5 Scientific summary

5.1 General

The purpose of the conference was to enable an efficient communication of most recent results of the mainstream od quantum group theory and noncommutative geometry and to give an up to date insight into such aubjects as locally compact quantum groups, quantum Lie algebras, Hopf algebras, Poisson-Lie groups, noncommutative differential calculus, symmetries of physical models and q-special functions.

These rapidly growing branches of mathematics require knowledge of differential geometry, algebraic geometry, group theory, algebra, functional analysis and K-theory. The diversity of prerequisite mathematical tools and operator algebra techniques used in the development of the subject make direct assistance of the experts extremely important in advancing research in this area. The conference topics focus on research themes which seem to be of strong interest to many mathematicians and provide guidance and inspiration to both newcommers and scientists already active in the field.

5.2 Scientific highlights

5.2.1 Operator algebras

Marek Bożejko (Wrocław): ***Groups and probability.***

ABSTRACT: It is shown how models of group algebras (quantum groups) with von Neumann trace (Haar measure), free product groups and natural positive definite functions like-Haagerup functions, regular free product functions (now called Boolean product) and others give construction of conditional free probability. As a special case one gets constructions of (i) *free probability of Voiculescu*, (ii) *Boolean probability* and (iii/iv) *monotone/antimonotone probability of Muraki*. Classical (commutative) model of probability is done using groups. Also central limit theorems are presented with some applications to Harmonic Analysis on free groups.

Joachim Cuntz (Münster): ***C^*-algebras and number fields.***

ABSTRACT: Some properties of the regular C^*-algebra associated with a cancellative semigroup (following X.Li) are discussed. Such C^*-algebras are considered in the special case where the semigroup comes from the ring of algebraic integers in a number field.

Sergio Doplicher (Roma): ***Superselection structure in local quantum theories with (neutral) massless particles.***

ABSTRACT: The superselection structure in theories with massless particles which do not carry superselection quantum numbers is studied. The results valid in the massive case do extend, in particular we have the intrinsic notion of particle statistics of superselection sectors, and the existence of a unique compact group dual to the superselection structure. Problems with covariance and spectrum condition remain in the case of non simple sectors i.e. obeying a parastatistics of order grater than one.
(This is a report on joint work in progress with D. Buchholz and J.E. Roberts).

Alexander Helemskii (Moscow): ***Metric projectivity and freedom for normed and operator modules.***

ABSTRACT: In functional analysis there are several reasonable approaches to the notion of a projective module. The so-called relative projectivity (1970) and topological projectivity take into account the norm topology of the modules in question, whereas extreme projectivity (ascending to Grothendieck, 1955) and the metric projectivity (a new notion) take into account the exact value of the norm.

At the beginning a general-categorical frame-work is suggested that, as particular cases, contains all known versions. The basic notion is the so-called rigged category, generalizing the relative abelian category of MacLane. The best of rigged categories, called freedom-loving, possess a sufficient family of free objects. As a very workable corollary, all projective objects are exactly retracts of free objects. A similar frame-work contains known types of injective (and cofree) normed modules.

As to the two rigged categories, providing metrically projective and, respectively, extremely projective normed modules, it turns out that the first one is freedom-loving, whereas the second is not. Moreover, metrically free A-modules are characterized as $A \underset{p}{\otimes} \ell_1^0(M)$, where $\underset{p}{\otimes}$ is the non-completed projective tensor product, M an index set, and $\ell_1^0(M)$ the normed subspace of $\ell_1(M)$, consisting of finitely supported functions. Cofree A-modules are exactly those of the form $\mathcal{B}(A, \ell_\infty(M))$.

Next the following concrete and natural question: *what can be said about metrically projective modules in the simplest case of normed spaces?* is discussed.

Theorem. *Metrically projective normed spaces are exactly $\ell_1^0(M)$.*

We do not know whether the same description is valid for extremely projective normed modules.

This theorem means that in the case of normed spaces the metric projectivity coincides with freedom. Note also that there are more metrically injective normed spaces than cofree spaces.

