Spectral theory of dynamical systems
4-8 October 2010

Abstracts

Jon Aaronson, *Tel Aviv University, Israel*

Title: Rational weak mixing in infinite measure spaces

Abstract: Motivated by an example of E. Hopf (1937), K. Krickeberg (1967) introduced a topological ratio mixing property now called `HK mixing` and enjoyed by Markov shifts with the `strong ratio limit property`. The theory of `weakly wandering sets` as developed by Hajian and Kakutani (1964) shows that there is no measure theoretic version of HK mixing. I'll show that Hopf's example is `rationally weak mixing` in a sense which implies that for every pair of measurable sets in a hereditary ring, `HK mixing` is satisfied along a subsequence of density one.

Houcein Abdalaoui, *Universite de Rouen, France*

Title: On Approximately Transitive property and simple spectrum

Abstract: In my talk, I will present my recent joint work with Mariusz Lemańczyk on the Approximately transitive property (AT). It is shown that there exists a finite measure-preserving actions with simple spectrum and without AT property. We exhibit a spectral sufficient condition to insure that the finite measure-preserving actions is not AT. As a consequence, we prove that Rudin-Shapiro substitution is not AT.

Oleg Ageev, *Moscow State Technical University, Russia*

Title: Spectral analysis of some joinings dynamical systems

Abstract: Nonsimplicity of a typical transformation implies a lot of its joinings sitting on finitely many graphs. Taking such a joining, the corresponding dynamical system is not so well understood from the spectral point of view. Trying to study the spectrum of it, we explain how it is related to some well-known constructions, and, in particular, how likely its spectrum is homogeneous.

Jean-Pierre Conze, *Universite de Rennes I, France*

Title: Groups of affine transformations on compact nilmanifolds: ergodicity, spectral gap property

Abstract: Let N be a connected, simply connected, nilpotent Lie group and let D be a lattice in N. Denote by m the measure induced on the nilmanifold N/D by a Haar measure of N. Let Γ be a group of affine transformations of N/D. The general idea is that the ergodic properties of the action of Γ on $(N/D, m)$ can be reduced to the ergodic properties of its factor action on the torus factor $T := N/N^1.D$. If Γ is generated by a single affine map, W. Parry proved in 1969 that the ergodicity of the action on the torus implies the ergodicity of the action on the nilmanifold. We will show that the same statement holds for any group of affine maps. It does not reduce to the case of a single map, since there are examples of ergodic groups of automorphisms of nilmanifolds, with each automorphism in the group non ergodic. We will also discuss quantitative spectral results, recently proved by B. Bekka and R. Heu. Take for N the $(2d + 1)$-dimensional real Heisenberg group. Let μ be a probability measure on the group of automorphisms of N/D. The convolution operator by μ has a spectral gap on $L_0^2(N/D, m)$, iff the corresponding operator on the torus factor T has a spectral gap. This leads to the question of a "quenched" behaviour for random products of automorphisms on nilmanifolds, extending known properties of toral automorphisms.

Alexandre Danilenko, *National Academy of Sciences of Ukraine*

Title: ON SPECTRAL MULTIPLICITIES OF ERGODIC ABELIAN GROUP ACTIONS

Abstract: I will survey recent progress in realizations of subsets of positive integers as spectral multiplicities of ergodic (or weakly mixing or mixing) actions of Abelian locally compact groups.

Tomasz Downarowicz, *Wrocław University of Technology, Poland*

Title: CLASSES OF LARGE SETS IN $\mathbb{Z}$ AND THEIR CONNECTION WITH MIXING PROPERTIES (joint work with Vitaly Bergelson)

Abstract: We study notions of large sets in $\mathbb{Z}$ such as syndetic sets, IP*-sets, Central*-sets, Δ*-sets, etc. in the context of sets of "fat intersectons" $\{n \in \mathbb{Z} : \mu(A \cap T^n B) > \mu(A)\mu(B) - \epsilon\}$. This allows for a characterization of mixing, mild mixing and weak mixing, as well as to identify a new intermediate class (between weak and mild mixing).

Bassam Fayad, *Universite de Paris 13, CNRS, France*

Title: ERGODIC DYNAMICS IN THE SOLID TORUS

Abstract: In a joint work with H. Eliasson and R. Krikorian, we construct smooth conservative ergodic maps in the solid torus $\mathbb{T} \times \mathbb{D}$ with arbitrary (in particular Diophantine) translation frequencies on the boundary. We discuss the construction and some of its relations with the problem of existence of invariant tori near a quasi-periodic Diophatine torus of a hamiltonian flow.

