Intrinsic ultracontractivity and ground state estimates of Feynman-Kac semigroups for a class of Lévy processes

Kamil Kaleta

Institute of Mathematics and Computer Science
Wrocław University of Technology

6th ICSAA
Będlewo, 10 September 2012
***

The talk is based on joint work with J. Lőrinczi (Loughborough)

Classical background

(1) $V : \mathbb{R}^d \to \mathbb{R}$, $V \in L^\infty_{\text{loc}}(\mathbb{R}^d)$, $V \geq 0$ and $V(x) \to \infty$ as $|x| \to \infty$
(2) Schrödinger semigroup: (T_t), $T_t := \exp(-(-\Delta + V))$, $t > 0$, on $L^2(\mathbb{R}^d)$
(3) Kernel: $T_t f(x) = \int q(t,x,y) f(y) dy$, $t > 0$
(4) T_t are compact $\Rightarrow$ there is orthonormal basis $\{\varphi_n\}_{n \geq 0} \subset L^2(\mathbb{R}^d)$ and
$0 < \lambda_0 < \lambda_1 \leq \lambda_2 ... \to \infty$, $T_t \varphi_n = e^{-\lambda_n t} \varphi_n$, φ_n are continuous, $\varphi_0 > 0$.

Classical background

(1) $V : \mathbb{R}^d \to \mathbb{R}$, $V \in L^\infty_{\text{loc}}(\mathbb{R}^d)$, $V \geq 0$ and $V(x) \to \infty$ as $|x| \to \infty$
(2) Schrödinger semigroup: (T_t), $T_t := \exp(-(-\Delta + V))$, $t > 0$, on $L^2(\mathbb{R}^d)$
(3) Kernel: $T_t f(x) = \int q(t,x,y) f(y) dy$, $t > 0$
(4) T_t are compact $\Rightarrow$ there is orthonormal basis $\{\varphi_n\}_{n\geq 0} \subset L^2(\mathbb{R}^d)$ and
$0 < \lambda_0 < \lambda_1 \leq \lambda_2 ... \to \infty$, $T_t \varphi_n = e^{-\lambda_n t} \varphi_n$, φ_n are continuous, $\varphi_0 > 0$.

Define intrinsic semigroup $(\widetilde{T}_t)_{t>0}$ on $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$:

$$\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x) = \int_{\mathbb{R}^d} f(y) \frac{e^{\lambda_0 t} q(t,x,y)}{\varphi_0(x)\varphi_0(y)} \varphi_0^2(y) dy$$

Classical background

(1) $V : \mathbb{R}^d \to \mathbb{R}$, $V \in L^\infty_{loc}(\mathbb{R}^d)$, $V \geq 0$ and $V(x) \to \infty$ as $|x| \to \infty$
(2) Schrödinger semigroup: (T_t), $T_t := \exp(-(-\Delta + V))$, $t > 0$, on $L^2(\mathbb{R}^d)$
(3) Kernel: $T_t f(x) = \int q(t,x,y) f(y) dy$, $t > 0$
(4) T_t are compact $\Rightarrow$ there is orthonormal basis $\{\varphi_n\}_{n \geq 0} \subset L^2(\mathbb{R}^d)$ and
$0 < \lambda_0 < \lambda_1 \leq \lambda_2 ... \to \infty$, $T_t \varphi_n = e^{-\lambda_n t} \varphi_n$, φ_n are continuous, $\varphi_0 > 0$.

Define intrinsic semigroup $(\widetilde{T}_t)_{t>0}$ on $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$:

$$\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x) = \int_{\mathbb{R}^d} f(y) \frac{e^{\lambda_0 t} q(t,x,y)}{\varphi_0(x)\varphi_0(y)} \varphi_0^2(y) dy$$

Definition (Davies and Simon, 1984)

(T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$, $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

Classical background

(1) $V: \mathbb{R}^d \to \mathbb{R}$, $V \in L^\infty_{\text{loc}}(\mathbb{R}^d)$, $V \geq 0$ and $V(x) \to \infty$ as $|x| \to \infty$
(2) Schrödinger semigroup: (T_t), $T_t := \exp(-(-\Delta + V))$, $t > 0$, on $L^2(\mathbb{R}^d)$
(3) Kernel: $T_t f(x) = \int q(t,x,y) f(y) dy$, $t > 0$
(4) T_t are compact $\Rightarrow$ there is orthonormal basis $\{\varphi_n\}_{n \geq 0} \subset L^2(\mathbb{R}^d)$ and
$0 < \lambda_0 < \lambda_1 \leq \lambda_2 ... \to \infty$, $T_t \varphi_n = e^{-\lambda_n t} \varphi_n$, φ_n are continuous, $\varphi_0 > 0$.