Conclusion: the natural version of the metric projectivity for operator modules is considered and the rigged category that provides this type of projectivity is introduced. It is shown that it is freedom-loving. In particular, metrically free operator spaces turn out to be $\bigoplus_1$-sums of copies of the operator space $\mathbb{C} \bigoplus_1 \mathcal{B}(\mathbb{C}^2) \bigoplus_1 \cdots \bigoplus_1 \mathcal{B}(\mathbb{C}^n) \bigoplus_1 \cdots$. This part of the talk is strongly influenced by the paper of Blecher on standard duals of operator spaces (1992).

Nigel Higson (Penn State): *K-homology and $[Q,R]=0$.*

Abstract: If M is a symplectic manifold equipped with a Hamiltonian action of a Lie group, then its reduction is obtained by constraining all momenta associated to the group action to be zero, and then dividing out by the action. Under favorable circumstances the reduction is a symplectic manifold, and under favorable circumstances both M and its reduction may be quantized so as to obtain Hilbert spaces. The quantization commutes with reduction problem (or $[Q,R]=0$ for short) is to identify the quantization of the reduced manifold with the G-fixed part of the quantization of M. The problem was posed by Guillemin and Sternberg in the early 1980s. It was solved by Meinrenken in the mid-1990s using geometric techniques, and shortly afterwards by Tian and Zhang using analysis. In the talk an outline of some of the close links that exist between $[Q,R]=0$ and the geometric and analytic versions of K-homology theory is presented.

Yasuyuki Kawahigashi (Tokyo): *Superconformal field theory and noncommutative geometry.*

Abstract: An operator algebraic approach to chiral superconformal field theory, particularly the $N=2$ SCFT is described. After presenting representation theoretic aspects and classification theory, some analogy to classifical differential geometry and connections to noncommutative geometry is also discussed. In particular, the index pairing arising from representations of the super Virasoro algebra and JLO cocyles for the entire cyclic cohomology is described.

David Kyed (Göttingen): *Amenability for subalgebras in finite von Neumann algebras.*

Abstract: Since Atiyah's seminal work on group actions, the notion of L^2-Betti numbers has been generalized to a vast number of different contexts, including equivalence relations, discrete measured groupoids and quantum groups. In each of these instances, it turns out that the L^2-Betti numbers can be described using certain operator algebras naturally associated with the situation at hand, and furthermore that the L^2-Betti numbers always vanish in the presence of amenability. A version of amenability suited for arbitrary dense subalgebras in finite von Neumann algebras is described and it is explained how it can be used to unify all the above mentioned vanishing results. Moreover, it is discussed how above notion of amenability is linked with hyper-finiteness of the enveloping von Neumann algebra.
(This is joint work with Vadim Alekseev).

Romuald Lenczewski (Wrocław): *Matricial freeness and matricial R-transform.*

Abstract: A new concept of noncommutative independence called matricial freeness and some recent developments in that area is discussed. This concept extends freeness of Voiculescu in a non-trivial way and unifies the main notions of noncommutative independence. In particular, some asymptotic results which establish a relation between blocks of large random matrices and matricial freeness are presented. Recent results on the unified noncommutative analog of the logarithm of the Fourier transform called the matricial R-transform are also discussed.

Władysław A. Majewski (Gdańsk): *On the structure of the set of positive maps.*

Abstract: A natural characterization of the structure of positive maps is presented. Arguments are based on the concept of exposed points, links between tensor products and mapping spaces, and smoothness of unit balls in certain Banach spaces. This seems to be an answer to an old open problem studied both in Quantum Information and Operator Algebras.

Yuri Savchuk (Leipzig): *Examples of C^*-algebras generated by unbounded elements.*

Abstract: C^*-algebras generated by unbounded elements were introduced by S.L. Woronowicz as a framework for the theory of non-compact quantum groups. Given generators and relations it is not easy to verify if they generate a C^*-algebra and then to describe it. We will show that partial crossed product C^*-algebras is a useful construction in this context.

Konrad Schmüdgen (Leipzig): ***Unbounded operators and C^*-Algebras.***

ABSTRACT: The first part of the lecture is concerned with two notions that relate unbounded closed operators to C^*-algebras. The first concept is the famous affiliation relation invented by Baaj and studied by S.L. Woronowicz in a series of fundamental papers. The second is a new but weaker notion. The second half of the talk deals with the characterization of well-behaved unbounded representations of $*$-algebras by means of bounded representations of C^*-algebras generated by algebras of fractions. An important class of algebras are group graded $*$-algebras with commutative component of the unit element. Guiding examples are quantum algebras and enveloping algebras of Lie algebras.

