Conference on Several Complex Variables
on the occasion of Professor Józef Siciak's 80th birthday
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Abstracts of short talks

Lev Aizenberg (Ramat Gan)
Duality in the Hardy spaces of functions holomorphic in strictly convex domains of $\mathbb{C}^n$

Let D be a strictly convex domains in $\mathbb{C}^n$ and $\widetilde{D} = \{w : \langle wz\rangle \neq 1, z \in D\}$, $\langle wz\rangle = w_1z_1 + \cdots + w_nz_n$, be its dual. Then $(H^p(D))^* = H^q(\widetilde{D}))$, with $1/p + 1/q = 1$, $p > 1$.

Mirosław Baran, Leokadia Białas-Cież (Kraków)
Comparison principles for compact sets in $\mathbb{C}^N$ *with HCP and Markov properties*

If $\|z\|$ is a given norm in $\mathbb{C}^N$ and $f : \mathbb{C}^N \longrightarrow [-\infty, \infty)$ is a plurisubharmonic function then we can consider its growth function

$$M_f(z) = \sup_{\|w\|\leq\|z\|} f(w).$$

Some properties (e.g. convexity) of $M_f(z)$ are of special interest in the study of psh functions. A similar construction can be applied to the Siciak extremal function $\Phi(E, z)$ associated to a compact set $E \subset \mathbb{C}^N$. In this fashion we can obtain new comparison principles related to (HCP) and Markov property of the set E. We shall also prove that L-capacity in $\mathbb{C}^N$, which is a generalization of logarithmic capacity in $\mathbb{C}$ and is defined as

$$C(E) = \liminf_{\|z\|\to\infty} \frac{\|z\|}{\Phi^*(E, z)}, \quad \|z\| = (|z_1|^2 + \ldots |z_N|^2)^{1/2},$$

has the following *p*roduct property

$$C(E_1 \times E_2) = \min(C(E_1), C(E_2)), \quad E_1 \subset \mathbb{C}^{N_1},\ E_2 \subset \mathbb{C}^{N_2}.$$

Marcin Bilski (Kraków)
Approximation of holomorphic maps from Runge domains to algebraic varieties

I will discuss a new proof of Lempert's theorem on approximation of holomorphic maps from Runge domains to algebraic varieties.

Sergii Favorov (Kharkov)
Uniformly spread discrete sets

We consider discrete infinite sets in $\mathbb{R}^p$, i.e. infinite sets without finite limit points. Such sets appear in various branches of analysis (zero and pole sets of almost elliptic functions, various models in the mathematical theory of quasicrystals, and so on). In my talk I am going to introduce some notions and to present some theorems connected with such sets.
For arbitrary infinite discrete sets $A = (a_n)_{n\in I}$ and $B = (b_n)_{n\in I}$ put $d(A, B) = \inf \sup_n |a_n - b_{\sigma(n)}|$, where $\inf$ is taken over all bijections σ of the index set I. Actually, the condition $d(A, B) < \infty$ means that B is a bounded perturbation of the set A.

Theorem 1. *If the function* $f(t) = d(A, A + t)$ *is bounded uniformly in* $t \in \mathbb{R}^p$*, then* A *has a uniform density* $\Delta = \Delta(A)$, $0 < \Delta < \infty$, *i.e.*

$$\exists \lim_{T\to\infty} \frac{\operatorname{card}\{n : \|a_n - c\| < T\}}{(2T)^p} = \Delta,$$

uniformly with respect to $c \in \mathbb{R}^p$*, where* $\|\cdot\|$ *is the* l^∞*-norm in* $\mathbb{R}^p$*. Moreover,* $d(A, \Delta^{-1/p}\mathbb{Z}^p) < \infty$.

Corollary. *Let* L_1, L_2 *be arbitrary full-rank lattices in* $\mathbb{R}^p$*. Then* L_1 *is a bounded perturbation of* L_2 *iff* L_1 *and* L_2 *have the same density.*

Next, a discrete set A is Bohr's almost periodic, if its counting measure $\sum_{x\in A}\delta_x$ is Bohr's almost periodic in the weak sense. We find a geometric criterium for any discrete set to be Bohr's almost periodic. Also, we prove the following result.

Theorem 2. *If a discrete set $A\subset\mathbb{R}^p$ is almost periodic and the set of differences $A-A$ is discrete, then $A=L+F$, where F is finite, and L is a full-rank lattice in $\mathbb{R}^p$.*

A similar result was obtained for sets whose counting measure is Besicovitch's almost periodic in the weak sense.

