Deterministic and Stochastic Dynamics, Fractals, Turbulence

IM PAN Warsaw, May 14-18, 2009.

Workshop Program

Lectures in room 403

Thursday, May 14

9.50–10.00 OPENING

10.00–11.00 P. Imkeller **Martingale optimality, BSDE and cross hedging of insurance derivatives**

11.15–12.15 E. Priola **Well-posedness of the transport equation by stochastic perturbation**

12.30–13.30 Yu. Lyubich **Single locus population dynamics**

15.15–16.15 A. Zdunik **Holomorphic endomorphisms on complex projective spaces – ergodic theory**

16.30–17.30 M. Rams **Lyapunov spectrum for rational maps**

17.45–18.45 V. Ryazanov **On the Dirichlet problem for the Beltrami equations**

Friday, May 15

9.30–10.30 A. Żuk **Ihara zeta function for infinite graphs**

11.00–12.00 M. Roginskaya **Orbits of Read-type operators**

12.15–13.15 M. Wojciechowski **Bounded Approximation Property of Sobolev spaces of functions on arbitrary simply connected two dimensional domain**

15.00–16.00 P. Domański **Splitting of short exact sequences and differential equations**

16.20–16.50 B. Bárány **Hausdorff dimension of iterated function system with overlap**

17.00–17.30 O. Dovgoshey **Tangent spaces to metric spaces**

17.40–18.10 V. Potyemkin **On the noncloseness of the David classes**

Saturday, May 16

9.45–10.30 P. Góra **Random compositions of homeomorphisms of $[0,1]$ and market assets game**

11.00–12.00 K. Oleszkiewicz **On fake Brownian motions**

12.15–13.15 T. Komorowski **Passive tracer in a slowly decorrelating random flow with a large mean**

Monday, May 18

9.30–10.30 P. Mucha **The maximal regularity for the Stokes system in the critical function space**

11.00–12.00 G. Łukaszewicz **Shear flows and their attractors**

12.15–13.15 M. Arnold **How big can be the singularity of the Navier-Stokes system?**

15.00–15.45 W. Zajączkowski **Global regular solutions to the Navier-Stokes equations**

15.50–16.35 V. A. Solonnikov **On some problems of mhd in multi-connected domains**

16.55–17.40 J. Rencławowicz **Solutions to the Poisson equation in L_p weighted spaces for $p \geq 2$**

17.45–18.30 G. Łysik **Formal and analytic solutions of the Burgers equation**

Abstracts

Maxim Arnold (McMaster University)

How big can be the singularity of the Navier-Stokes system?

Abstract. Cafarelli, Kohn and Nirenberg in their well-known joint work obtain an upper bound on the Hausdorf dimension of the singular set of three-dimensional Navier-Stokes System. I would like to present some ideas of construction Hausdorf-like dimension of the wave-front set of 3d-NSS which is bounded from below.

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Balázs Bárány (ESR, IM PAN)

Hausdorff dimension of iterated function system with overlap

Abstract. We investigate the properties of the Hausdorff dimension of the attractor of the iterated function system (IFS) $ax, bx, bx + 1$. We give a formula for Lebesgue almost every (a, b), $a < b$, and a dense subset, where the dimension drops.

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Zdzisław Brzeźniak (University of York)

On the stochastic Strichartz estimates and the stochastic nonlinear Schrödinger equation on a two dimensional manifold equation

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Paweł Domański (Adam Mickiewicz University, IM PAN)

Splitting of short exact sequences and differential equations

Abstract. This is an expository talk on applications to partial differential equations of the results on the splitting of short exact sequences

$$0 \to E \to F \to G \to 0,$$

i.e., theorems describing when q has a linear continuous right inverse.
We present several splitting results and illustrate them by applying to various problems on differential equations. We show the classical Vogt's $(DN) - (\Omega)$ splitting theorem and apply it to the division problem for differential forms. We show graded splitting theorem and apply it to Schwartz problem on the existence of a solution operator for linear PDE on the space of smooth functions. We give a lifting criterion for operators on the the co-called PLS-spaces and apply it to the problem of analytic dependence of the solutions of PDE on distributions. Finally, we present tame splitting theorem of Vogt and Poppenberg and apply it to solvability of convolution equations in analytic functions.

