CODY Autumn in Warsaw

1 October – 10 December 2010

Low-dimensional dynamics

Warsaw, 15–19 November 2010

Titles and abstracts of talks

Magnus Aspenberg (Jacobs University, Bremen)

Small divisors and binomial recursions

(joint work with Rodrigo Perez)

The background of the problem goes back to a famous paper by C. L. Siegel in 1942 which briefly can be stated as follows. Suppose $f : \mathbb{C} \to \mathbb{C}$ is analytic function and $f(z_0) = z_0$, $f'(z_0) = e^{2\pi i\theta}$ where θ is irrational. We say that θ is Diophantine if, roughly spoken, it is badly approximable by rational numbers, so that for each rational number p/q we have

$$|\theta - p/q| \geq \frac{c}{q^{\mu}},$$

where c, μ are positive parameters. Siegel showed that given f as above with θ Diophantine, then there is a conformal map (called the conjugating function or linearising map) φ mapping a neighbourhood of z_0 to a neighbourhood of 0 such that

$$\varphi \circ f(z) = \lambda\varphi(z), \tag{1}$$

where we have put $\lambda = e^{2\pi i\theta}$. In other words, f looks like a conformally distorted rigid rotation around z_0 . The largest (simply connected) domain S where the conjugation (1) holds is nowadays called a *Siegel disk*.

To show the existence of φ one has to prove that the formal power series satisfying (1) converges. The terms in the series for φ are defined recursively, and when θ is irrational, they may blow up due to so called "small divisors" in the recursion formula, thereby threatening the convergence of the series. Siegel surmounted this problem by first noting that each term is a sum of sub-terms, and then showing by an ingenious argument (given that θ is Diophantine) that this sum is majorised by the largest sub-term, which grows exponentially. However, series of this kind may still converge even if this largest sub-term grows super-exponentially. But for this to happen it is necessary to detect large cancellations between the sub-terms. The result which I present is focused on a simplified recursion formula from the setting above and is tailored for tackling such problems. The result also has a striking connection to the Catalan numbers.

Krzysztof Barański (University of Warsaw)

Omega-limit sets for the Stein-Ulam Spiral map

(joint work with Michał Misiurewicz)

In the late 1950's, using computers in the Los Alamos National Laboratory, Stanisław Ulam and Paul Stein performed a comprehensive research on a class of quadratic maps of the 2-dimensional simplex Δ to itself. Those maps arise in the theory of population genetics. One of them has the behavior much different than the 96 other ones. We call it the Stein-Ulam Spiral map. In 1972, S. Vallander asked whether the ω-limit set of any interior point of Δ, except its center, is equal to the boundary of Δ. We prove that this is the case for the points from a residual subset of Δ. On the other hand, we show that for any closed invariant subset E of the boundary of Δ intersecting all three sides of Δ, the set of points having E as the ω-limit set is relatively large.

Athanasios Batakis (Université d'Orléans)

On intermittent random mass transport models in fractal domains

Michael Benedicks (KTH Royal Institute of Technology)

Attractors for invertible and non-invertible dissipative maps

There has been a lot of interest of the theory of attractors of two-dimensional non uniformly hyperbolic maps, in particular of Hénon maps and similar dynamical systems. In this talk I will survey the existing theory of Hénon maps: existence of SRB-measures with positive Lyupunov exponent and statistical properties and also discuss whether it carries over to a non-invertible setting.

Andrzej Biś (University of Łódź)

Partial variational principle for groups of homeomorphisms

We apply the general theory of Carathéodory dimension characteristic, introduced by Pesin, to a finitely generated group G of homeomorphisms of

a closed Riemannian manifold M and define its topological entropy $h_{top}(G)$ as a Caratheodory dimension characteristic. The Brin-Katok local measure entropy introduced for a single map $f : X \to X$ is generalized to a finitely generated group of homeomorphisms. Moreover, we show the relation between local measure entropy of G and its topological entropy.

