CODY Autumn in Warsaw

1 October – 10 December 2010

Transcendental dynamics

Warsaw, 8–12 November 2010

TITLES AND ABSTRACTS OF TALKS

Krzysztof Barański (University of Warsaw)

Bowen's formula for non-hyperbolic meromorphic maps

(joint work with Bogusława Karpińska and Anna Zdunik)

Let f be a transcendental meromorphic function in the class $\mathcal{S}$. We prove that the pressure $P(f,t)$ equals the supremum of the pressures of f over all hyperbolic subsets of the Julia set, and we prove Bowen's formula for f, i.e. we show that the Hausdorff dimension of the radial Julia set of f is equal to the infimum of the set of t, for which $P(f,t)$ is non-positive. Similar results hold for (non-exceptional) transcendental meromorphic functions f in the class $\mathcal{B}$, for which the closure of the post-singular set does not contain the Julia set.

Anna Miriam Benini (Stony Brook University)

Accessibility of repelling periodic orbits for postsingularly finite exponential maps

We will show that repelling periodic points are landing points of periodic dynamic rays for a function $f(z) = e^z + c$ where the orbit of the singular value c is bounded. This also gives a new proof of the same theorem for polynomials, originally due to Douady.

Ruben Berenguel (Universitat de Barcelona)

Approximating bifurcation loci by zeros of functions (poster)

(joint work with Núria Fagella)

In this poster we present a (rigorous) numerical algorithm to approximate bifurcation loci for one-parameter families of entire functions, depending analytically on a parameter. The algorithm is based on finding some sets which are dense on the bifurcation loci or on parts of it. These sets are described as the zeros of some dynamically defined functions.

Jordi Canela (Universitat de Barcelona)

Trans-quasiconformal surgery in cubic Milnor polynomials (*work in progress*) (poster)

By performing quasiconformal surgery on certain family of degree 4 Blaschke products, one can get a cubic Milnor polynomial. We try to make an inverse of the previous surgery by using trans-quasiconformal surgery.

Kuldeep Singh Charak (University of Jammu)

Normal families of bicomplex meromorphic functions

Robert Devaney (Boston University)

Rings around the McMullen domain

Goran Drazic (Jacobs University Bremen)

Measurable dynamics of the exponential (poster)

The poster talks about the typical (from the measure theoretical point of view) orbit, and describes how well does it follow the orbit of 0 with some interesting periodic behaviour. It is a review of a paper by M. Lyubich.

Adam Epstein (University of Warwick)

Deformation theory for finite type maps

We apply fundamental constructions of Teichmuller Theory to the construction and study of deformation spaces of finite type transcendental maps.

Núria Fagella (Universitat de Barcelona)

A Separation Theorem for entire transcendental functions

(joint work with Anna Benini)

Let f be an entire transcendental map in class B. Suppose also that f is of finite order (or a finite composition of finite order functions) and additionally assume that all fixed rays land. By the snail lemma, they must do so at repelling or parabolic fixed points. The set of fixed rays which land at repelling fixed points, together with their landing points, separate the plane into unbounded Jordan domains called "basic regions" of f. We prove that every basic region of f contains exactly one interior fixed point of f. The theorem holds for periodic points of a given period p, substituting f by f^p. A (slightly stronger) version of this theorem was proven for polynomials by Goldberg, Milnor and Kiwi. Such result has several applications, like for example that there cannot be any Cremer periodic point in the boundary of a periodic Siegel disk.

Alastair Fletcher (University of Warwick)

Quasiregular dynamics

In this talk, we will investigate some of the features that occur when quasiregular mappings in $\mathbb{R}^n$ are iterated. The main focus will be on iterating quasiregular mappings of polynomial type, and similarities and differences when compared to the iteration of polynomials. There may be pictures.

Xavier Jarque (Universitat Rovira i Virgili)

Non landing hairs in Sierpiński curve Julia sets of transcendental entire maps

We consider the family of transcendental entire maps given by $f_a(z) = a(z-(1-a))\exp(z+a)$ where a is a complex parameter. Every map has a superattracting fixed point at $z=-a$ and an asymptotic value at $z=0$. For $a>1$ the Julia set of f_a is known to be homeomorphic to the Sierpiński universal curve (Morosawa, 1999), thus containing embedded copies of any one–dimensional plane continuum. In this paper we study subcontinua of the Julia set that can be defined in a combinatorial manner.

