WEYL'S LAPLACIAN EIGENVALUE ASYMPTOTICS FOR THE MEASURABLE RIEMANNIAN STRUCTURE ON THE SIERPIŃSKI GASKET

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On the Sierpiński gasket K, Kigami [3] has introduced the notion of the measurable Riemannian structure, with which the "gradient vector fields" of functions, the "Riemannian volume measure" μ and the "geodesic metric" $\rho_{\mathcal{H}}$ are naturally associated. Kigami has also proved in the same paper the two-sided Gaussian bound for the corresponding heat kernel $p_t^{\mathcal{H}}(x,y)$, and I have further shown several detailed heat kernel asymptotics, such as Varadhan's asymptotic relation

$$\lim_{t\downarrow 0} 4t \log p_t^{\mathcal{H}}(x,y) = -\rho_{\mathcal{H}}(x,y)$$

in a recent paper [1].

In the talk, Weyl's Laplacian eigenvalue asymptotics is presented for this case. Specifically, let d be the Hausdorff dimension of K and $\mathcal{H}^d$ the d-dimensional Hausdorff measure on K, both with respect to the "geodesic metric" $\rho_{\mathcal{H}}$. Then for some $c_N > 0$ and for any non-empty open subset U of K with $\mathcal{H}^d(\partial U) = 0$,

$$\lim_{\lambda\to\infty} \frac{N_U(\lambda)}{\lambda^{d/2}} = c_N \mathcal{H}^d(U),$$

where $N_U(\lambda)$ is the number of the eigenvalues, less than or equal to λ, of the Dirichlet Laplacian on U. Moreover, we will also see that the Hausdorff measure $\mathcal{H}^d$ is Ahlfors regular with respect to $\rho_{\mathcal{H}}$ but that it is singular to the "Riemannian volume measure" μ. A renewal theorem for functionals of Markov chains due to Kesten [2] plays a crucial role in the proof of the above asymptotic behavior of $N_U(\lambda)$.

References

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