Paul Blanchard (Boston University)

Checkerboard Julia sets

Sylvain Bonnot (University of Toronto)

Thurston maps and Galois actions

The theory of Thurston mappings (post-critically finite branched covering maps of the 2-sphere) lies at the intersection of many domains: algebraic topology, holomorphic dynamics, Teichmüller theory. In this talk, we show how Galois theory for covering maps can be used towards a classification of such mappings, by providing some useful representations of fundamental groups into Galois groups.

Henk Bruin (University of Surrey)

Thermodynamics of piecewise linear Fibonacci maps

(joint work with Mike Todd)

The unimodal Fibonacci interval map is known to be Lebesgue dissipative whenever the critical order is sufficiently large. In order to study the thermodynamic properties of such maps, we introduce a countable piecewise linear model which induces to a countably piecewise Markov map without the "Big Image property". It turns out that the pressure function and equilibrium measures of such systems can still be investigated in great detail, giving more insight on how dissipativity is reflected in the properties of the pressure function.

Davoud Cheraghi (University of Warwick)

Typical orbits of quadratic polynomials with a neutral fixed point: Brjuno type

We will be considering quadratic polynomials with a Siegel disk, whose rotation number at the fixed point has "high returns". We will describe the behaviour of the orbits of the points in the Julia set with respect to the Lebesgue measure.

Trevor Clark (Stony Brook University)

Regular or stochastic dynamics in families of unimodal maps

About fifteen years ago, Palis conjectured that typical dynamical systems should possess good statistical properties. Through the work of Avila, Lyubich, de Melo and Moreira, this has been proven for unimodal maps with a non-degenerate critical point. I will show how to remove the condition on the critical point in analytic families of unimodal maps; along the way proving that that the hybrid classes in the space of unimodal maps yield a lamination near all but countably many maps in the family. The essential difference in the higher degree case is the presence of non-renormalizable maps without decay of geometry.

Kleyber Cunha (Universidade de São Paulo)

Renormalization for some piecewise smooth homeomorphism of the circle

(joint work with Daniel Smania)

In this work, we find sufficient conditions for two piecewise $C^{2+\nu}$ homeomorphism f and g of the circle to be C^1 conjugate. Besides the restrictions on the combinatorics of the maps (we assume that maps have bounded "rotation number"), and necessary conditions on the one-side derivatives of points where f and g are not differentiable, we also assume zero mean nonlinearity for f and g.

The proof is based on the study of Rauzy-Veech renormalization of genus one generalized interval exchange maps with certain restrictions on its combinatorics.

Artem Dudko (University of Toronto)

Ecalle-Voronin invariants and Resurgent functions

(joint work with David Sauzin)

In my talk I will present Ecalle's resurgent approach to study of the dynamics of simple parabolic germs near the origin. Consider a simple parabolic germ $f(z) = z + z^2 + \sum_{n\geq 3} a_n z^n$ at the origin. By definition, Fatou coordinates for such germ are the conformal solutions of the equation $\varphi(f(z)) = \varphi(z)+1$ in two regions near the origin, called the attracting and the repelling Fatou petals. This equation has a formal solution $\widetilde{\varphi}$ in terms of divergent series. The series $\widetilde{\varphi}$ gives an asymptotic expansion of the Fatou coordinates. In my talk I will explain the procedure of Borel-Laplace summation which allows to obtain the Fatou coordinates from the series $\widetilde{\varphi}$. The Borel transform of $\widetilde{\varphi}$ has branched singularities at points $2\pi ik, k \in \mathbb{Z}$. If time allows I will explain a remarkable relation between these singularities and Ecalle-Voronin invariants.

Alexander Fel'shtyn (University of Szczecin)

How to categorify dynamical zeta functions

I will discuss Weil type dynamical zeta functions. These zeta functions count periodic points of dynamical system homologically or in the presence of fundamental group of a manifold and give rise to the Reidemeister torsion.

In the talk a categorification of Weil type dynamical zeta functions is proposed. The first indication of a possibility of such categorification is a formula of Milnor's that relates Weil zeta function of a monodromy map to the Alexander polynomial of a algebraic knot and Ozsváth-Szabo categorifications of the Alexander polynomial and Turaev torsion function.

