Some property of convex and $\mathbb{C}$-convex sets

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For domain $D \subset \mathbb{R}^n$ we denote $\Gamma(x)$ a subset of $(n-2)$-dimensional planes of Grassmanian manifold $G'(n, n\text{-}2)$, which pass through the point x, but do not cross the D it will be a subset of compact submanifold $G(n, n\text{-}2)$.

Then, the following criterion of domain convexity is true. .

Theorem 1. A bounded domain $D \subset \mathbb{R}^n$ is convex if and only if the sets $\Gamma(x)$ are non empty and connected for all points of boundary ∂D.

This theorem generalizes to the case of real Euclidean space a complex theorem 10. 2 from monograph of Yu. Zelinsky [1] This result is concerning to one of the important characteristics of $\mathbb{C}$-convex sets and solves one of the problems raised by L. Aisenberg [2].

Theorem 2(Zelinsky). Bounded domain $D \subset \mathbb{C}^n, \Theta \in D$ is the $\mathbb{C}$-convex iff the sets $\Gamma(z)$ are not empty to all points of $z \in \partial D$.

Differences from the quoted results are firstly that in the case of a multidimensional complex space, not each plane of real codimension two will be complex hyperplane, secondly, because the real planes of codimension two are more than complex hyperplanes comprehensive, due to this we get a stronger result namely the convexity of domain. The next two results reinforce the results in accordance G. Aumanna [3] for the real Euclidean space, and Yuri Zelinsky [1] for the complex Euclidean space to the case when a priori known acyclisity of investigated compacts.

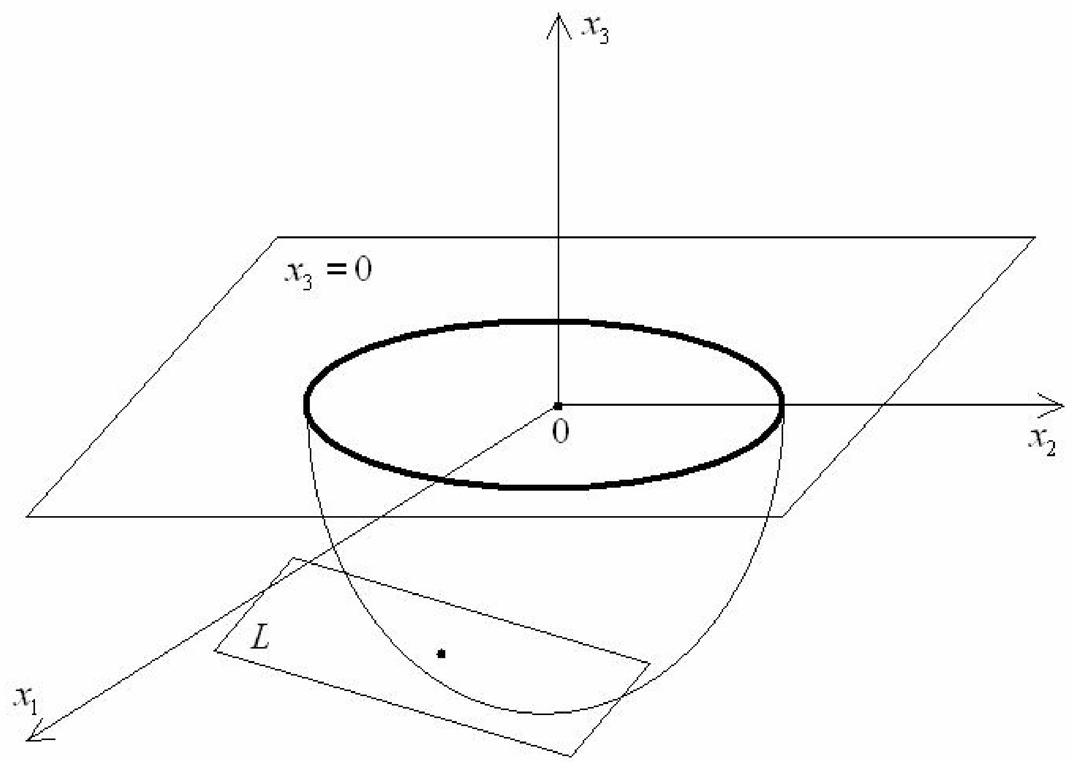
Theorem 3. To acyclic compact $K \subset \mathbb{R}^n$ be convex was necessary and sufficient that all its sections by supporting *m*–planes for fixed *m,* $1 \leq m \leq n$-1, is acyclic.

An example illustrating the need (minimality) imposed conditions.

Example 1. Hemisphere

$$S^{-} = \left\{ (x_1, x_2, x_3) \middle| x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2} = 1, x_3 \leq 0 \right\}.$$

The supporting plane $x_3 = 0$ *cross her on the one-dimensional cycle (circle). Intersection with any other supporting plane, such as* L, *is the only relevant point of a hemisphere.*


x_3
$x_3 = 0$
0
x_2
L
x_1

Theorem 4. . *To acyclic compact* $K \subset \mathbb{C}^n$ *with not empty the inside of be* $\mathbb{C}-$*convex necessary and sufficient that all its sections by supporting complex m-planes for fixed m,* $1 \leq m < n-1$, *are acyclic and in the case where* $m = n-1$, *that they were* $\mathbb{C}-$*convex.*

References

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