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Oleksiy Dovgoshey (NASU, Donetsk)

Tangent spaces to metric spaces

Abstract. The recent achievements in the metric space theory are closely related to some generalizations of differentiation. See, for instance, the concept of weak gradient or Cheeger's notion of differentiability for Rademacher's theorem in certain metric measure spaces. A tangent space to an arbitrary metric space X at a point $p \in X$ was defined in as a factor space of some family of sequences of points $x_n \in X$ which converge to p. This approach makes possible to define a metric space valued derivative of functions $f : X \to Y$; X and Y are metric spaces, as mappings between tangent spaces to X and, respectively, to Y. In the lecture will be discussed some examples of tangent spaces to metric spaces and, moreover, will be described the metric spaces which have bounded or proper or compact tangent spaces and metric spaces having ultrametric tangent spaces.

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Peter Imkeller (Humbolt University)

Martingale optimality, BSDE and cross hedging of insurance derivatives
(Joint work with S. Ankirchner and G. Reis)

Abstract. A financial market model is considered on which agents (e.g. insurers) are subject to an exogenous financial risk, which they trade by issuing a risk bond. Typical risk sources are climate or weather. Buyers of the bond are able to invest in a market asset correlated with the exogenous risk. We investigate their utility maximization problem, and calculate bond prices using utility indifference. This hedging concept is interpreted by means of martingale optimality, and solved with BSDE and Malliavin's calculus tools. Prices are seen to decrease as a result of dynamic hedging. The price increments are interpreted in terms of diversification pressure.

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Tomasz Komorowski (MCSU, IM PAN)

Passive tracer in a slowly decorrelating random flow with a large mean
(Joint work with L. Ryzhik (U. of Chicago) and E. Nieznaj (Politechnika Lubelska))

Abstract. We consider the movement of a particle advected by a random flow with a large mean that dominates the fluctuations of the flow. When the field is sufficiently strongly mixing Kesten and Papanicolaou have proved that scaled motion of the advected particle approximates a Brownian motion with the covariance matrix given by the Kubo formula. In our case the flow does not decorrelate rapidly but is locally self similar and Gaussian. We identify the Hurst exponent for the particle trajectory and prove that the limit of one dimensional marginals for the properly scaled trajectory process is a fractional Brownian motion.

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Grzegorz Łukaszewicz (Warsaw University)

Shear flows and their attractors (Joint work with M. Boukrouche)

Abstract. We consider the problem of the existence and finite dimensionality of attractors for some classes of two-dimensional turbulent boundary driven flows which naturally appear in lubrication theory. The flows admit mixed, non-standard boundary conditions and time-dependent driving forces. We are interested in the dependence of the dimension of the attractors on the geometry of the flow domain and on the boundary conditions.

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Grzegorz Łysik (IM PAN)

Formal and analytic solutions of the Burgers equation

Abstract. In the first part of the lecture we shall study formal power series solutions to the initial value problem for the Burgers type equation $\partial_t u - \Delta u = X\big(f(u)\big)$ with polynomial nonlinearity f and a vector field X, and we prove that they belong to the formal Gevrey class G^2. Next we give necessary and sufficient conditions for the formal power series solutions to the initial value problem of the Burgers equation $\partial_t u - \partial_x^2 u = \partial_x\big(u^2\big)$ to be convergent or Borel summable.

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Yuri Lyubich (Technion, Haifa)

Single locus population dynamics

Abstract. Some classical models of the population genetics will be considered, and a survey of results concerning the evolution to equilibrium will be presented in the talk. Reference: Yu. Lyubich, Mathematical Structures in the Population Genetics, Springer, 1992.

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Piotr Mucha (Warsaw University)

The maximal regularity for the Stokes system in the critical function space (A joint work with Raphael Danchin.)

Abstract. I would like to present some new results for the evolutionary Stokes system in the half space with the no slip boundary condition. The main gaol is to construct the maximal regularity estimate in the Besov space $L_(0,T;\dot{B}^0_{p,1}(R^n_+))$ with time independent constant. The key difficulty is the fact that we are not allowed to apply the standard tools of harmonic analysis, because of properties of the L_1 regularity (in time). The talk will be based on the joint paper with Raphael Danchin [J. Fan. Anal. 256 (2009), 881-927].

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Krzysztof Oleszkiewicz (IM PAN)

On fake Brownian motions

Abstract. We will describe a direct construction of a non-Gaussian continuous martingale having the same one-dimensional distributions as the standard Brownian motion. Similar objects were considered recently by Hamza and Klebaner, and by Albin. The new approach seems more explicit and elementary.