Keiko Fujita (Saga)
An application of a quotient function in the time-frequency space

The aim of this talk is to show that a quotient function is useful in a method of blind source separation problem.
On the blind source separation problem, there is a method to use the quotient function of complex valued time-frequency informations of two observed signals. A characteristic of the method is that we first try to estimate the number of sources by studying the quotient function in the time-frequency space. Considering such an application, we have studied a complex valued quotient function in $\mathbb{R}^n, n\geq 2$, or $\mathbb{C}^n$.
In this talk, we will review a basic theorem related to the method of estimating the number of sources and will present some problems of the method.

Alexander Goncharov (Ankara)
Equilibrium Cantor-type sets

Equilibrium Cantor-type sets are suggested. This allows to obtain Green functions with preassigned moduli of continuity and compact sets with preassigned growth of Markov's factors.

Bernhard Gramsch (Mainz)
Oka's principle and the division of distributions for analytic Fredholm functions

The complex analytic homotopy theory for Fredholm operators depending analytically or continuously on parameters is applied to the division of vector valued distributions. This new connection is also based on some classical contributions (e.g. Math. Ann. **214** (1975), 95–147) and some work with W. Kaballo (Math. Nachr. **204** (1999), 83–100). The Oka-Grauert-Gromov principle is discussed for special Fréchet manifolds of semi Fredholm operators and idempotent elements in algebras of pseudodifferential operators. As source spaces for the Oka principle we can admit in some cases holomorphy regions in DFN-spaces with basis. The Hörmander class $(1,1)$ is known to be not spectrally invariant in any $L(H)$, where H is a Hilbert space. But some amount of a holomorphic Fredholm theory can be derived also in this case using lifting methods. A series of problems is mentioned for the operator valued Oka principle in connection with submanifolds in Fréchet algebras.

Lisa Hed (Umeå)
Approximation of plurisubharmonic functions

We shall consider different kinds of approximation of plurisubharmonic functions on a hyperconvex domain Ω. Especially, we shall look at approximation by plurisubharmonic functions defined on strictly larger domains Ω_j, $\Omega\subset\subset\Omega_j$. In a joint work with U. Cegrell we proved that if we can approximate one function from a certain class of plurisubharmonic functions from outside, then we can approximate all such functions. We shall discuss what geometric conditions Ω should have for this approximation to work. Finally, we will consider the connection between this approximation and the notion of plurisubharmonic functions defined on compact sets that was introduced by Poletsky.

Krzysztof Jarosz (Edwardsville)
Riemann Mapping Theorem in $\mathbb{C}^n$?

The classical Riemann Mapping Theorem states that a nontrivial simply connected domain Ω in $\mathbb{C}$ is holomorphically homeomorphic with the open unit disc $\mathbb{D}$. Furthermore, if the boundary of Ω is homeomorphic

with the unit circle, then that homeomorphism from $\mathbb{D}$ onto Ω can be extended to the boundary. It is very well known that simply connected domains in $\mathbb{C}^n$, for $n > 1$, are generally not holomorphically equivalent.
Are "similar" domains "almost" holomorphically equivalent? That may of course depend on the meaning of these two words. For Banach spaces A, B the closeness is normally measured by the Banach-Mazur distance: $d_{B-M}(A,B) = \inf\{\|T\|\|T^{-1}\| : T : A \to B\}$. For domains Ω_1, Ω_2 in $\mathbb{C}$ the quasiconformal distance $d_q(\Omega_1, \Omega_2)$ is the most natural measure of closeness. For one dimensional surfaces two concepts coincide.

Theorem. *Let S_i, $i = 1, 2$, be bordered one dimensional Riemann surfaces and $A(S_i)$ be the algebras of functions continuous on S_i and analytic on* $\operatorname{int} S_i$. *Then $d_q(S_1, S_2) < 1+\varepsilon$ iff $d_{B-M}(A(S_1), A(S_2)) < 1+\varepsilon'$, where ε and ε' tend to zero simultaneously.*

For example the domains $S_\varepsilon = \{z \in \mathbb{C} : 1 < |z| < 2 + \varepsilon\}$ are not holomorphically equivalent, but $d_q(S_0, S_\varepsilon) \simeq d_{B-M}(A(S_0), A(S_\varepsilon))$ as $\varepsilon \to 0$.
We discuss the general setting of this problem for abstract uniform algebras and also what is known for the domains in $\mathbb{C}^n$.

