In this talk we will discuss a recent joint work with Andrei Lerner. We prove, within the context of spaces of homogeneous type, L^p and exponential type self-improving properties for measurable functions satisfying a very weak form of the Poincaré type. In particular we obtain an improvement of some known recent results. Our method is based on rearrangements and avoids completely the "good-λ" inequality technique and any kind of representation formulae.

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Andrei Pokrovskii (NAS Ukraine)

Removable singularities for solutions to quasilinear elliptic equations

The first part of the talk deals with removable singularities for p-harmonic functions in the class of all continuously differentiable functions with Hölder gradient. In terms of the Hausdorff measure we give a sufficient condition for the removability which is necessary for small Hölder exponents. The main result of the second part of the talk is the similar theorem for solutions to the minimal surface equation.

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Kai Rajala (University of Jyväskylä)

Spatial quasiregular mappings with distortion close to one

We discuss a quantitative proof for the following result of Martio, Rickman and Väisälä: for each dimension $n > 2$, there exists a number $K(n) > 1$, so that all non-constant n-dimensional K-quasiregular mappings with $K < K(n)$ are local homeomorphisms.

◊ ◊ ◊

Francesco Serra Cassano (University of Trento)

Intrisic graphs and minimal surfaces in Heisenberg group

I will speak of a new characterization of a notion of intrinsic regular graphs in the Heisenberg group equipped with its Carnot-Caratheodory metric. This notion was originally introduced for the study of the rectifiability in this setting. Moreover this characterization will be applied to the study of non parametric minimal surfaces in intrinsic sense.

◊ ◊ ◊

Andrzej Stós (Wrocław Technology University, CNRS-MAPMO Orleans)
Poisson kernel in hyperbolic spaces
(joint work with T. Byczkowski and P. Graczyk)

Our starting point is the formula for the Fourier transform of the poisson kernel of half-space in the (real) hyperbolic space. We investigate asymptotical behaviour of the kernel itself. We give a description of the general formula for the kernel in even dimensions. In some particular cases we give closed expressions.

♢ ♢ ♢

Karl-Theodor Sturm (University of Bonn)
Mass transportation and nonlinear diffusions (3 lectures)

We study nonlinear diffusions on a Riemannian manifold M as gradient flows with respect to suitable free energy functionals S on the space $P(M)$ of probability measures over M. The space $P(M)$, equipped with the $L2$-Wasserstein metric, will be regarded as an infinite dimensional manifold. Geodesics in $P(M)$ are the paths of optimal mass transportation. Uniform convexity properties of S on $P(M)$ will imply contraction and equilibration properties for the nonlinear diffusions on M, e.g. Talagrand inequality, logarithmic Sobolev inequality, exponential convergence to equilibrium. Of particular interest is the relative entropy with respect to the Riemannian volume. Its gradient flow turns out to be the heat equation. One of the main results is that a number K is a lower bound for the Hessian of the relative entropy on $P(M)$ if and only if it is a lower bound for the Ricci curvature on M. This gives rise to a new definition of lower bounds for the Ricci curvature for general metric measure spaces. Also curvature-dimension conditions can be formulated. These conditions will be discussed in detail.

♢ ♢ ♢

Marc Troyanov (Ecole Polytechnique Federale de Lausanne)
Royden algebras on MMD spaces

The p-Royden algebra of a Riemannian manifold is the Banach algebra of continuous bounded functions vanishing at infinity and having a p-integrable gradient. I will explain in this talk how this algebra can be defined on a metric measure space and I will show how a metric space is characterized up to bilipshitz equivalence by its p-Royden algebra under suitable hypothesis.

♢ ♢ ♢

Heli Tuominen (University of Jyväskylä)
Orlicz-Sobolev spaces on metric measure spaces

In the talk, I will define an Orlicz-Sobolev space $N^{1,\psi}X$ on metric measure space X using weak upper gradients and discuss the basic properties of the space $N^{1,\psi}X$.

There are several ways to generalize the classical theory of the Sobolev spaces to the setting of metric measure spaces. In spaces $N^{1,p}(X)$, defined by Shanmugalingam in [1], an upper gradient is a substitute of the classical weak gradient. Recall that a Borel measurable function $g \geq 0$ is *an upper gradient* of a real-valued function u *in* X if

$$|u(x) - u(y)| \leq \int_\gamma g\, ds \tag{1}$$

for each pair of points x, y in X, and all rectifiable curves γ joining x and y in X. The space $N^{1,p}(X)$ consists of functions $u \in L^p(X)$ having an p-weak upper gradient $g \in L^p(X)$, that is, the inequality 1 holds for u and g except for a family Γ of compact, rectifiable curves with zero p-modulus.