Nikos Frantzikinakis, *University of Crete, Greece*

Title: CONVERGENCE OF MULTIPLE ERGODIC AVERAGES, RECENT ADVANCES AND A SPECTRAL APPROACH

Abstract: Recent advances in ergodic theory have enabled us to deal with various special cases of a conjecture of Bergelson and Leibman concerning the mean convergence of multiple ergodic averages of commuting transformations taken along polynomial iterates. The case of linear iterates was dealt a few years ago by T. Tao using a finitary argument, and ergodic variants were given by T. Austin, and subsequently by B. Host, using the machinery of "pleasant/magic" extensions. The case of iterates with distinct degrees was very recently dealt in joint work with Q. Chu and B. Host using the machinery of "nil" factors. I am going to discuss these somewhat different approaches, and also mention a different, still unexplored, spectral approach, that could potentially help us resolve the general conjecture.

Krzysztof Frączek, *Nicolaus Copernicus University, Poland*

Title: ON MARKOV QUASI-SIMILARITY OF KOOPMAN OPERATORS

Abstract: The talk is devoted to the isomorphism problem in ergodic theory: in which cases two measure–preserving systems are essentially the same? We will deal with an equivalence, called Markov quasi-similarity, introduced by Vershik for polymorphisms. In the setting of measure–preserving systems this yields an equivalence which is stronger than spectral equivalence and weaker than weak isomorphism. The talk is based on a joint work with M. Lemańczyk.

Melanie Guenais, *Universite de Paris XI, France*

Title: ON SPECTRAL EIGENVALUES OF SOME TRANSFORMATIONS LINKED TO AN IRRATIONAL ROTATION

Abstract: This talk is based on a common work with F. Parreau. For a given irrational rotation T on the torus $\mathbb{T}$, the main result settles necessary and sufficient conditions on step functions ϕ and $t \in \mathbb{T}$ that satisfy the cohomological equation

$$\exp\left(2i\pi\phi\right) = e^{2i\pi t} f/f \circ T.$$

for a measurable function f. This yields a characterization of eigenvalues and eigenfunctions for several transformations arising from irrational rotations and step functions : cylinder flows, special flows, induced maps... From there I will present some special flows and three-interval exchange transformations with exotic spectral properties.

Eugene Gutkin, *Nicolaus Copernicus University, Poland*

Title: CONSERVATIVENESS AND ERGODICITY FOR THE BILLIARD ON NONCOMPACT POLYGONS

Abstract: Well known models of statistical physics lead to the study of billiard orbits on non-compact billiard tables. Since the Liouville measure is infinite, not only the ergodicity but even the conservativeness of billiard dynamics on these tables cannot be taken for granted. We study the billiard on several families of Z-periodic and Z^2-periodic polygons. Among them is the famous wind-tree model of Paul and Tatiana Ehrenfest.

Anatole Katok, *Pennsylvania State University, USA*

Title: ON ERGODIC AND SPECTRAL PROPERTIES OF REAL-ANALYTIC DIFFEOMORPHISMS

Abstract: In the smooth category there is an old and well-developed approximation by conjugation (earlier often called Anosov-Katok) method of constructing diffeomorphisms with prescribed ergodic and topological properties. Extension of at least some aspects of this method to the real-analytic category has been topical since the method was introduced over forty years ago.
I will report on a substantial progress that has been achieved beginning about ten years ago. Basic results were announced in the 2004 joint paper with Bassam Fayad. Detailed proofs and further extensions are being worked out now.
A model result: Arbitrary close to any rotation of the sphere of dimension 2n-1 there exist uniquely ergodic weakly mixing diffeomorphisms either volume-preserving or preserving a singular measure. Topology is the uniform topology of the complexification on arbitrary large compact sets in the complex 2n-dimensional space.

Svetlana Katok, *Pennsylvania State University, USA*

Title: CALCULATION OF THE ENTROPY FOR A CLASS OF GAUSS-LIKE MAPS RELATED TO A NEW KIND OF CONTINUED FRACTIONS

Abstract: I will discuss Gauss-like one-dimensional maps related to a new family of (a,b)-continued fractions and studied jointly with Ilie Ugarcovici. Their natural extension maps have attractors with finite rectangular structure that can be "computed" from the data (a,b), and the geodesic flow on the modular surface can be represented as a special flow over the corresponding cross-sections parametrized by these attractors. I will explain how the density of the smooth invariant measure and the measure-theoretic entropy of these one-dimensional maps is calculated explicitly based on the shape of the attractors.

Andrey Kochergin, *Lomonosov Moscow State University, Russia*

Title: THE TIME CHANGE, SMOOTHNESS, SINGULARITIES AND MIXING PROPERTIES FOR FLOWS.

Abstract: I will talk about some general results concerning time change and mixing, and about some results concerning mixing properties for flows on surfaces.

Joanna Kułaga, *Nicolaus Copernicus University, Poland*

Title: ON THE ABSENCE OF METRIC AND SPECTRAL SELF-SIMILARITIES

Abstract: We begin with short survey of what is known as far as the self-similarities are concerned. We sketch a method of proving the absence of metric and spectral self-similarities for measure-preserving flows. We apply it to a class of special flows.