I will show that the Weil zeta function of a symplectomorphism of surface is a graded Euler characteristic in periodic Floer homology or a graded Euler characteristic of mapping tori in sutured Floer homology.

It is interesting to generalise this result to higher dimensions.

Pablo Guarino (Instituto Nacional de Matemática Pura e Aplicada)

Rigidity for smooth critical circle maps

In this talk we will discuss rigidity problems for critical circle maps. In a joint work with my advisor Welington de Melo, we recently proved that two C^∞ critical circle maps with the same irrational rotation number and the same odd degree of the critical point are C^1 conjugate. If their common rotation number belongs to a full Lebesgue measure set the conjugacy is $C^{1+\alpha}$ for some $\alpha > 0$ that only depends on the rotation number. The main tool for these results is the study of the dynamics of a suitable defined renormalization operator.

Joanna Jaroszewska (University of Warsaw)

On some aspects of dynamics of one-dimensional random dynamical systems

Our presentation is devoted to the Sharkovsky's-type results for random dynamical systems. We do the survey of what is known so far for random dynamical systems acting on intervals and provide new results on higher order random difference equations as well as for some two dimensional random systems. We also discuss the Klunger's conjecture on a structure of periodic orbits of random subshifts of finite type.

Dominik Kwietniak (Jagiellonian University)

Dynamics of spacing shifts

Spacing shift given by a set of positive integers P is a shift-invariant closed subset of the binary full shift consisting of all sequences ω such that if $\omega_i = \omega_j = 1$ and $i \neq j$, then $|i - j|$ is in P. In other words: all distances between two occurrences of a symbol 1 in ω are in P. Spacing shifts were introduced by Lau and Zame, and were used in [1, 2, 3, 4, 5] as a tool for constructing various (counter)examples.

During the talk we present results on topological entropy of spacing shifts, and apply them to prove some properties of difference sets (sets of the form $\{k-l : k, l \in A,\ k > l\}$, where $A \subset \mathbb{N}$ is a given set). If time permits, we will show how to use spacing shifts to construct some new examples of dynamical systems with curious properties. Among the others, we introduce

a new method of constructing ω chaotic sets and we prove that there are examples of ω-chaotic maps which are not ω^*-chaotic, where both notions are due to Li [6].

References

[1] Lau, Kenneth; Zame, Alan. *On weak mixing of cascades*. Math. Systems Theory **6** (1972/73), 307–311.

[2] Banks, John. *Regular periodic decompositions for topologically transitive maps*. Ergodic Theory Dynam. Systems **17** (1997), no. 3, 505–529

[3] Jiménez López, Víctor; Snoha, L'ubomír. *Stroboscopical property, equicontinuity and weak mixing*. Iteration theory (ECIT '02), 235–244, Grazer Math. Ber., 346, Karl-Franzens-Univ. Graz, Graz, 2004.

[4] Banks, John; Nguyen, T.T.D.; Oprocha, Piotr and Trotta, Belinda. *Dynamics of Spacing Shifts, preprint*. 2010.

[5] Kwietniak, Dominik; Oprocha, Piotr. *On weak mixing, minimality and weak disjointness of all iterates*. `arXiv:1009.2277v1[math.DS]` 2010.

[6] Li, Shi Hai. *Dynamical properties of the shift maps on the inverse limit spaces*. Ergodic Theory Dynam. Systems **12** (1992), no. 1, 95–108.