Erling Størmer (Oslo): ***Positive linear maps of operator algebras.***

ABSTRACT: In the mid 1970's Lech made important contributions to the theory of positive maps on C^*-algebras. The lecture gives a survey of parts of the theory with special emphasis on results relating to his work.

Masamichi Takesaki (Berkeley): ***Invariants and model construction of group actions and outer actions on a factor.***

ABSTRACT: Recently, Toshihiko Masuda added a new unified approach to the cocycle conjugacy classification theory of a countable discrete amenable group actions on an AFD factor, completing successfully the mission which spans more than 30 years starting with Alain Connes ground breaking work in the mid 70's. In the talk the invariants and a model construction of group actions and outer actions on an AFD factor is discussed.

Stefaan Vaes (Leuven): ***Rigidity and classification for von Neumann algebras.***

ABSTRACT: Rigidity and classification for von Neumann algebras Thanks to Sorin Popa's deformation/rigidity theory there has been a huge progress in the classification of group von Neumann algebras and group measure space II_1 factors associated with ergodic probability measure preserving group actions. A survey, focusing on a number of W^*-superrigidity theorems is given.

Mariusz Wodzicki (Berkeley): ***Credit card renormalization.***

Joachim Zacharias (Nottingham): ***Higher Rokhlin properties of actions on C^*-algebras.***

ABSTRACT: A generalised Rokhlin property for actions of finite groups and single automorphisms on C^*-algebras involving several towers consisting of positive elements rather than a single tower consisting of projections is introduced. This notion is more flexible than the usual Rokhlin property not requiring the existence of many projections in the coefficient algebra. It is automatic for minimal dynamical systems compact spaces with finite covering dimension and also perserves finiteness of nuclear dimension and Z-stability.

Laszló Zsidó (Roma): ***On Woronowicz's approach to the Tomita-Takesaki Theory.***

ABSTRACT: The theory of M. Tomita of the standard form of general von Neumann algebras was a turning-point in the theory of Operator Algebras and is up to this day one of the most important tools by working with von Neumann algebras. It became accessible in 1970 in the exposition of M. Takesaki, which contains so many fundamental contributions also of him that the whole theory is usually referred as the "Tomita-Takesaki Theory".

The Tomita-Takesaki Theory is very involved and can be contemplated from different points of view. In the decade 1970 – 1980 several approaches appeared to the theory, each one seeking to attain more transparency. One of them was the paper of S.L. Woronowicz "Operator Systems and their application to the Tomita-Takesaki Theory" appeared in 1979.

In the talk a short-cut version of Woronowicz's approach to the Tomita-Takesaki Theory is presented.

5.2.2 Topological quantum groups

Marat Aukhadiev (Kazan): ***Infinite compact quantum semigroup.***

ABSTRACT: The compact quantum semigroup structure on the Toeplitz algebra $\mathcal{T}$, endowed with coproduct $\Delta: \mathcal{T} \to \mathcal{T} \otimes \mathcal{T}$ is discussed. It is shown that $(\mathcal{T}, \Delta)$ becomes a compact quantum semigroup.

Bialgebra $\mathcal{A}$ is called a *weak Hopf algebra*, if there exists a linear map $S: \mathcal{A} \to \mathcal{A}$ satisfying the following conditions

$$\mu(\mathrm{id} \otimes S \otimes \mathrm{id})\Delta^2 = \mathrm{id}, \ \mu(S \otimes \mathrm{id} \otimes S)\Delta^2 = S$$

where $\Delta^2 = (\Delta \otimes \mathrm{id})\Delta$, and $\mu: \mathcal{A} \odot \mathcal{A} \to \mathcal{A}$ is a linear associative map given by

$$\mu(a \otimes b) = ab.$$

It is proven that the compact quantum semigroup $(\mathcal{T}, \Delta)$ contains a dense weak Hopf algebra $(\mathcal{P}, \Delta, S)$.

The comultiplication Δ induces a multiplication on the dual algebra to $\mathcal{T}$. With this operation the dual algebra becomes a unital commutative Banach algebra. The existence of Haar functional h on $\mathcal{T}$ is shown.