In particular, we show the existence of non–landing hairs with prescribed combinatorics embedded in the Julia set for all parameters $a \geq 3$. We also study the relation between non–landing hairs and the immediate basin of attraction of $z = -a$. Even as each non–landing hair accumulates onto the boundary of the immediate basin at a single point, its closure, nonetheless, becomes an indecomposable subcontinuum of the Julia set.

Bogusława Karpińska (Warsaw University of Technology)

Pressure for non-hyperbolic meromorphic maps

(joint work with Krzysztof Barański and Anna Zdunik)

In this talk we shall discuss some techniques of the thermodynamic formalism for certain classes of meromorphic functions and show how they can be applied to study the geometry of the Julia set.

Marta Kosek (Jagiellonian University)

Changes at each step. Generalizations of Julia sets (poster)

Helena Mihaljević-Brandt (Christian-Albrechts-Universität Kiel)

On transcendental entire functions with escaping singular sets and no wandering domains

(joint work with Lasse Rempe)

This will be a talk on recent work, where we show that certain transcendental entire functions for which all singular values escape to infinity have no wandering domains. This includes all maps of the form $z \mapsto \lambda \frac{\sinh(z)}{z} + a$ with $\lambda > 0$ and $a \in \mathbb{R}$.

Tarakanta Nayak (Indian Institute of Technology Bhubaneswar)

Omitted values and dynamics

Let M be the class of all transcendental meromorphic functions $f : \mathbb{C} \to \mathbb{C} \cup \{\infty\}$ with at least two poles or one pole that is not an omitted value, and $M_o = \{f \in M : f$ has at least one omitted value$\}$. A complete classification in terms of forward orbits of all the multiply connected Fatou components is made. As a corollary, it follows that the Julia set is not totally disconnected unless all the omitted values are contained in a single Fatou component. Non-existence of both Baker wandering domains and invariant Herman rings are proved. Eventual connectivity of each wandering domain is proved to exist. For functions with exactly one pole, we show that Herman rings of period two also do not exist. A necessary and sufficient condition for the existence of a dense subset of singleton buried components in the Julia set is established for functions with two omitted values. The conjecture that a meromorphic function has at most two completely invariant Fatou components is confirmed for all $f \in M_o$ except in the case when f has a single omitted value, no critical value and is of infinite order. Some relevant examples are discussed.

Daniel Nicks (The Open University)

Wandering domains in quasiregular dynamics

Sullivan famously proved that the Fatou set of a rational function has no wandering components. In this talk, we investigate a question asked by Lasse Rempe about the existence of wandering components of the Fatou set for a certain class of quasiregular mappings. These maps are a natural generalisation of non-linear complex polynomials. In order to tackle this question, we must first consider what is meant by the 'Fatou set' of such mappings.

John Osborne (The Open University)

The structure of spider's web fast escaping sets

Rippon and Stallard have recently shown that, for certain transcendental entire functions, the fast escaping set is connected and consists of an intricate structure of loops and unbounded continua (a form they call a 'spider's web'). In this talk I will give an overview of this phenomenon, and describe some new results on the topology and the dynamics of the 'holes' in the spider's web for such functions (i.e. the components of the complement of the fast escaping set).

Jörn Peter (Christian-Albrechts-Universität Kiel)

Hausdorff measure of escaping sets

Phil Rippon (The Open University)

Dynamical properties of entire functions of regular growth

(joint work with Gwyneth Stallard and Dave Sixsmith)

Let f be a transcendental entire function with maximum modulus $M(r, f)$, where $r > 0$. We describe various regularity properties that $M(r, f)$ can have and show how these regularity properties are related to two topics:

1. the existence of multiply connected Fatou components of f;
2. the existence of a certain spider's web structure for the fast escaping set of f,

which implies that f has many strong dynamical properties.

Manjula Samarasinghe (Queen Mary, University of London)

Quasi-Fuchsian correspondences (poster)

We consider the action of (holomorphic) correspondences or algebraic functions acting on the Riemann sphere $\overline{\mathbb{C}}$ and their limit sets. A holomorphic correspondence, or an algebraic function, is a polynomial relation $P(z, w) = 0$ where $P(z, w) = z^n A_n(w) + z^{n-1}A_{n-1}(w) \ldots + A_0(w)$ for some polynomials A_i's in w with coefficients in $\mathbb{C}$. We say that P is an $(n : m)$ holomorphic correspondence if the degrees of P in variables z and w are n and m respectively.

