

CONSERVATION PROPERTY OF SYMMETRIC JUMP-DIFFUSION PROCESSES

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We say that a Markov process is conservative if the associated particle stays at the state space forever. This property is one of important global path properties of Markov processes. In particular, there are many results on the conservativeness criterion of symmetric diffusion processes, in terms of the volume growth of the underlying measure and the growth of the “coefficient”, established by Grigor’yan, Davies, Ichihara, Takeda, Oshima, Sturm,....

Motivated by the recent progress on the analysis of jump processes, there also have been results on the conservativeness criterion of symmetric jump(-diffusion) processes generated by regular Dirichlet forms ([1, 2, 3, 4]). In [1, 2, 3], the volume of the underlying measure is allowed to grow exponentially, but the coefficients are assumed to be bounded. In contrast with this, we allow in [4] the coefficients to be unbounded; however, since the explicit form of the L^2 -generator is needed for the proof, we assume that the state space is \mathbb{R}^d and the underlying measure is the Lebesgue measure on \mathbb{R}^d . Furthermore, we also need the assumption on the “drift” parts which may entail the continuity on the coefficients.

A purpose in this talk is to establish a conservativeness criterion for symmetric jump-diffusion processes generated by regular Dirichlet forms, in terms of the volume growth of the underlying measure and the growth of the coefficients. Moreover, by using this criterion, we remove the conditions in [4] as we mentioned before. We also generalize the results in [1, 2, 3, 4] so that we allow the volume of the underlying measure to grow exponentially and the coefficients to be unbounded at the same time. We do not know about the sharpness of our criterion in general; however, we can show the sharpness for a class of time changed Dirichlet forms.

We finally give examples related to symmetric stable-like processes and censored stable-like processes.

REFERENCES

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