Piotr Jucha (Kraków)
On the dimension of the Bergman space

It is known that the space of square integrable holomorphic functions (the Bergman space) of a planar domain is either trivial or infinite dimensional. On the other hand, there are non-pseudoconvex domains in $\mathbb{C}^n$ for $n > 1$ which have Bergman spaces of finite dimension. The problem whether there exist such pseudoconvex domains is open. We solve the problem for some Hartogs domains using Hörmander's L^2–techniques. In particular, the dimension of the Bergman space of a domain $\{(z, w) : |w| < e^{-u(z)}\}$ is fully characterized by the properties of a subharmonic function u.

Damir Kinzebulatov (Toronto)
Oka-Cartan type theory for certain algebras of holomorphic functions on coverings of complex manifolds

Given an algebra of holomorphic functions on a covering of a complex manifold bounded over fibres of the covering (e.g. Bohr's holomorphic almost periodic functions), we obtain analogues of Cartan theorems A and B for coherent-type sheaves on the maximal ideal space of this algebra. This yields a number of results on interpolation within the algebra, divisors determined by the functions in the algebra etc.

Agnieszka Kowalska (Kraków)
Polynomial approximation on semialgebraic sets

The Markov and Bernstein inequality estimated derivatives of polynomials in terms of their degrees and values on an interval or a circle. These inequalities play an important role in the constructive theory of functions. Many its generalization are still the subject of investigations. In particular, are known generalization of these inequalities for some compact subset of $\mathbb{R}^n$ or $\mathbb{C}^n$ and some curves. It is known that for $\mathbb{C}$ determining sets Markov's property is equivalent to some very interesting conditions. Some generalizations of Markov inequality on some semialgebraic sets will be presented.

Viacheslav Krivokolesko (Krasnoyarsk)
Integral representation of holomorphic functions in bounded n-circular linear convex domain with piecewise regular boundary

In [1] we have obtained a new integral representation for holomorphic functions on bounded linearly convex domains with piecewise regular boundary (linearly convex polyhedra). In this case, the natural question is how can the condition of piecewise regular boundaries of the region be weakened? To answer this question, it seems natural to first consider a simpler situation: the case of n-circular linearly convex polyhedra.
Article [2] shows a detail of the integral representation of [1] for this situation. The integrals on the "faces", "edges", "tops" of linear convex polyhedron [1] are reduced to repeated integrals of the projections of the "faces", "edges", "tops" on the Reinhardt and integrals on skeleton of the unit polycylinder. In this study

on the linearly convex n-circular domain is reduced to the investigation of the projection of this region on a Reinhardt diagram [3].
Note that the right side of the integral representation from [1] is the sum of some terms. In the integration of holomorphic monomials over the boundary of particular area we will get some identities [2], related both to the degree of the monomial, and with the geometrical parameters of the treated area. In [1] the concept of mixed Levian is introduced. For example, mixed Levian of the first order is the determinant of Levy and is associated with the curvature of a surface for which it is located. In the mixed Levians of the second-order the functions involved define the intersection between two surfaces. So far, we do not know the answer to the question: what is the geometric meaning of mixed Levians of order $k > 1$?

References

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[3] M. Forsberg, M. Passare, A. Tsikh, *Laurent Determinants and Arrangements of Hyperplane Amoebas*, Advances in Mathematics textbf151 (2000), 45–70.

Michael Langenbruch (Oldenburg)

Bases in spaces of analytic functions

For certain weighted spaces of holomorphic functions defined on strips or conic neighborhoods of $\mathbb{R}$ we calculate linear topological invariants of (DN)- and (Ω)-type. In many situations this implies that these spaces admit a bases and that they are tamely isomorphic to the dual of a power series space of finite type which can often be calculated. Especially our results apply to certain Gelfand/Shilov spaces and to spaces of Fourier hyperfunctions or modified Fourier hyperfunctions.

Benedikt Magnusson (Reykjavik)

Disc formulas for ω-plurisubharmonic functions

Poletsky's famous theorem for the envelope of the Poisson functional gives a disc formula for the largest plurisubharmonic function dominated by a given upper semicontinuous function. We will see how to formulate and prove the Poletsky theorem in the case of ω-plurisubharmonic functions, where ω is the difference of two postive, closed $(1,1)$-currents.
This result enables us to get some new results about the classical case $\omega = 0$. More specifically it combines two well-known disc formulas, for the Poisson functional and the Riesz functional, into one.

Tejinder Neelon (San Marcos)

Restrictions of power series and functions to algebraic curves

The analogs of the following theorems in more general set up of classes of smooth functions and subrings formal power series will be presented.

(i) (Hartogs) A function of several complex variables is analytic if it is analytic separately in each variable.
(ii) (Lelong) A formal power series in n variables that converges on a nonpolar set of lines through the origin is necessarily convergent.
(iii) (Bochnak-Siciak) If an infinitely differentiable function f is real-analytic on every line segment through a point p then f is real-analytic in a neighborhood of p.