We consider two particular non empty classes of $(2 : 2)$ correspondences and show that their limit sets are topological circles: these correspondences can be viewed as a deformation of the action of modular group $PSL(2,\mathbb{Z})$ on $\overline{\mathbb{C}}$.

Dierk Schleicher (Jacobs University Bremen)

TBA

Mitsuhiro Shishikura (Kyoto University)

Smoothness of hairs for some entire functions

(joint work with Masashi Kisaka)

For complex exponential functions, Devaney and Krych have shown that there are "hairs" which consist of escaping points. Viana showed that these hairs are smooth (infinitely differentiable). We try to extend this result to a larger class of entire functions. If the time permits, we will also discuss a possible application to the Julia sets of Cremer polynomials.

Anand Prakash Singh (Central University of Rajasthan)

Escaping sets of composite entire functions

Let f be a transcendental entire function and f^n, $n \in \mathbb{N}$ denote the n^{th} iterate of f. Eremenko defined an escaping set as

$$I(f) := \{z \in \mathbb{C} : f^n(z) \to \infty \text{ as } n \to \infty\}.$$

A subset of $I(f)$ in which the iterates of a transcendental entire function tend to infinity arbitrarily fast was considered by Bergweiler and Hinkkanen who defined the set

$$A(f) := \{z \in \mathbb{C} : \text{ there exists } L \in \mathbb{N} \text{ such that } |f^n(z)| > M(R, f^{n-L}) \text{ for } n > L\}$$

where $M(R,f) = \max_{|z|=R} |f(z)|$, R is any value such that $R > \min_{z \in J(f)} |z|$, and $J(f)$ is the Julia set of f. An alternate definition of $A(f)$ was given by Rippon and Stallard who defined the set

$$B(f) := \{z \in \mathbb{C} : \text{ there exists } L \in \mathbb{N} \text{ such that } f^{n+L}(z) \in \widetilde{f^n(D)}, n \in \mathbb{N}\}$$

where D is an open disk meeting Julia set of f and $\widetilde{U}$ denotes the union of U and its bounded complementary components. In this paper we obtain several properties of $B(h)$ where h is a composite transcendental entire function.

Gwyneth Stallard (The Open University)

Properties of multiply connected Fatou components of entire functions

(joint work with Walter Bergweiler and Phil Rippon)

Let f be a transcendental entire function with a multiply connected Fatou component. We show that, for large n, the Fatou components $f^n(U)$ contain much larger annuli than was previously known to be the case. In proving this result we obtain further results about the behaviour under iteration of points in U – showing, for example, that each such point eventually maps inside an annulus inside $f^n(U)$.

Sebastian van Strien (University of Warwick)

Density of hyperbolicity for classes of real transcendental entire functions and circle maps

(joint work with Lasse Rempe)

In this talk I want to report on a result in which we prove density of hyperbolicity in spaces of

(i) real transcendental entire functions, bounded on the real line, whose singular set is finite and real and

(ii) transcendental functions $f\colon \mathbb{C}\setminus\{0\} \to \mathbb{C}\setminus\{0\}$ which preserve the circle and whose singular set (apart from $0, \infty$) is contained in the circle.

In particular, we prove density of hyperbolicity in the famous Arnol'd family of circle maps and its generalizations, and solve a number of other open problems for these functions.

We also prove density of (real) hyperbolicity for certain families as in (i) but without the boundedness condition; in particular our results apply when the function in question has only finitely many critical points and asymptotic singularities.

Anna Zdunik (University of Warsaw)

Very nice inducing

We describe an inducing procedure, relying on an invariant measure. Several results proved with a use of this inducing will be presented.

Jian-Hua Zheng (Tsinghua University)

Some problems on transcendental dynamics

Along with some problems, I introduce some results on several aspects of dynamics of transcendental meromorphic functions. These results should be a background or a development of those problems. The problems deal with topological dynamics (e.g., Herman rings, Baker domains and Wandering domains) and measurable dynamics (e.g., Entropy, Pressure, Dimension and Lyapunov exponents). I myself am very interested in them because they stem from my study of transcendental meromorphic function dynamics.