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Vladimir Potyemkin (NASU, Donetsk)

On the noncloseness of the David classes

(Joint work with Vladymir I. Ryazanov)

Abstract. There exist many interesting aspects in generalizing quasiconformal mappings to mappings whose dilatations are not uniformly bounded but are controlled in some other sense. Various classes of the mean quasiconformal mappings on the plane were studied in articles of L. Ahlfors, I.N. Pesin, S.L. Krushkal', R. Kühnau, V.M. Miklyukov, G.D. Suvorov and others. Note that every sense preserving homeomorphism f on the plane with partial derivatives satisfies a Beltrami equation. One of the major achievements in this field was the existence theorem of Guy David (1988) who proved that the Beltrami equation has a homeomorphic $W^{1,1}_{loc}$ solution if the dilatation has an exponential restriction of the measure type. The study of compactness properties for David homeomorphisms was initiated by P. Tukia (1991). We have established that the classes of normalized $W^{1,1}_{loc}$ homeomorphisms with any measure restrictions for the distribution function of the dilatation are not closed in general and therefore not sequentially compact except the case when the class reduces to the usual class of quasiconformal mappings.

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Enrico Priola (Università di Torino)

Well-posedness of the transport equation by stochastic perturbation

(A joint work with F. Flandoli and M. Gubinelli.)

Abstract. We consider the linear transport equation with a globally Hölder continuous and bounded vector field. While this deterministic PDE may not be well-posed, we prove that a multiplicative stochastic perturbation of Brownian type is enough to render the equation well-posed.

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Michał Rams (IM PAN)

Lyapunov spectrum for rational maps

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Joanna Rencławowicz (IM PAN)

Solutions to the Poisson equation in L_p weighted spaces for $p \geq 2$

Abstract. We examine the Poisson equation with boundary condition on cylinder in R^3 in weighted spaces of L_p type, with $p \geq 2$. The weight is given as a distance from a distinguished plane in some positive power. We treat in different way $p = 2$ and $p \geq 3$ cases. The motivation of the problem is the analysis of the inflow-outflow motion described with Navier-Stokes equations.

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Maria Roginskaya (Chalmers University, Göteborg)

Orbits of Read-type operators

(This is a joint work with Sophie Grivaux.)

Abstract. I will explain part of the ideas used by Ch. Read in the construction of operator without non-trivial invariant subsets in ℓ_1, and explain how these ideas can be adjusted to a wider class of Banach spaces, including the Hilbert space. While the resulting operators still have non-trivial invariant subspaces, the structure of their orbits is peculiar.

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Vladymir I. Ryazanov (NASU, Donetsk)

On the Dirichlet problem for the Beltrami equations

(Joint work with Yuri Dybov)

Abstract. Boundary problems for the Beltrami equation with $\mu = 0$ are due to the famous dissertation of Riemann and to the known works of Hilbert (1904, 1924) and Poincaré (1910) and, for the uniformly elliptic case, with $\|\mu\|_\infty < 1$ to Vekua. It's given a series of criteria for the existence of regular solutions of the Dirichlet problem in terms of majorants for the distorsion function; in particular, of the class BMO, bounded mean oscillation by John and Nirenberg, FMO, finite mean oscillation by Ignat'ev-Ryazanov, and of the class $W^{1,2}_{loc}(D)$, Martio-Miklyukov. These advances became possible thanking to the recent development of the theory of the so-called ring Q-homeomorphisms by Ryazano-Srebro-Yakubov.

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Michał Wojciechowski (IM PAN)

Bounded Approximation Property of Sobolev spaces of functions on arbitrary simply connected two dimensional domain

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Wojciech Zajączkowski (IM PAN)

Global regular solutions to the Navier-Stokes equations.

Abstract. We present global regular solutions to the Navier-Stokes equations which are close either to two-dimensional or to axially symmetric solutions at the initial time. The existence of solutions which are close to 2d solutions is proved in cylindrical domains which are nonaxially symmetric in general, but solutions close to the axially symmetric solutions are considered in axially symmetric domains. We consider slip boundary conditions which are crucial for the proofs. This is connected with the fact the problem for vorticity plays a very important role in the proofs and the slip boundary conditions imply appropriate boundary conditions for the vorticity.

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Anna Zdunik (Warsaw University)

Holomorphic endomorphisms on complex projective spaces – ergodic theory

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Andrzej Żuk (Wrocław University)

Ihara zeta function for infinite graphs

Abstract. For a finite graph one can define the Ihara zeta function which is related to the lengths of cycles of the graph. It is closely related to the random walk operator. There are several ways to define it for infinite graphs. We will present several examples in case of graphs related to groups.