Alexander Rashkovskii (Stavanger)

Approximation of plurisubharmonic singularities

We consider classes of isolated singularities of plurisubharmonic functions that can be approximated by analytic singularities with control over the residual Monge-Ampère masses. They are characterized in terms of Green functions for Demailly's approximations and relative types. The types relative to such singularities are represented as lower envelopes of weighted divisorial valuations and analytic disk functionals.

Tomasz Rodak (Łódź)
Equivalence of mappings at infinity

We give, in terms of the Łojasiewicz inequality, a sufficient condition for germs of holomorphic mappings (respectively $\mathcal{C}^2$ real mappings) at infinity to be isotopical.

Małgorzata Stawiska (Ann Arbor)
Lelong classes on toric manifolds and a theorem of Siciak

We characterize Lelong classes on a toric manifold with an ample torus invariant line bundle, generalizing an approximation theorem due to Siciak. We include a counterexample to the theorem when the line bundle is globally generated, but not ample.
This is joint work with Maritza Branker.

Stevo Stević (Beograd)
Isometries on the Bergman-Privalov class on the unit ball

Bergman-Privalov class $AN_\alpha(\mathbb{B})$ consists of all holomorphic functions on the unit ball $\mathbb{B} \subset \mathbb{C}^n$ such that

$$\|f\|_{AN_\alpha} := \int_{\mathbb{B}} \ln(1 + |f(z)|) dV_\alpha(z) < \infty,$$

where $\alpha > -1$, $dV_\alpha(z) = c_{\alpha,n}(1-|z|^2)^\alpha dV(z)$ ($dV(z)$ is the normalized Lebesgue volume measure on $\mathbb{B}$ and $c_{\alpha,n}$ is the normalization constant, that is, $V_\alpha(\mathbb{B}) = 1$). Under a mild condition, we characterize surjective isometries (not necessarily linear) on $AN_\alpha(\mathbb{B})$, and prove that T is a surjective multiplicative isometry (not necessarily linear) on $AN_\alpha(\mathbb{B})$ if and only if it has the form

$$Tf = f \circ \psi \quad \text{or} \quad Tf = \overline{f \circ \overline{\psi}},$$

for every $f \in AN_\alpha(\mathbb{B})$, where ψ is a unitary transformation of the unit ball. The corresponding results for the case of the Bergman-Privalov space on the unit polydisk $\mathbb{D}^n$ are also presented.

Shigeru Takeuchi (Gifu)
Lie algebras of polynomial vector fields and the invariant holomorphic functions

The purpose of this talk is to consider finite dimensional complex Lie algebras of polynomial vector fields on the complex affine space $\mathbb{C}^n$ and to investigate its Lie algebra structure. We would like to discuss some characteristics of the invariant holomorphic functions with respect to the action of the associated Lie groups G. This is an extended version of the one variable case (2010), where we have classified all the possible Lie algebras, and hence the realization as Lie groups acting effectively on the affine line. In our case the situations are more complicated if dimensions become greater. We could express G invariance of a function f by a system of linear partial differential equations with holomorphic coefficients, specifically algebraic ones in our case. Then we will show the possible Lie algebras. It is desirable to treat more general vector fields, i.e. with non-algebraic holomorphic coefficients. We will discuss the possibility to obtain any results in this direction.

Chia-chi Tung (Mankato)
On Hilbert exponent, Noether stability and global Nullstellensatz

An objective of this work is to find conditions characterizing the membership of the ideal of a subvariety $\mathfrak{S}$ in a product complex space and give an explicit expression for a Hilbert exponent of $\mathfrak{S}$. A main result obtained is the following: Assume that X, Y are normal complex spaces and $S \subset X$ a subvariety admitting a weakly q-flat defining map $g : X \to \mathbb{C}^p$. Then for each relatively compact open set $D \subset X$, a Hilbert relation over Y holds for all holomorphic functions on $Y \times D$ vanishing on the subvariety $\mathfrak{S} = Y \times (S \cap D)$, with an explicitly determined Hilbert exponent $\mathfrak{h}_{D,S}$. Generalizing results of A. Płoski and P. Tworzewski, the Noether stability of relative regular functions on an algebraic variety is proved (under a weakened Noether condition) and, consequently, a global Nullstellensatz is established for subvarieties lying in $Y \times C^N$, respectively, $Y \times \mathbb{P}^N(\mathbb{C})$. Also obtained are conditions for the ideal of a divisor in a product space to admit a principal generator and characterizations of solid pseudospherical harmonics on a semi-Riemann domain.