Define intrinsic semigroup $(\widetilde{T}_t)_{t>0}$ on $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$:

$$\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x) = \int_{\mathbb{R}^d} f(y) \frac{e^{\lambda_0 t} q(t,x,y)}{\varphi_0(x)\varphi_0(y)} \varphi_0^2(y) dy$$

Definition (Davies and Simon, 1984)

(T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$, $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$ or, equivalently,

$$\frac{e^{\lambda_0 t} q(t,x,y)}{\varphi_0(x)\varphi_0(y)} \leq C_t, \quad x, y \in \mathbb{R}^d$$

Classical background

E. B. Davies, B. Simon, *Ultracontractivity and heat kernels for Schrödingers operators and Dirichlet Laplacians*, J. Funct. Anal. 59, 1984, 335-395

If $V(x) = |x|^a$, $a > 0$, or $V(x) = |x|^2 \log(2 + |x|)^b$, $b > 0$,
then IUC occurs iff $a > 2$ and $b > 2$.
For $V(x) = |x|^a$, $a > 2$, we have $\varphi_0(x) \asymp \exp\left(-\frac{2}{2+a}|x|^{1+\frac{a}{2}}\right)(1+|x|)^{\frac{a}{4}-\frac{d-1}{2}}$.

Classical background

E. B. Davies, B. Simon, *Ultracontractivity and heat kernels for Schrödingers operators and Dirichlet Laplacians*, J. Funct. Anal. 59, 1984, 335-395

If $V(x) = |x|^a$, $a > 0$, or $V(x) = |x|^2 \log(2+|x|)^b$, $b > 0$,
then IUC occurs iff $a > 2$ and $b > 2$.
For $V(x) = |x|^a$, $a > 2$, we have $\varphi_0(x) \asymp \exp\left(-\frac{2}{2+a}|x|^{1+\frac{a}{2}}\right)(1+|x|)^{\frac{a}{4}-\frac{d-1}{2}}$.

B. Alziary, P. Takáč: *Intrinsic ultracontractivity of a Schrödinger semigroup in* $\mathbb{R}^N$, J. Funct. Anal. 256 (12), 2009, 4095–4127.

If $\int_{r_0}^{\infty} \frac{1}{\sqrt{V(r)}} dr < \infty$ for some $r_0 > 0$, then IUC occurs.
Moreover, if $V(x) = V(|x|)$, then IUC also implies this condition.

Classical background

E. B. Davies, B. Simon, *Ultracontractivity and heat kernels for Schrödingers operators and Dirichlet Laplacians*, J. Funct. Anal. 59, 1984, 335-395

If $V(x) = |x|^a$, $a > 0$, or $V(x) = |x|^2 \log(2 + |x|)^b$, $b > 0$,
then IUC occurs iff $a > 2$ and $b > 2$.
For $V(x) = |x|^a$, $a > 2$, we have $\varphi_0(x) \asymp \exp\left(-\frac{2}{2+a}|x|^{1+\frac{a}{2}}\right)(1 + |x|)^{\frac{a}{4} - \frac{d-1}{2}}$.

B. Alziary, P. Takáč: *Intrinsic ultracontractivity of a Schrödinger semigroup in $\mathbb{R}^N$*, J. Funct. Anal. 256 (12), 2009, 4095–4127.

If $\int_{r_0}^{\infty} \frac{1}{\sqrt{V(r)}} dr < \infty$ for some $r_0 > 0$, then IUC occurs.
Moreover, if $V(x) = V(|x|)$, then IUC also implies this condition.
Hence, for

$$V(x) = |x|^2 (\log|x|)^2 (\log\log|x|)^2 \ldots (\underbrace{\log\ldots\log|x|}_{(m-1)\text{-times}})^2 (\underbrace{\log\ldots\log|x|}_{m\text{-times}})^{2+\delta}, \quad m \in \mathbb{N},\ \delta \geq 0,$$

IUC holds iff $m \in \mathbb{N}$ and $\delta > 0$.

