GRADIENT ESTIMATES OF q-HARMONIC FUNCTIONS OF FRACTIONAL SCHRÖDINGER OPERATOR

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We will present results concerning gradient estimates of q-harmonic functions u of the fractional Schrödinger operator $-(-\Delta)^{\alpha/2} + q$, $\alpha \in (0, 1]$ in bounded domains $D \subset \mathbf{R}^d$. For nonnegative u we show that if q is Hölder continuous of order $\eta > 1 - \alpha$ then $\nabla u(x)$ exists for any $x \in D$ and $|\nabla u(x)| \le cu(x)/(\mathrm{dist}(x, \partial D) \wedge 1)$. The exponent $1-\alpha$ is critical i.e. when q is only $1 - \alpha$ Hölder continuous $\nabla u(x)$ may not exist. The above gradient estimates were known for $\alpha \in (1, 2]$ under the assumption that q belongs to the Kato class $\mathcal{J}^{\alpha-1}$. The case $\alpha \in (0, 1]$ is different. To obtain results for $\alpha \in (0, 1]$ probabilistic methods are used. As a corollary, one obtains for $\alpha \in (0, 1)$ that a weak solution of $-(-\Delta)^{\alpha/2}u + qu = 0$ is in fact a strong solution.