LÉVY DRIVEN BSDES: L_2-REGULARITY AND FRACTIONAL SMOOTHNESS

CHRISTEL GEISS

Motivated by the results about the L_p-variation of *backward stochastic differential equations* (BSDEs) obtained in [1], we are interested in the L_2-regularity of the solution (Y, Z) to the BSDE

$$\begin{aligned} Y_t &= H + \int_t^T f\Big(s, Y_s, \int_{\mathbb{R}} Z_{s,x}\kappa(dx)\Big)ds - \sigma \int_{(t,T]} Z_{s,0} dW_s \\ &\quad - \int_{(t,T]\times\mathbb{R}} Z_{s,x}\, x\tilde{N}(dt, dx), \quad 0 \le t \le T. \end{aligned}$$

Here (σW_s) is the Brownian part and $\tilde{N}$ denotes the compensated Poisson random measure associated with a square integrable Lévy process (L_t). We assume that the generator f is Lipschitz (κ is a certain measure on $\mathbb{R}$) while the terminal condition $H \in L_2$ is supposed to be a function of finitely many increments of (L_t). Examples are $H = \mathbb{1}_{[k,\infty)}(X_T^n)$, where X_T^n is the Euler approximation of the SDE

$$X_t = x_0 + \int_0^t a(X_{s-})dL_s, \quad 0 \le t \le T,$$

or $H = g(L_{t_1} - L_{t_0}, ..., L_{t_n} - L_{t_{n-1}})$ with $0 \le t_1 < t_2 < ... < t_n \le T$ and $g : \mathbb{R}^n \to \mathbb{R}$ being a Borel function.

We introduce the concept of (path-dependent) fractional smoothness in the Malliavin sense, i.e. a version of anisotropic Besov spaces defined by the real interpolation method, and relate the fractional smoothness of H to the L_2-regularity properties of the solution (Y, Z).

This is joint work with A. Steinicke.

References

[1] C. Geiss, S. Geiss and E. Gobet: Generalized fractional smoothness and L_p-variation of BSDEs with non-Lipschitz terminal condition. Accepted *Stoch. Proc. Appl.* 2012.