A class of Lévy processes – notation and assumptions

- X_t – **symmetric jump Lévy process** in $\mathbb{R}^d$, $d \geq 1$
- $\mathbf{P}^x$, $\mathbf{E}^x$ – measure and expected value of X_t starting at $x \in \mathbb{R}^d$

A class of Lévy processes – notation and assumptions

- X_t – **symmetric jump Lévy process** in $\mathbb{R}^d$, $d \geq 1$
- $\mathbf{P}^x$, $\mathbf{E}^x$ – measure and expected value of X_t starting at $x \in \mathbb{R}^d$
- characteristic exponent:

$$\psi(\xi) = A\xi \cdot \xi + \int_{\mathbb{R}^d} (1 - \cos(\xi \cdot z))\nu(dz)$$

A — symmetric non-negative definite $d \times d$ matrix (*Gaussian coefficient*)
ν — measure on $\mathbb{R}^d \setminus \{0\}$, $\int_{\mathbb{R}^d} (1 \wedge |z|^2)\nu(dz) < \infty$ (*Lévy measure*)
(ν is symmetric, i.e., $\nu(B) = \nu(-B)$)

A class of Lévy processes – notation and assumptions

- X_t – **symmetric jump Lévy process** in $\mathbb{R}^d$, $d \geq 1$
- $\mathbf{P}^x$, $\mathbf{E}^x$ – measure and expected value of X_t starting at $x \in \mathbb{R}^d$
- characteristic exponent:

$$\psi(\xi) = A\xi \cdot \xi + \int_{\mathbb{R}^d} (1 - \cos(\xi \cdot z))\nu(dz)$$

A — symmetric non-negative definite $d \times d$ matrix (*Gaussian coefficient*)
ν — measure on $\mathbb{R}^d \setminus \{0\}$, $\int_{\mathbb{R}^d} (1 \wedge |z|^2)\nu(dz) < \infty$ (*Lévy measure*)
(ν is symmetric, i.e., $\nu(B) = \nu(-B)$)

- generator: for $\varphi \in C_c^2(\mathbb{R}^d)$

$$L\varphi(x) = \sum_{j,k=1}^{d} A_{jk} \frac{\partial^2 \varphi}{\partial x_j \partial x_k}(x) + \lim_{\varepsilon \searrow 0} \int_{|y|>\varepsilon} (\varphi(y+x) - \varphi(x))\nu(dy)$$

A class of Lévy processes – notation and assumptions

Assumption 1

We have $\nu(dx) = \nu(x)dx$, $\nu(x) > 0$, $x \in \mathbb{R}^d$, and

(a) $\forall_{r \in (0,1/2]} \; \exists_{C_1 = C_1(r)} \quad \nu(x) \asymp C_1 \nu(y), \quad r \leq |y| \leq |x| \leq |y| + 1$

(b) $\exists_{C_2} \quad \nu(x) \leq C_2 \nu(y), \quad 1/2 \leq |y| \leq |x|$

(c) $\exists_{C_3} \int_{|z-x|>1/2, \; |z-y|>1/2} \nu(x-z)\nu(z-y)dz \leq C_3 \, \nu(x-y), \quad |y-x| \geq 1$

A class of Lévy processes – notation and assumptions

Assumption 1

We have $\nu(dx) = \nu(x)dx$, $\nu(x) > 0$, $x \in \mathbb{R}^d$, and

(a) $\forall_{r \in (0,1/2]} \ \exists_{C_1 = C_1(r)} \quad \nu(x) \asymp C_1 \nu(y), \quad r \leq |y| \leq |x| \leq |y| + 1$

(b) $\exists_{C_2} \quad \nu(x) \leq C_2 \nu(y), \quad 1/2 \leq |y| \leq |x|$

(c) $\exists_{C_3} \ \int_{|z-x|>1/2, \ |z-y|>1/2} \nu(x-z)\nu(z-y)dz \leq C_3 \, \nu(x-y), \quad |y-x| \geq 1$

(a)-(b): $\nu(x) \asymp \kappa(|x|)$, $\kappa : R_+ \to R_+$ nonincreasing, $\kappa(s) \leq C\kappa(s+1)$, $s > 1/2$

(c) has structural importance!

A class of Lévy processes – notation and assumptions

Assumption 1

We have $\nu(dx) = \nu(x)dx$, $\nu(x) > 0$, $x \in \mathbb{R}^d$, and

(a) $\forall_{r\in(0,1/2]}\ \exists_{C_1=C_1(r)}\quad \nu(x) \asymp C_1\nu(y), \qquad r \le |y| \le |x| \le |y|+1$

(b) $\exists_{C_2}\quad \nu(x) \le C_2\nu(y), \qquad 1/2 \le |y| \le |x|$

(c) $\exists_{C_3}\ \int_{|z-x|>1/2,\ |z-y|>1/2} \nu(x-z)\nu(z-y)dz \le C_3\,\nu(x-y), \qquad |y-x| \ge 1$

(a)-(b): $\nu(x) \asymp \kappa(|x|)$, $\kappa : R_+ \to R_+$ nonincreasing, $\kappa(s) \le C\kappa(s+1)$, $s > 1/2$

(c) has structural importance!