Hoang Nhat Le Thanh (Université d'Orléans)

Minkowski dimension of the image of the circle by the conformal maps generated by Bloch class (poster)

Genadi Levin (The Hebrew University of Jerusalem)

Common limits of Fibonacci circle maps

(joint work with Grzegorz Świątek)

Jelena Ligere (Riga Technical University)

TBA

Mykola Matviichuk (Taras Shevchenko National University of Kyiv)

On the Birkhoff center and topological entropy of induced systems

For a dynamical system (I, f) given by a continuous interval map $f : I \to I$ we will consider the following induced dynamical systems:

1. $(\mathcal{I}, \mathcal{F})$, where $\mathcal{I}$ is the space of all compact intervals in I endowed with the Hausdorff metric and $\mathcal{F} : \mathcal{I} \to \mathcal{I}$ is a continuous map such that $\mathcal{F}(J) := \{f(x) : x \in J\}$ for all $J \in \mathcal{I}$;

2. $(S_H(I), F)$, where $S_H(I)$ is the space of all continuous maps from I to itself endowed with the Hausdorff metric applied to the graphs of maps and $F : S_H(I) \to S_H(I)$ is a continuous map given by $F(\phi) = f \circ \phi$ for all $\phi \in S_H(I)$.

Both of these systems are larger than the original (one-dimensional) one, because each of them contains a copy of (I, f). On the other hand the system $(\mathcal{I}, \mathcal{F})$ is a factor system of $(S_H(I), F)$ (the last one is called the functional envelope of (I, f) (see [1] for details)). There is a natural surjective map $\pi : \phi \mapsto \text{range}(\phi)$ for all $\phi \in S_H(I)$ such that the diagram

$$\begin{array}{ccc} S_H(I) & \xrightarrow{F} & S_H(I) \\ \downarrow \pi & & \downarrow \pi \\ \mathcal{I} & \xrightarrow{\mathcal{F}} & \mathcal{I} \end{array}$$

commutes.

We study the relations between the dynamical properties of the systems, in particular, the Birkhoff center and the topological entropy. The talk will be based on the recent papers [2, 3, 4].

References

[1] J. Auslander, S. Kolyada and L. Snoha, *Functional envelope of a dynamical system*, Nonlinearity *20* (2007), no. 9, 2245–2269.

[2] S. Kolyada and M. Matviichuk, *On the Birkhoff center*, in preparation.

[3] M. Matviichuk, *Entropy of induced maps for one-dimensional dynamics*, Grazer Math. Ber., Bericht 354 (2009). In: A.N. Sharkovky et al. (Eds.) Proc. ECIT '08, 180-185.

[4] M. Matviichuk, *On the set of periodic intervals of an interval map*, Regul. Chaotic Dyn., 2010, 15 (2-3), 378-381.

Yauhen Mikulich (Jacobs University Bremen)

Classification of postcritically finite Newton maps

We consider Newton maps arising as complex dynamical systems from the well known Newton's method. Our goal is to classify postcritically finite Newton maps of arbitrary degree in terms of the so-called Newton graphs. We show that there is a bijective mapping from the set of postcritically finite Newton maps to the set of abstract Newton graphs modulo the corresponding equivalence relations on the sets of maps and graphs under consideration.

Olena Mul (Ternopil National Ivan Pul'uj Technical University)

Analysis of dynamics of some discrete systems

(joint work with Ilona Dzenite)

Dynamics of nonlinear systems of controlled machine units with discrete parameters is analyzed. The objectives of the investigation are to study possible vibrations in such discrete dynamical systems, as well as to find ways how to decrease the harmful effect of vibrations on the normal functioning of the systems. The used mathematical model is a system of ordinary differential equations of the fifth order with a cubic nonlinearity. For the investigations averaging method is first applied to such advanced model, that allows to determine conditions of stability of both one-frequency stationary modes and biharmonic ones. Amplitudes of the vibrations, as well as the approach to equilibrium on fast time scales, are obtained. Dependence of stationary modes on the different parameters of the system is analyzed. Using feedback, one can optimize functioning of the machine units with discrete parameters. The results may be applied in industry for the design of new improved technical systems.

Thuy Nga Nguyen Thi (Université d'Orléans)

Simulation of growth of cities (poster)

Christopher Penrose (Queen Mary, University of London)

Uniformly continuous conformal metrics and equicontinuity for mixed iteration of correspondences

A holomorphic correspondence from the Riemann sphere to itself is defined by an algebraic equation in two complex variables. When iterated, holomorphic correspondences generalise rational maps and finitely generated subgroups of $PSL(2,\mathbb{C})$. We consider "global" (that is mixed forward and backward) iteration and the question of equi-continuity.