Orlicz-Sobolev spaces in $\mathbb{R}^n$, $W^{1,\Psi}(\mathbb{R}^n)$, consist of $L^\Psi(\mathbb{R}^n)$-functions with the weak partial derivatives in the Orlicz space $L^\Psi(\mathbb{R}^n)$, and are used, for example, in the theory of partial differential equations and in variational problems.

Our definition of an Orlicz-Sobolev space on a metric measure space X is based on upper gradients in a similar way as the definition of the space $N^{1,p}(X)$. For these spaces, we need tools such as modulus, capacity, and Poincaré-type inequalities connected to Young function Ψ. For a Young function Ψ, the space $N^{1,\psi}X$ consists of functions $u \in L^\Psi(X)$ having a Ψ-weak upper gradient $g \in L^\Psi(X)$, that is, the inequality 1 holds for u and g except for a family Γ of compact, rectifiable curves with zero Ψ-modulus.

The space $N^{1,\psi}X$ is always a Banach space and its functions are absolutely continuous on almost all curves. If the Young function Ψ is doubling, then the space $N^{1,\psi}X$ has more important properties, for example density of bounded functions and existence of a minimal Ψ-weak upper gradient. If the underlying metric space X supports a Ψ-Poincaré inequality, then Lipschitz functions are dense both in norm and "in the Lusin sense". The talk is based on my PhD thesis [2].

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[1] Shanmugalingam, N.: Newtonian spaces: An extension of Sobolev spaces to metric measure spaces. - Rev. Mat. Iberoamericana, 16(2000), 243–279.

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♢ ♢ ♢

Ignacio Uriarte-Tuero (Universidad Autonoma de Madrid)

On Marcinkiewicz integrals and harmonic measure

Jones and Makarov gave sharp density estimates for harmonic measure using a modified version of Marcinkiewicz integrals called $\tilde{I}_0$. It was also used by Jones and Smirnov to substantially advance in the Sobolev and quasiconformal removability problems. We generalize $\tilde{I}_0$ to make it account for different densities of sets over which to integrate, in particular giving a different proof than Jones' and Makarov's of its key properties. This should have applications to the aforementioned Sobolev and quasiconformal removability problems.

♢ ♢ ♢

Dachun Yang (Beijing Normal University)

Besov and Triebel-Lizorkin spaces on spaces of homogeneous type

Let $(X, \rho, \mu)_{d,\theta}$ be a space of homogeneous type which includes metric measure spaces and some fractals, namely, X is a set, ρ is a quasi-metric on X satisfying that there are constants $\theta \in (0, 1]$ and $C_0 > 0$ such that for all $x,\ x',\ y \in X$,

$$|\rho(x, y) - \rho(x', y)| \le C_0 \rho(x, x')^\theta [\rho(x, y) + \rho(x', y)]^{1-\theta},$$

and μ is a nonnegative Borel regular measure on X satisfying that for some $d > 0$ and all $x \in X$,

$$\mu(\{y \in X :\ \rho(x, y) < r\}) \sim r^d.$$

We introduce the fractional integrals and derivatives, and prove that $B^s_{pq}(X)$ and $F^s_{pq}(X)$ have the lifting properties for $|s| < \theta$. We establish the frame decompositions of $B^s_{pq}(X)$ and $F^s_{pq}(X)$. Applying these frame characterizations, we obtain the estimates of entropy numbers of compact embeddings between $B^s_{pq}(X)$ or $F^s_{pq}(X)$ when $\mu(X) < \infty$. Parts of these results are new even when $(X,\ \rho,\ \mu)_{d,\theta}$ is just the n-dimensional Euclidean space, or a compact d-set, Γ, in $\mathbb{R}^n$, which includes various kinds of fractals. We also introduce new spaces of Lipschitz type and show that these spaces are just the Besov spaces or Triebel-Lizorkin spaces when the smooth index is less than θ. Moreover, we clarify the relationships amongst these Lipschitz-type spaces, Hajłasz-Sobolev spaces, Korevaar-Schoen-Sobolev spaces, Newtonian Sobolev space and the Cheeger-Sobolev spaces on metric-measure spaces, showing that they are the same space with equivalence of norms. Furthermore, we prove a Sobolev embedding theorem, namely that the Lipschitz-type spaces with large orders of smoothness can be embedded in Lipschitz spaces. For metric-measure spaces with heat kernel, we establish a Hardy-Littlewood-Sobolev theorem, and hence prove that Lipschitz-type spaces with small orders of smoothness can be embedded in Lebesgue spaces.

