GEODESIC DISTANCES AND INTRINSIC DISTANCES ON SOME FRACTAL SETS

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The off-diagonal Gaussian asymptotics of the heat kernel density associated with local Dirichlet form is often described by using the intrinsic distances (cf. [4, 3] and the references therein). When the underlying space has a Riemannian structure, the geodesic distance is defined as well, and it coincides with the intrinsic distance in good situations.

Then, what if the underlying space is a fractal set? In typical examples, the heat kernel asymptotics is *sub*-Gaussian; accordingly, the intrinsic distance vanishes identically. However, if we take (a sum of) energy measures as the underlying measure, we can define the nontrivial intrinsic distance as well as the geodesic distance, and can pose a problem whether they are identical. For the two-dimensional standard Sierpinski gasket, the affirmative answer has been obtained ([1, 2]) by using some detailed information on the transition density. In this talk, I will discuss this problem in a more general framework and provide some partial answers based on elementary arguments.

References

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