WEIGHTED-POINCARÉ INEQUALITIES FOR NONLOCAL DIRICHLET FORMS

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Let V be a locally bounded measurable function such that e^{-V} is bounded and belongs to $L^1(dx)$, and let $\mu_V(dx) := C_V e^{-V(x)}\,dx$ be a probability measure. We present the criterion for the weighted Poincaré inequality of the non-local Dirichlet form

$$D_{\rho,V}(f,f) := \iint (f(y)-f(x))^2 \rho(|x-y|)\,dy\,\mu_V(dx)$$

on $L^2(\mu_V)$. Taking $\rho(r) = e^{-\delta r} r^{-(d+\alpha)}$ with $0 < \alpha < 2$ and $\delta \geq 0$, we get some conclusions for general fractional Dirichlet forms, which can be regarded as a complement of our recent work [2], and an improvement of the main result in [1]. In this especial setting, concentration of measure for the standard Poincaré inequality is also derived.

Our technique is based on the Lyapunov conditions for the associated truncated Dirichlet form, and it is considerably efficient for the weighted Poincaré inequality of the following non-local Dirichlet form

$$D_{\psi,V}(f,f) := \iint (f(y)-f(x))^2 \psi(|x-y|)\, e^{-V(y)}\,dy\, e^{-V(x)}\,dx$$

on $L^2(\mu_{2V})$, which is associated with symmetric Markov processes under Girsanov transform of pure jump type.

References

[1] Mouhot, C., Russ, E. and Sire, Y.: Fractional Poincaré inequalities for general measures, *J. Math. Pures Appl.* **95** (2011), 72–84.

[2] Wang, F.-Y. and Wang, J.: Functional inequalities for stable-like Dirichlet forms, *Preprint*, 2012, also see arXiv:1205.4508