Assumption 2

X_t is strong Feller, i.e., there is $p(t,x,y) = p(t,y-x)$ such that $\mathbf{E}^x f(X_t) = \int f(y)p(t,x,y)dy$. Moreover, there is $t_b > 0$ such that $0 < p(t_b, x) \le C_4$, $x \in \mathbb{R}^d$.

A class of Lévy processes – notation and assumptions

Let $D \subset \mathbb{R}^d$ be a bounded and open set and let $G_D(x,y)$ denote the **Green function** of X_t in D:

$$\mathbf{E}^x \int_0^{\tau_D} f(X_t)dt = \int_D G_D(x,y)f(y)dy, \quad x \in D, \quad \text{with} \quad \tau_D = \inf\{t \geq 0 : X_t \notin D\}.$$

A class of Lévy processes – notation and assumptions

Let $D \subset \mathbb{R}^d$ be a bounded and open set and let $G_D(x,y)$ denote the **Green function** of X_t in D:

$$\mathbf{E}^x \int_0^{\tau_D} f(X_t)dt = \int_D G_D(x,y)f(y)dy, \quad x \in D, \quad \text{with} \quad \tau_D = \inf\{t \geq 0 : X_t \notin D\}.$$

Assumption 3 [BKK]

For all $0 < p < q < R \leq 1$

$$\sup_{x \in B(0,p)} \sup_{y \in B(0,q)^c} G_{B(0,R)}(x,y) < \infty$$

A class of Lévy processes – notation and assumptions

Let $D \subset \mathbb{R}^d$ be a bounded and open set and let $G_D(x, y)$ denote the **Green function** of X_t in D:

$$\mathbf{E}^x \int_0^{\tau_D} f(X_t)dt = \int_D G_D(x, y) f(y) dy, \quad x \in D, \quad \text{with} \quad \tau_D = \inf\{t \geq 0 : X_t \notin D\}.$$

Assumption 3 [BKK]

For all $0 < p < q < R \leq 1$

$$\sup_{x \in B(0,p)} \sup_{y \in B(0,q)^c} G_{B(0,R)}(x, y) < \infty$$

[BKK] K. Bogdan, T. Kumagai, M. Kwaśnicki: *Boundary Harnack inequality for Markov processes with jumps*, preprint, 2012, arXiv:1207.3160v1

A class of Lévy processes – specific classes of examples

Examples include:

(1) subclass of Subordinate Brownian motions, e.g.
– rotationally symmetric α-stable process, $\alpha \in (0, 2)$,
– mixture of such processes with different indices α and β,
– rotationally symmetric ralativistic α-stable process, $\alpha \in (0, 2)$,
– jump-diffusion,
– rotationally symmetric geometric α-stable process, $\alpha \in (0, 2)$

(2) subclass of symmetric Lévy processes with non-degenerate Brownian part: $\psi(\xi) = c|\xi|^2 + \int_{\mathbb{R}^d}(1 - \cos(z \cdot \xi))\nu(dz)$, $c > 0$

(3) symmetric stable-like Lévy processes

(4) symmetric Lévy processes with sub-exponentially localized Lévy measures: $\nu(x) \asymp e^{-a|x|^{\beta}}|x|^{-d-\delta}(1 + |x|)^{d+\delta-\gamma}$, where $a > 0$, $\beta \in (0, 1]$, $\delta \in [0, 2)$ and $\gamma > (d + 1)/2$

A class of Lévy processes – specific classes of examples

Examples include:

(1) subclass of Subordinate Brownian motions, e.g.
– rotationally symmetric α-stable process, $\alpha \in (0,2)$,
– mixture of such processes with different indices α and β,
– rotationally symmetric ralativistic α-stable process, $\alpha \in (0,2)$,
– jump-diffusion,
– rotationally symmetric geometric α-stable process, $\alpha \in (0,2)$

(2) subclass of symmetric Lévy processes with non-degenerate Brownian part: $\psi(\xi) = c|\xi|^2 + \int_{\mathbb{R}^d} (1 - \cos(z \cdot \xi))\nu(dz)$, $c > 0$