For $\mathcal{K}$, the space of compact operators in $\mathcal{T}$, the space of functionals orthogonal to $\mathcal{K}$ is isomorphic as a Banach algebra to the algebra $M(S^1)$ of regular Borel measures on the circle.

Teodor Banica (Cergy-Pontoise): ***Quantum permutation groups: an overview.***

ABSTRACT: One problem with the finite quantum groups is that there is no analogue of the symmetric group S_n among them. However, by using Woronowicz's axiomatization of compact quantum groups, Wang constructed in 1998 a certain compact quantum group S_n^+, having properties similar to that of S_n. A number of recent results regarding the study of S_n^+, and of its quantum subgroups is presented.
(Talk is based on joint work with Bichon, Collins, Curran, Natale, Skalski, Speicher).

Bhowmick Jyotishman (Trondheim): ***Quantum group of unitaries and quantum automorphisms.***

ABSTRACT: Motivated by the significance of the gauge group and the group of inner automorphisms in the noncommutative geometry approach to standard model, the notion of quantum group of unitaries of a finite dimensional C*-algebra is introduced. Using an intrinsic definition, we obtain Wang's universal quantum groups $A_u(Q)$ as the quantum group of unitaries. Using this, we get the compact quantum group version of the gauge group of a finite spectral triple, which in turn gives rise to 'quantum inner fluctuations' of the metric, just like the classical case.
(This is an ongoing work with L. Dabrowski, F. D'Andrea and B. Das).

Michel Enock (Paris): ***Measured quantum groupoids. Definition, actions, Morita equivalence, deformation by 2-cocycles.***

ABSTRACT: Following J. Kustermanns and S. Vaes' locally compact quantum groups, a theory of measured quantum groupoids was cnstructed, which contains both locally compact quantum groups and measured groupoids. This theory has been used as a technical tool by K. De Commer when he studied Morita equivalence between locally compact quantum groups. It is shown that De Commer's constructions can be extended as well in the measured quantum groupoid framework. Application to the deformation problem of measured quantum groupoids by 2-cocycle is given.

Byung-Jay Kahng (Buffalo): ***Fourier transform and duality in the locally compact quantum group setting.***

ABSTRACT: In abstract harmonic analysis, the notion of Fourier transform is defined at the level of abelian locally compact groups, where Pontryagin duality holds. For further generalization, the category of quantum groups, where Pontryagin-type, self duality holds is studied. The quantum groups under consideration are locally compact quantum groups, in the C*-algebra or von Neumann algebra framework. Motivated by Van Daele's work in the multiplier Hopf algebra framework, it is possible to define the (generalized) Fourier transform in the locally compact quantum group setting. Then some of its properties, like the analogues of the Fourier inversion theorem, Plancherel theorem, and the convolution product are investigated. The study of the Fourier transform enhances also our understanding of the duality picture, including the dual pairing map.

Erik Koelink (Nijmegen): ***The quantum group analogue of $SU(1,1)$ and special functions.***

ABSTRACT: Shortly after the introduction of quantum groups it was realised that quantum groups form the natural habitat for special functions of basic hypergeometric type, and these interpretations led to a wealth of results for such special functions. This was essentially one-way traffic, until special functions played an essential role in the construction of the quantum group analogue of $SU(1,1)$ in the sense of

Kustermans and Vaes. Some of the results in both fields are discussed.
(Based on joint work with J. Kustermans (CMP 2003) and W. Groenevelt & J. Kustermans (IMRN 2010)).

Magnus Landstad (Trondheim): *Totally disconnected quantum groups.*

ABSTRACT: It is well known that there are no non-zero continuous functions on $\mathbb{R}$ with the property that both the function and its Fourier transform have compact support. *If we replace $\mathbb{R}$ with any locally compact abelian group, is the same thing true?*
It is shown that the answer to this question is the same as to the following:

1. It is folklore that the product of a multiplication operator and a convolution operator is compact. *When is this product a finite rank operator?*
2. *When does a multiplication operator and a convolution operator commute?*

These facts are used in the discussion how a theory of totally disconnected quantum groups should look like.
(Joint work with A. Van Daele).