♢ ♢ ♢

Anna Zatorska-Goldstein (Warsaw University)
Very weak solutions of nonlinear subelliptic equations

Consider an equation

$$X^*A(x,u,Xu)+B(x,u,Xu)=0, \tag{2}$$

with x in a bounded region $\Omega\subset\mathbb{R}^n$ and $X=(X_1,\ldots,X_k)$ – a family of smooth vector fields in $\mathbb{R}^n$ defined on a neighborhood of Ω, satisfying Hörmander condition, and $X^*=(X_1^*,\ldots,X_k^*)$ – a family of operators adjoint to X_i in L^2. In classical situation, i.e. $X=\nabla=\left(\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2},\ldots,\frac{\partial}{\partial x_n}\right)$, we obtain familiar A-harmonic equation. A and B are both Carathéodory functions and satisfy standard growth conditions: $A(\cdot,\cdot,\xi)\approx|\xi|^{p-2}\xi$.

I prove a generalization of a theorem of Iwaniec, Sbordone and Lewis on higher integrability of the so–called very weak solutions of the A-harmonic equation. We say that u is a weak solution of the equation (2) if for every $\phi\in C_0^\infty(\Omega)$

$$\int_\Omega A(x,u,Xu)\cdot X\phi(x)dx+\int_\Omega B(x,u,Xu)\phi(x)dx=0 \tag{3}$$

and the function u belongs to the Sobolev space $W^{1,p}$. The last assumption comes from the variational formulation of the problem. If function A satisfies standard growth conditions ($|A(x,s,\xi)|\approx|\xi|^{p-1}$), then the L^p-integrability condition on u and its derivatives allows us to take as a test function an appropriate power of u multiplied by a smooth cut-off function (or another local construction of a test function based on u). In such a way we can obtain better properties of solutions (e.g. Hölder continuity).

On the other hand, the integrals in (3) are well defined for $|Xu|^{p-1}\in L^1$. It is natural to ask if one can work with weaker regularity assumptions for weak solutions. In classical situation, with $X=\nabla$, T. Iwaniec and C. Sbordone ([4], [6]) proved that if u satisfies (3), but its derivatives are à priori integrable with some exponent strictly lower than the natural exponent p, then in fact they are integrable with the exponent p and therefore u belongs to Sobolev space $W^{1,p}$.

Definition. A function u is called a very weak solution to (2) if u satisfies (3) but it belongs to the Sobolev space $W^{1,r}$, where the exponent r is strictly lower than the natural exponent p.

Theorem 1. *There exists $\delta>0$, such that if u is a very weak solution to 2, $u\in W^{1,p-\delta}_{X,loc}(\Omega)$, then $u\in W^{1,p+\tilde{d}}_{X,loc}(\Omega)$ for some $\tilde{d}>0$, and hence it is a classical weak solution to 2.*

The idea of Sbordone and Iwaniec was to use the Hodge decomposition in construction of a test function. Later J. Lewis [7] published another proof, where a construction of a test function was based on a Hardy–Littlewood maximal function. We follow the idea of Lewis. We follow also the way of Iwaniec, Sbordone and Lewis to show the higher integrability of the gradient by application of Gehring Lemma. We use it in a version formulated by Giaquinta [2], introducing changes that are necessary to adapt it to arbitrary metric spaces with a doubling measure. Up to the author's best knowledge, this lemma is not available in mathematical literature in such generality. We need a metric version of the theorem because of the change of metric in $\mathbb{R}^n$, as a result of working with a differential operator X instead of a classical gradient. The idea of proof is analogous to that in euclidean case, however, we cannot use tools which are strictly connected to the euclidean geometry: decomposition into diadic cubes, classical Calderon–Zygmund Theorem etc. Therefore in general metric spaces one has to use different arguments which replace the classical Calderon–Zygmund decomposition. The metric version of Gehring Lemma might be of special interest, as it is a vital tool in many applications, for example in the theory of quasi–regular mappings and nonlinear p.d.e.'s (see [1], [3], [5], [8]).