Sebastian Król, *Nicolaus Copernicus University, Poland*

Title: THE BLUM-HANSON PROPERTY VIA CONVERGENCE IN MEASURE

Abstract: I am going to present a sufficient condition for a contraction T on L_p to possess the Blum-Hanson property, i.e.

$$\lim_{n \to \infty} \frac{1}{N} \sum_{k=1}^{N} T^{n_k} f$$

exists in L_p for every increasing sequence $(n_k)_{k \in \mathbb{N}} \subset \mathbb{N}$ and $f \in L_p$. As corollaries, I obtain results due to Ackoglu and Sucheston and to Müller and Tomilov characterizing the Blum-Hanson property for positive T and for an arbitrary contractions on l_p, respectively.

Jan Kwiatkowski, *University of Warmia and Mazury, Poland*

Title: A SURVEY OF SPECTRAL PROPERTIES OF MORSE FLOWS

Abstract: We present main results concerning spectral properties of Morse dynamical systems obtained by M. Keane, T. Downarowicz and J. Kwiatkowski. Those results include among others:
I. Either the class of spectral isomorphism coincides with the class of metric isomorphism or it consists of continuum metrically non-isomorphic classes of Morse flows.
II. Rank one is a spectral invariant in the class of Morse flows.

Mariusz Lemańczyk, *Nicolaus Copernicus University, Poland*

Title: SPECTRAL THEORY AND JOININGS

Abstract: We focus on the problem of convolutions of the maximal spectral type of a measure-preserving system. A spectral method of constructing systems with "exotic" measure-preserving properties will be provided. I will also describe a recent development toward so called joining primeness property and its relation with the simple convolution property. Examples of smooth flows with the simple convolution property will be presented.

Aleksander Prikhodko, *Moscow State University, Russia*

Title: ON RANK ONE FLOWS WITH LEBESGUE SPECTRUM AND FLAT POLYNOMIALS WITH COEFFICIENTS $\{0,1\}$ OVER THE GROUP $\mathbb{R}$

Abstract: In 1966 J.Littlewood asked a question concerning complex polynomials $P(z) = c_0 + c_1 z + ... + c_n z^n$ with the property $|c_j| = 1$ for all j: Is it possible to find a polynomial in this class which is flat on the unit circle S^1? A variation of the Littlewood's question is the flatness phenomenon in the class of polynomials with coefficients $\{0,1\}$. In this paper we show that for given $0 < a < b$ and $\epsilon > 0$ there exist trigonometric sums $P(t) = n^{-1/2}(e(tw(0)) + e(tw(1)) + ... + e(tw(n-1)))$ on R, $e(x) = \exp(2\pi i x)$, with a real frequency function $w(j)$ such that $P(t)$ is ϵ-flat on $[a,b]$ according to the norm in $L^1[a,b]$. We apply this method to construct a rank one flow having simple Lebesgue spectrum.

Jacek Serafin, *Wrocław University of Technology, Poland*

Title: PHENOMENA IN RANK-1 $\mathbb{Z}^2$ ACTIONS

Abstract: I will present an example of rank-one partially mixing $\mathbb{Z}^2$-action, which has a non-rigid factor and for which the Weak Closure Theorem fails. This is in contrast to one-dimensional actions, where this type of behavior cannot occur.

Jean-Paul Thouvenot, *Universite de Paris VI, France*

Title: LOOKING FOR THE STRUCTURE

Abstract: We shall describe the algorithm which allows the production of small entropy factors that split in a large class (e.g. loosely Bernoulli) transformations.

Yuri Tomilov, *IM PAN and Nicolaus Copernicus University, Poland*

Title: SPECTRUM AND WANDERING

Abstract: Let T be a bounded linear operator on a Hilbert space H. A vector $x \in H$ is called weakly wandering for T if there is an increasing sequence $\{n_k\} \subset \mathbb{N} \cup \{0\}$ such that the vectors $\{T^{n_k} x\}$ are mutually orthogonal. By a well-known result due to Krengel, every unitary operator on H without point spectrum has a dense subset of weakly wandering vectors. We will present several far-reaching extensions of the Krengel result. In particular, we will show that if T is a power bounded operator on H with infinite peripheral spectrum and with empty peripheral point spectrum then the set of weakly wandering vectors for T is dense in H. Our spectral assumptions on T are in a sense best possible. This is joint work with V. Müller (Prague).

Benjamin Weiss, *Hebrew University of Jerusalem, Israel*

Title: HILBERT DYNAMICAL SYSTEMS

Abstract: Among the closed algebras of functions on the integers the the Hilbert algebra, which is the closure of B(Z), the Fourier transforms of bounded measures, is fundamental. It was Walter Rudin who first showed that there exists a weakly almost periodic function on the group of integers Z which is not in the Hilbert algebra. I will describe such a function which is also has non trivial dynamics - in particular it is recurrent. A related construction will give an example of a Polish monothetic group which is reflexively, but not Hilbert, representable.