An invariant (admissible) conformal metric is known to exist on the (global) Fatou domains of a rational map and is conjectured to be uniformly continuous with respect to the spherical metric. Such an invariant conformal metric is uniformly continuous on "orbi-compact" subsets. A similar result holds for a domain fully invariant under a correspondence where the action lifts to a group resolution.

Remus Radu (Cornell University)

Topological models for a class of complex Hénon maps

(joint work with John Hubbard and Raluca Tanase)

We will describe how to build topological models ("pinched ball models") for a certain class of complex Hénon maps, including small perturbations of z^2+c, with $|c+1| < 1/4$. The underlying model where the nontrivial pinching takes place is the unit 3-sphere with a solenoid removed; the pinching is done inside the unit 4-ball. The one-dimensional analogue is the pinched disk model for polynomial Julia sets.

Juan Rivera Letelier (Pontificia Universidad Católica de Chile)

Stochastic potentials of interval maps

(joint work with Irene Inoquio-Renteria)

Since the 1980's there have been a wealth of results on the thermodynamic formalism of an interval map f and a potential φ, that extend and go much beyond the classical result of Lasota and Yorke. The uniform expansion hypothesis corresponds to $\sup \varphi < P(f, \varphi)$, or more generally that for some integer $n \geq 1$ the function $S_n(\varphi) = \varphi + \varphi \circ f + \cdots + \varphi \circ f^{n-1}$ satisfies

$$\sup \frac{1}{n} S_n(\varphi) < P(f, \varphi). \tag{1}$$

In particular, under different mild assumptions, the work of Hofbauer and Keller, Rychlik, Baladi and Keller and Liverani, Saussol and Vaienti imply that the existence of an exponentially mixing equilibrium state of f for the potential φ. We show that for a sufficiently regular f and φ this result is optimal: the existence of an exponentially mixing equilibrium state implies (1). We also give other characterizations of this property and discuss its (lack of) genericity.

Ana Rodrigues (Universidade do Porto)

Simple conjugacy invariants for braids

(joint work with Michał Misiurewicz)

We define simple conjugacy invariants of braids, which we call turning numbers, and investigate their properties. Since our motivation comes from the investigation of periodic orbits of orientation preserving disk homeomorphisms, turning numbers work best for braids with the cyclic permutation, especially for positive permutation cyclic braids.

Nikita Selinger (Jacobs University Bremen)

On the boundary behavior of Thurston's pullback map

We define explicitly the extension to the boundary of the augmented Teichmüller space in a way that is similar to the definition of the action of

Thurston's pullback map on the Teichmüller space. We characterize then the dynamics of Thurston's pullback map near invariant strata on the boundary of the augmented Teichmüller space. We use the obtained classification to simplify the proofs of Thurston's theorem and Canonical Obstruction theorem due to Pilgrim as well as to prove some further results about the behavior of the Thurston's pullback map on the boundary.

Weixiao Shen (National University of Singapore)

On stochastic stability of interval maps

We study the derivative growth along random orbits for an interval map with the summability condition of exponent one. As an application, we show such an interval map is strongly stochastically stable.

Mitsuhiro Shishikura (Kyoto University)

Siegel disk boundaries and hairs for irrationally indifferent fixed points

A fixed point of a holomorphic function is called irrationally indifferent if its multiplier (derivative) has modulus one but no a root of unity. We study such fixed points via near-parabolic renormalization. Based on a joint work with H. Inou, for a class of maps containing quadratic polynomials, we can analyze semi-local invariant sets around the fixed point, such as the boundary of Siegel disks and Perez-Marco hedgehogs, provided that the rotation number is of sufficiently high type.