As an application of the theorem we have the following compactness theorem:

Theorem 2. *Let F be a compact subset of Ω and δ be a constant defined by Theorem 1. Let $\{u_i\}_{i\in\mathbb{N}}$ be a family of very weak solutions to 2 such that $u_i \in W^{1,r}_X(\Omega)$ for some $p-\delta < r < p$. If the family is bounded in $W^{1,r}_X(\Omega)$, then it is compact in $W^{1,p}_X(F)$.*

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[1] Gehring, F.W., The L^p-integrability of the partial derivatives of a quasiconformal mapping. Acta Math. **130** (1973), 265–277.

[2] Giaquinta, M., Multiple integrals in the calculus of variations and nonlinear elliptic systems. Annals of Mathematics Studies, 105. Princeton University Press, Princeton, NJ, 1983.

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[4] Iwaniec, T., p–Harmonic tensors and quasilinear mappings. Annals of Math. **136** (1992), 589–624.

[5] Iwaniec, T., The Gehring Lemma., Duren, P. (ed.) et al., Quasiconformal mappings and analysis. A collection of papers honoring F.W. Gehring. Proceedings of the international symposium, Ann Arbor, MI, USA, August 1995. New York, NY, Springer, 1998, 181–204.

[6] Iwaniec, T., Sbordone, C., Weak minima of variational integrals. J. reine angew. Math. **454** (1994), 143–161.

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[8] Meyers, N.G, Elcrat A., Some results on regularity for solutions of nonlinear elliptic systems and quasi-regular functions. Duke Math. J. **42** (1975), 121–136.

Programme of the Conference

Thursday, July 15

16.00–22.00 ARRIVAL

Friday, July 16

Morning Chairperson: Bogdan Bojarski

9.15–9.30 Opening: Bogdan Bojarski, Olli Martio

9.40–10:30 Juha Kinnunen (University of Oulu)
Pointwise behaviour of Sobolev functions on metric spaces

Tee & coffee

11.00–11.50 Carlos Perez (Universidad de Sevilla)
Self-improving properties of generalized Poincaré type inequalities through rearrangements

12.00–12.50 Zoltan M. Balogh (University of Bern)
Stepanov's differentiability theorem in metric measure spaces

Afternoon Chairperson: Juha Kinnunen

15.00–15.50 Tadeusz Iwaniec (Syracuse University)
Weekly differentiable mappings between manifolds

Tee & coffee

16.20–16.50 Alexander Egorov (Sobolev Institute of Mathematics)
Stability of classes of mappings and quasiconvex functions

17.00–17.30 Maria Karmanova (Novosibirsk State University)
Area & coarea formulas on rectifiable sets in metric spaces and Sobolev mappings

Saturday, July 17

Morning Chairperson: Tadeusz Iwaniec

9.30–10.20 Nicola Garofalo (Purdue University)
Minimal surfaces in Carnot groups, I

Tee & coffee

10.50–11.40 Stephen Keith (Australian National University)
Differentiable structures for metric measure spaces, I

11.50–12.40 Kazuhiro Kuwae (Yokohama City University)
On Calabi's strong maximum principle via local Dirichlet forms

Afternoon Chairperson: Steve Buckley

15.00–15.50 Dachun Yang (Beijing Normal University)
Besov and Triebel-Lizorkin spaces on spaces of homogeneous type

Tee & coffee

16.20–16.50 Valentino Magnani (Pisa University)
Blow-up and transverse mass of submanifolds in the Heisenberg group

17.00–17.30 Gerard Misiołek (University of Notre Dame)
Exponential map on diffeomorphism groups

19.00–21.00 CONFERENCE DINNER

Sunday, July 18

Morning Chairperson: Olli Martio

9.30–10.20 Nicola Garofalo (Purdue University)
Minimal surfaces in Carnot groups, II

Tee & coffee

10.50–11.40 Stephen Keith (Australian National University)
Differentiable structures for metric measure spaces, II