Sutanu Roy (Göttingen): *Homomorphisms of quantum groups.*

ABSTRACT: Several equivalent notions of homomorphism between locally compact quantum groups compatible with duality are introduced. In particular, it is shown that these homomorphisms are equivalent to functors between the respective categories of coactions. The reduced bicharacter are lifted to universal quantum groups for any locally compact quantum group defined by a modular multiplicative unitary, without assuming Haar weights. These is in the general setting of modular multiplicative unitaries.

Pekka Salmi (Waterloo): *Idempotent states and contractive idempotents on quantum groups.*

ABSTRACT: An idempotent state on a locally compact group is just a probability measure that is an idempotent with respect to the convolution. The Kawada-Ito theorem characterises such idempotent states as the normalised Haar measures of compact subgroups.

On the dual side, idempotents states on group C^*-algebras are characteristic functions of open subgroups. In the talk idempotent states on coamenable locally compact quantum groups are considered and the connections between idempotent states, quantum subgroups and invariant C^*-subalgebras are discussed. The more general case of contractive idempotents is also explored.
(The talk is based on joint works with A. Skalski and with M. Neufang, A. Skalski and N. Spronk).

Thomas Timmermann (Münster): *Transformation quantum groupoids in the setting of operator algebras.*

ABSTRACT: A central paradigm of non-commutative geometry is to regard algebras as functions on virtual or "quantum" spaces. Given an action of a group on a commutative algebra, and hence on a corresponding classical space, the associated crossed product can be regarded as the groupoid algebra of a natural transformation groupoid. If the commutative algebra is replaced by a non-commutative one, or if the group is replaced by a quantum group, one could therefore expect the associated crossed product to carry the structure of a quantum groupoid. As shown by Lu and Brzezinski-Militaru in the algebraic setting, this interpretation is possible only if the action satisfies additional assumptions and is part of a Yetter-Drinfeld structure on the underlying algebra.
In the lecture presents a review of the algebraic construction and explaination how it can be adapted to actions of locally compact quantum groups on von Neumann algebras, yielding measured quantum groupoids in the sense of Enock and Lesieur is given.

Jean-Michel Vallin (Orléans): *On examples of finite quantum groupoids and subfactors.*

ABSTRACT: It is known that the structure of finite depth subfactors is closely related to the actions of quantum groupoids on von Neumann algebras. In the lecture an approach to the construction of finite quantum groupoids and of subfactors associated with them is described and some concrete examples are presented.
(Joint work with L.-Vainerman).

Shuzhou Wang (Georgia): *Simple compact quantum groups.*

ABSTRACT: In the theory of compact quantum groups in the sense of Woronowicz, two classes of examples have been intensively studied. In the first one there are that obtained as various deformations of compact Lie groups which were the main objects of study in the theory of quantum groups in the later 1980s and early 1990s. In the second one there are examples constructed as universal objects in appropriate categories of quantum transformation groups and they are in general not obtainable as deformations of compact Lie groups but contain the former class as quantum subgroups. The existence of the latter class of quantum groups leads to a fundamental task to classify simple compact quantum groups.

In the lecture the notion of simple compact quantum groups is proposed and indications how to prove simplicity of

1. the universal orthogonal groups,
2. the quantum automorphism of finite spaces including the quantum permutation groups, and
3. deformations of simple Lie groups

are given.

5.2.3 Hopf algebras

The theories of Hopf algebras, multiplier Hopf algebras, Hopf algebroids, etc. was implicitly present in most, if not all, of the lectures devoted to quantum groups and some of those dealing with noncommutative geometry. This compensated for the relatively small number of lectures addressing Hopf algebras in a more direct way.

Ulrich Krähmer (Glasgow): ***Noncommutative differential calculi from Hopf algebroids.***

ABSTRACT: The talk is a survey on cohomology theories that are governed by Hopf algebroids. It is explained that the cohomology ring of a left Hopf algebroid (assumed to be projective over the base algebra) is a Gerstenhaber algebra, and when the dual homology theory is a Batalin-Vilkovisky module over it, or in the words of Nest, Tamarkin and Tsygan, when the two form a differential calculus. A main considered example concerns twisted Calabi-Yau algebras such as the standard compact groups of Woronowicz, or the standard Podleś sphere.
(Report on joint work with N. Kowalzig).

