ASYMPTOTIC INDEPENDENCE AND LIMIT LAWS FOR WIENER CHAOS

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Let $(X_i)_{i\in I}$ be random variables such that each X_i is a homogeneous Wiener chaos of order q_i relative to a fixed Brownian motion. Consider $(X_i)_{i\in I_k}$, where $I_k \subset I$ are disjoint nonempty blocks. Then $(X_i)_{i\in I_k}$ are independent between blocks if and only if $\mathrm{Cov}(X_i^2, X_j^2) = 0$ for all i, j which are not in the same block. This extends the criterion for the independence of a sequence of multiple Wiener-Itô integrals given in Rosiński and Samorodnitsky (1999) (the case $\mathrm{Card}(I_k) = 1$). It also generalizes the well-known covariance criterion for the independence of jointly Gaussian random variables ($q_i = 1$).

We extend this criterion to the asymptotic moment-independence of a homogeneous Wiener chaos. As a consequence, we derive a multidimensional version of the celebrated fourth moment theorem of Nualart and Peccati (2005) and show new bounds on the rate of convergence. Other related results on the multivariate convergence of multiple Wiener-Itô integrals, that involve Gaussian and non Gaussian limits, will be discussed. If time permits, an extension to a non-Gaussian discrete chaos will also be mentioned.

This talk is based on a recent joint work with Ivan Nourdin.

References

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[2] D. Nualart and G. Peccati (2005). Central limit theorems for sequences of multiple stochastic integrals. *Ann. Probab.* **33**, no. 1, 177-193.

[3] J. Rosiński and G. Samorodnitsky (1999). Product formula, tails and independence of multiple stable integrals. *Advances in stochastic inequalities* (Atlanta, GA, 1997), 169-194, *Contemp. Math.* **234**, Amer. Math. Soc., Providence, RI.