Afternoon

13.00–19.00 EXCURSION to Poznań

Monday, July 19

Morning	Chairperson: Zoltan Balogh
9.30–10.20	Nicola Garofalo (Purdue University) **Minimal surfaces in Carnot groups, III**
	Tee & coffee
10.50–11.40	Stephen Keith (Australian National University) **Differentiable structures for metric measure spaces, III**
11.50–12.40	Steve Buckley (NUI Maynooth) **Sphericalization and flattening**
Afternoon	Chairperson: Jana Björn
15.00–15.50	Francesco Serra Cassano (University of Trento) **Intrisic graphs and minimal surfaces in Heisenberg group**
	Tee & coffee
16.20–16.50	Jani Onninen (University of Jyväskylä) **Mappings of finite distortion**
17.00–17.30	Andrei Pokrovskii (NAS Ukraine) **Removable singularities for solutions to quasilinear elliptic equations**
17.40–18.10	Andrzej Stós (Wrocław Technology University, CNRS-MAPMO Orleans) **Poisson kernel in hyperbolic spaces** (joint work with T. Byczkowski and P. Graczyk)

Tuesday, July 20

Morning	Chairperson: Francesco Serra Cassano
9.30–10.20	Pekka Koskela (University of Jyväskylä) **Quasiconformal mappings on metric spaces, I**
	Tee & coffee
10.50–11.40	Karl-Theodor Sturm (University of Bonn) **Mass transportation and nonlinear diffusions, I**
11.50–12.40	Jan Malý (Charles University in Prague) **Fine topologies on metric spaces**
Afternoon	*Short communications, Session 1*
	Chairperson: Daniele Morbidelli
15.00–15.20	Luigi D'Onofrio (Universitá degli Studi di Napoli) **The p-harmonic transform beyond its natural domain of definition, interpolation and continuity** (joint work with Tadeusz Iwaniec)
15.25–15.45	Peter Hästö (University of Helsinki) **Arc-wise hyperbolic metrics**
15.50–16.10	Aigerim Kalybay (Kazakh National University) **On properties of multiweighted spaces and their applications to singular differential equations**
	Tee & coffee
	Short communications, Session 2
	Chairperson: Grzegorz Łysik
16.40–17.00	Alexander Meskhi (Scuola Normale Superiore di Pisa) **Fractional integrals on nonhomogeneous spaces**
17.10–17.30	Oleksandr Metelichenko (Odessa University & UCL) **Isodiametric problem in the groups with dilations**
17.40–18.00	Michele Miranda (University of Lecce) **Special functions of bounded variation in metric measure spaces**

Wednesday, July 21

Morning	Chairperson: Marc Troyanov
9.30–10:20	Pekka Koskela (University of Jyväskylä) **Quasiconformal mappings on metric spaces, II**
	Tee & coffee
10.50–11.40	Karl-Theodor Sturm (University of Bonn) **Mass transportation and nonlinear diffusions, II**
11.50–12.40	Jana Björn (Linköping University) **Boundary regularity for p-harmonic functions on metric spaces**
Afternoon	*Short communications, Session 3*
	Chairperson: Carlos Perez
15.00–15.20	Anna Zatorska-Goldstein (Warsaw University) **Very weak solutions of nonlinear subelliptic equations**
15.25–15.45	Kai Rajala (University of Jyväskylä) **Spatial quasiregular mappings with distortion close to one**
15.50–16.10	Heli Tuominen (University of Jyväskylä) **Orlicz-Sobolev spaces on metric measure spaces**
	Tee & coffee
	Short communications, Session 4
	Chairperson: Piotr Hajlasz
16.40–17.00	Ignacio Uriarte-Tuero (Universidad Autonoma de Madrid) **On Marcinkiewicz integrals and harmonic measure**
17.10–17.30	Mikhail Korobkov (Sobolev Institute of Mathematics) **Stability of a class of nonsmooth Lipschitz functions**
17.40–18.00	Uta Freiberg (University Jena) **Analysis on fractals**