Sergey Neshveyev (Oslo): ***Drinfeld twists and invariant cohomology of quantum groups***

ABSTRACT: By results of Drinfeld and Kazhdan-Lusztig, the comultiplication on the von Neumann algebra of the q-deformation G_q of a semisimple Lie group G is related to the classical one via an element F such that the coboundary of F^{-1} is a certain 3-cocycle on the dual of G, called Drinfeld's KZ-associator. In the talk it is explained what is known about F. In particular, it is shown that there exists a unique continuous in q family of such elements F_q satisfying some additional properties. Along the way we need to compute the second invariant cohomology of the dual of G_q. Combining this computation with a result of McMullen, as a byproduct we get a description of the group of monoidal autoequivalences of the representation category of G_q.
(Joint work with L. Tuset).

Alfons Van Daele (Leuven): ***Weak multiplier Hopf algebras.***

ABSTRACT: Consider a finite group G and the algebra A of complex functions on G with pointwise product. It is a Hopf algebra if the coproduct Δ is defined by $\Delta(f)(p,q) = f(pq)$ where $f \in A$ and $p, q \in G$. If the group is no longer assumed to be finite, we take for A the algebra of complex functions on G with finite support. The coproduct, defined as in the finite case, now makes A into a multiplier Hopf algebra. The coproduct does not map A into the tensor product $A \otimes A$ but into the multiplier algebra of this tensor product. If G is a finite groupoid, the algebra of complex functions on G with pointwise product, together with the coproduct as above, makes A into a weak Hopf algebra. The main difference is that the coproduct is no longer assumed to be unital. Finally, if G is a groupoid, finite or not, the above construction yields a weak multiplier Hopf algebra.

In the lecture a precise definition of a weak multiplier Hopf algebra and explanations where the various axioms come from are given. The main results of the theory are presented. If the weak multiplier Hopf algebra has (enough) integrals, the dual can be constructed within the same category, just as for multiplier Hopf algebras with integrals (algebraic quantum groups).

This theory is more general than the existing theory of weak Hopf algebras (as studied by Böhm, Nill & Szlachányi and others). On the other hand, if the underlying algebra is a *-algebra and if the integrals are positive, then it is a special case of the measured quantum groupoids as introduced and studied by Enock and Lesieur (and others).

5.2.4 Noncommutative geometry

Paul F. Baum (Penn State): ***Beyond ellipticity.***

ABSTRACT: A. Connes and H. Moscovici extended index theory to a class of hypoelliptic (but not elliptic) operators. By using K-homology, index theory can be further extended. K-homology is the dual theory to K-theory. The BD (Baum-Douglas) isomorphism of Kasparov K-homology and K-cycle K-homology can be taken as providing a framework within which the Atiyah-Singer index theorem can be extended to certain non-elliptic operators. This talk considers a class of non-elliptic differential operators on compact contact manifolds. These operators have been studied by a number of mathematicians. Roughly speaking, the difference between this class of operators and the Connes-Moscovici operators is the difference between nilpotent groups and abelian groups. Working within the BD framework — and applying the Connes-Thom isomorphism — the index problem will be solved for these operators.
(Joint work with Erik van Erp).

Tomasz Brzeziński (Swansea): ***Bundles over quantum projective spaces.***

ABSTRACT: New examples of quantum principal bundles or principal comodule algebras are presented. The first class of examples includes U(1) and quantum SU(2) bundles over quantum real projective spaces. All these bundles are obtained by prolongation of the defining $\mathbb{Z}_2$ actions. The second class contains bundles over quantum weighted projective spaces or quantum teardrop manifolds.

Michel Hilsum (Paris): ***Invariance of Godbillon-Vey map by absolutely continuous conjugacies.***

ABSTRACT: Godbillon-Vey class of a smooth codimension one foliation (V, F) is an element of $H^3(V, \mathbb{R})$. From works of A. Connes and H. Moscovici, this class is related to the cyclic homology of a Hopf algebra acting on the C*-algebra of the foliation. In the talk the topological invariance of this class is studied and a new result is explained. Its invariance under foliated isomorphism the transverse derivative of which belongs to Lebesgue measure class is discussed.