Daniel Smania (Universidade de São Paulo)

Twisted cohomological equation and renormalization

In the study of the spectrum of the renormalization operator for unimodal maps (or even more general maps), the twisted cohomological equation

$$v(x) = a(f(x)) - Df(x)a(x)$$

appears in a natural way. It also arises in the study of perturbations of quadratic-like maps (Lyubich) and linear response problems (Baladi-S.).

A better knowledge of the existence and regularity of the solution a(x) of this equation, when f is an infinitely renormalizable map, can potentially lead to bounds to the spectrum of the renormalization operator (spectral gap estimates).

Shrihari Sridharan (Chennai Mathematical Institute)

Non-vanishing derivatives of the Lyapunov exponents

We consider the family of hyperbolic quadratic polynomials and discuss about its Lyapunov characteristic exponents. We make use of the structural stability of this family and show that the results of Manning on Lyapunov exponents for Julia sets do not generalise to other Bernoulli measures.

Sebastian van Strien (University of Warwick)

TBA

Grzegorz M. Świrszcz (IBM Research)

Finding the optimal strategy in a tally game

Raluca Tanase (Cornell University)

Describing the closure of the escaping set for a class of complex Hénon maps

(joint work with John Hubbard and Remus Radu)

Consider the set U^+ of points that escape in forward time under the complex Hénon map. This set can be presented as a quotient of the cross product of the outside of the closed unit disc with $\mathbb{C}$ by a discrete group of automorphisms isomorphic to $\mathbb{Z}[1/2]/\mathbb{Z}$. We will show how to extend this description to $(\mathbb{C}-\mathbb{D})\times\mathbb{C}$ in certain cases, in order to capture the boundary J^+ of U^+. We will analyze this extension for Hénon maps that are small perturbations of hyperbolic polynomials with connected Julia sets.

Paulo Varandas (Universidade Federal da Bahia)

Robust exponential decay of correlations for singular-flows

(joint work with Vítor Araújo)

We construct open sets of C^k vector fields with singularities in dimension three that have robust exponential decay of correlations and satisfy the central limit theorem. In particular we use a criterium based on the work of Dolgopyat, detailed and expanded in the works of Baladi-Vallée and Avila-Gouezel-Yoccoz, on suspension semiflows over piecewise expanding full Markov maps to prove that the C^2-geometric Lorenz attractor has exponential decay of correlations with respect to the unique physical measure.

Denis Volk (Institute for Information Transmission Problems)

Thin attractors in skew products

We study partially hyperbolic skew products over invariant sets of Axiom A diffeomorphisms. Consider the systems with one-dimensional interval fibers such that these fibers form the central foliation of the diffeomorphism. We are interested in the existence of SRB (physical) measures and if they exist, in the Lyapunov stability of their supports (attractors).

In the work joint with V. Kleptsyn, for a typical such skew product, we obtained an explicit description of the dynamics. Namely, there exists a finite number of "thin" attractors and repellers. Each of them carries an SRB measure for a certain basin of attraction (repulsion). The union of these basins is the whole phase space modulo some set of zero measure. Each of the supports of these measures lies within some Lyapunov stable maximal attractor which is also "thin".

Michael Yampolsky (University of Toronto)

Thurston equivalence and algorithmic decidability

Michel Zinsmeister (Université d'Orléans)

Hausdorff dimension of real quadratic Julia sets

(joint work with Genadi Levin)

A result of Shishikura asserts that the supremum of dimensions of quadratic Julia sets $(z^2 + c)$ is equal to 2. The supremum for c real is unknown. In this talk I will prove that the supremum of the dimensions of the Julia set of $z^d + c$, c real is greater or equal to $2d/(d + 1)$.

PhD students Colloquium

Michel Zinsmeister (Université d'Orléans)

From Bieberbach to Schramm, a 20th century saga

The famous Bieberbach conjecture was stated in 1916. The first breakthrough towards its proof was done by Loewner in 1923. Loewner method have been revived in a totally different framework by Schramm in 1999. The aim of this talk is to revisit Bieberbach coefficient problem and the steps of its proof in the framework of Schramm SLE processes.