Thursday, July 22

Morning	Chairperson: Jan Malý
9.30–10:20	Pekka Koskela (University of Jyväskylä) **Quasiconformal mappings on metric spaces, III**
	Tee & coffee
10.50–11.40	Karl-Theodor Sturm (University of Bonn) **Mass transportation and nonlinear diffusions, III**
11.50–12.40	Daniele Morbidelli (Universitá di Bologna) **Conformal inversions for the Grushin metric and applications to nonlinear PDEs**
Afternoon	Chairperson: Gerard Misiolek
15.00–15.50	Marc Troyanov (Ecole Polytechnique Federale de Lausanne) **Royden algebras on MMD spaces**
	Tee & coffee
	Short communications, Session 5
16.20–16.40	Paweł Bechler (Polish Academy of Sciences) **Greedy wavelet projections and the spaces $BV(R^d)$**
16.50–17.10	Iya Ivanishko (Belarussian State University) **Sobolev type spaces on spaces of homogeneous type: different descriptions and embedding theorems**
17.20–17.40	Daria Isangulova (Sobolev Inst. Math. SB RAS) **Differentiability of mappings between Carnot - Carathéodory spaces in BV-topology**
17.40	CLOSURE

Friday, July 23

DEPARTURE

List of participants and their e-mail addresses

Adamowicz Tomasz: ta189358@students.mimuw.edu.pl
Aliashvili David: aliashvili@posta.ge
Balogh Zoltan: zoltan.balogh@math-stat.unibe.ch
Bechler Pawel: pbechler@impan.gov.pl
Björn Jana: jabjo@mai.liu.se
Bojarski Bogdan: B.Bojarski@impan.gov.pl
Borello Francesco: fborrello@dmi.unict.it
Buckley Steve: sbuckley@maths.may.ie
Capitanelli Rafaella: raffaela.capitanelli@uniroma1.it
D'Onofrio Luigi: ldonofri@unina.it
Egorov Alexander: yegorov@math.nsc.ru
Freiberg Uta: uta@mathematik.uni-jena.de
Garcia Rafael Espinola: espinola@us.es
Garofalo Nicola: garofalo@math.purdue.edu
Giannetti Flavia: giannett@unina.it
Goldstein Pawel: goldie@mimuw.edu.pl
Hajłasz Piotr: hajlasz@duch.mimuw.edu.pl
Hästo Peter: peter.hasto@helsinki.fi
Heikkinen Toni: toheikki@maths.jyu.fi
Hencl Stanislav: hencl@karlin.mff.cuni.cz
Isangulova Daria: dasha@math.nsc.ru
Ivanishko Iya: ijka@tut.by
Iwaniec Tadeusz: tiwaniec@mailbox.syr.edu
Kalybay Aigerim: aigerim_k@academset.kz
Karmanova Maria: maryka@online.nsk.su
Keith Stephen: stephen.keith@anu.edu.au
Kinnunen Juha: juha.kinnunen@oulu.fi
Korobkov Mikhail: korob@math.nsc.ru
Korte Rikka: rkorte@math.hut.fi
Koskela Pekka: pkoskela@math.jyu.fi
Kuwae Kazuhiro: kuwae@yokohama-cu.ac.jp
Lukkari Teemu: Teemu.Lukkari@hut.fi
Łysik Grzegorz: lysik@impan.gov.pl
Maasalo Outi Elina: outi.elina.maasalo@hut.fi
Magnani Valentino: magnani@dm.unipi.it
Malý Jan: maly@karlin.mff.cuni.cz
Marola Niko: nmarola@math.hut.fi
Martio Olli: olli.martio@helsinki.fi
Meskhi Alexander: a.meskhi@sns.it
Metelichenko Oleksandr: metel@math.ucl.ac.uk
Michalik Slawomir: slawek@impan.gov.pl
Miranda Michele: michele.miranda@unile.it
Misiołek Gerard: gmisiole@nd.edu
Morbidelli Daniele: morbidel@dm.unibo.it

Onninen Jani: jaonnine@maths.jyu.fi
Pere Mikko: mikko.pere@helsinki.fi
Perez Carlos: carlosperez@us.es
Pokrovskii Andrei: pokrovsk@imath.kiev.ua
Rajala Kai: kirajala@maths.jyu.fi
Rickly Matthieu: matthieu.rickly@math-stat.unibe.ch
Serra-Cassano Francesco: cassano@science.unitn.it
Stós Andrzej: stos@im.pwr.wroc.pl
Sturm Karl-Theodor: sturm@uni-bonn.de
Troyanov Mark: marc.troyanov@epfl.ch
Tuominen Heli: tuheli@maths.jyu.fi
Turunen Ville: Ville.Turunen@hut.fi
Uriarte-Tuero Ignacio: ignacio.uriarte-tuero@yale.edu
Yang Dachun: dcyang@bnu.edu.cn
Zatorska-Goldstein Anna: azator@mimuw.edu.pl