Richard Kerner (Paris): ***Spacetime symmetries from $\mathbb{Z}_3$-graded quark algebra.***

ABSTRACT: Certain $\mathbb{Z}_3$-graded associative algebras with cubic Z_3-invariant constitutive relations is investigated. The invariant forms on finite algebras of this type are given in the low dimensional cases with two or three generators. Non-associative ternary algebras of these forms are defined. It is shown how the Lorentz symmetry represented by the $SL(2, \mathbb{C})$ group emerges naturally without any notion of Minkowskian metric, just as the invariance group of the $\mathbb{Z}_3$-graded cubic algebra and its constitutive relations. Its representation is found in terms of Pauli matrices. The relationship of this construction with the operators defining quark states is also considered.

Jerzy Lukierski (Wrocław): ***Noncommutative quantum free fields and deformed Poincare symmetries.***

ABSTRACT: Quantum free fields on noncommutative Minkowski space and their covariance under deformed Poincare-Hopf symmetry are considered. These field will be described by braided tensor product of the algebra M of functions on noncommutative space-time and the algebra H of deformed field oscillators. Using the star-product realization of the algebra M the braided field commutator is given by standard Pauli-Jordan commutator function. The relation of this general framework with the descriptions of non-commutative free fields considered in the literature which employ the deformed relativistic symmetries (Aschieri et all, Balachandran et all, Fiore and Wess, Kulish) is discussed. As explicite examples of the canonical as well as κ deformations are considered.
(Joint results with M, Woronowicz).

Ryszard Nest (København): ***Towards quantum Hamiltonian action.***

ABSTRACT: Given a Poisson action of a Poisson-Lie group on a Poisson manifold, there is a well understood notion of the moment map and the associated Poisson reduction generalizing the classical Hamiltonian reduction of Marsden and Weinstein. The subject of this talk is the question of generalizing this to the case of quantum groups. It is described what a good candidate for the "quantum moment map" is and a few examples illustrating what happens with the reduction in this setting are given.

Vasyl Ostrovskyi (Kiev): ***On representations of real q-plane.***

ABSTRACT: Representations of real q-plane, pairs A, B of self-adjoint operators in a separable Hilbert space H, which satisfy the relation $AB = qBA$, $|q| = 1$, $q^4 \neq 1$ are discussed. For bounded operators, the relations imply $AB = BA = 0$ so there exist only degenerate representations. For unbounded operators,

one has to select the class of well-behaved representations; such class is known and all irreducible well-behaved pairs are classified up to a unitary equivalence. Relations between the resolvents of the operators A, B and some related problems for the arising unbounded operators are studied.

Andrzej Sitarz (Kraków): ***κ-Minkowski star product and its symmetries.***

ABSTRACT: A family of star products and involutions associated with κ-Minkowski space is investigated. Applying quantization maps it is shown that these star products restricted to a certain space of Schwartz functions have isomorphic Banach algebra completions. For two particular star products it is demonstrated that they can be extended to a class of polynomially bounded smooth functions, which allows a realization of the full Hopf algebra structure on κ-Minkowski. Furthermore, an explicit realization of the action of the κ-Poincaré algebra as an involutive Hopf algebra on this representation of κ-Minkowski space and construction of a twisted cyclic cocycle are presented.

Elmar Wagner (Mexico): ***Noncommutative spin geometry of the standard Podleś sphere.***

ABSTRACT: The aim of the talk is to show that Connes' Noncommutative Geometry can be applied successfully to quantum spaces arising in Quantum Group theory if one allows slight modifications of the original axioms. It is discussed that, in the case of the standard Podles sphere, the twisted versions of Hochschild and cyclic (co)homology fit better into the framework: There is a non-trivial twisted 2-cycle such that the known 0-summable spectral triple satisfies a modified orientability axiom. This twisted 2-cycle represents also the volume form of the algebraically defined covariant differential calculus. Moreover, there is a "twisted" Chern character mapping equivariant K0-classes into twisted cyclic homology such that the pairing with twisted cyclic cohomology computes the q-winding numbers of quantum line bundles. The Hochschild class of these cocycles can be expressed by residue formulas.

5.3 Proceedings

The conference proceedings will be published by Banach Center Publications.