(3) symmetric stable-like Lévy processes

(4) symmetric Lévy processes with sub-exponentially localized Lévy measures: $\nu(x) \asymp e^{-a|x|^\beta} |x|^{-d-\delta} (1+|x|)^{d+\delta-\gamma}$, where $a > 0$, $\beta \in (0,1]$, $\delta \in [0,2)$ and $\gamma > (d+1)/2$

Counterexamples: $\beta > 1$ or $\beta = 1$, $\gamma = (d+1)/2$ in (4)
(geometric 2-stable/gamma variance process)

Schrödinger perturbation of X_t

Definition (Kato class $\mathcal{K}^X$ for X_t)

We say that $V \in \mathcal{K}^X$ if V is a Borel function on $\mathbb{R}^d$ such that

$$\lim_{t\to 0} \sup_{x\in\mathbb{R}^d} \mathbf{E}^x \left[\int_0^t |V(X_s)| ds \right] = 0.$$

Also, $V \in \mathcal{K}^X_{\pm}$ (V is Kato decomposable) if

$$V = V_+ - V_- \quad \text{with} \quad V_+ \in \mathcal{K}^X_{\text{loc}} \quad \text{and} \quad V_- \in \mathcal{K}^X.$$

($V \in \mathcal{K}^X_{\text{loc}}$ if $V\mathbf{1}_B \in \mathcal{K}^X$ for every ball B)

Schrödinger perturbation of X_t

Definition (Kato class $\mathcal{K}^X$ for X_t)

We say that $V \in \mathcal{K}^X$ if V is a Borel function on $\mathbb{R}^d$ such that

$$\lim_{t \to 0} \sup_{x \in \mathbb{R}^d} \mathbf{E}^x \left[\int_0^t |V(X_s)| ds \right] = 0.$$

Also, $V \in \mathcal{K}^X_{\pm}$ (V is Kato decomposable) if

$$V = V_+ - V_- \quad \text{with} \quad V_+ \in \mathcal{K}^X_{\text{loc}} \quad \text{and} \quad V_- \in \mathcal{K}^X.$$

($V \in \mathcal{K}^X_{\text{loc}}$ if $V\mathbf{1}_B \in \mathcal{K}^X$ for every ball B) $\quad L^\infty_{\text{loc}}(\mathbb{R}^d) \subset K^X_{\text{loc}} \subset L^1_{\text{loc}}(\mathbb{R}^d)$

Schrödinger perturbation of X_t

Definition (Kato class $\mathcal{K}^X$ for X_t)

We say that $V \in \mathcal{K}^X$ if V is a Borel function on $\mathbb{R}^d$ such that

$$\lim_{t \to 0} \sup_{x \in \mathbb{R}^d} \mathbf{E}^x \left[\int_0^t |V(X_s)| ds \right] = 0.$$

Also, $V \in \mathcal{K}^X_{\pm}$ (V is Kato decomposable) if

$$V = V_+ - V_- \quad \text{with} \quad V_+ \in \mathcal{K}^X_{\text{loc}} \quad \text{and} \quad V_- \in \mathcal{K}^X.$$

($V \in \mathcal{K}^X_{\text{loc}}$ if $V\mathbf{1}_B \in \mathcal{K}^X$ for every ball B) $L^\infty_{\text{loc}}(\mathbb{R}^d) \subset K^X_{\text{loc}} \subset L^1_{\text{loc}}(\mathbb{R}^d)$

Definition (Feyman-Kac semigroup of X_t for V)

For $V \in \mathcal{K}^X_{\pm}$ we define:

$$T_t f(x) = \mathbf{E}^x \left[e^{-\int_0^t V(X_s) ds} f(X_t) \right], \quad f \in L^2(\mathbb{R}^d), \quad t > 0, \quad x \in \mathbb{R}^d$$

Feynman-Kac semigroup of X_t

Assumption 4

For a given Lévy process X_t let

(a) $V \in \mathcal{K}_{\pm}^{X}$

(b) $V(x) \to \infty$ as $|x| \to \infty$

Feynman-Kac semigroup of X_t

Assumption 4

For a given Lévy process X_t let

(a) $V \in \mathcal{K}_{\pm}^{X}$

(b) $V(x) \to \infty$ as $|x| \to \infty$

Under Assumptions 1-4:

- (T_t) is a strongly continuous semigroup of symmetric operators in $L^2(\mathbb{R}^d)$
- generator: $-H$, where $H = L + V$ (generalized Schrödinger operator)
- all $T_t : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$ are compact $\Rightarrow$ there exists an orthonormal basis consisting of $\varphi_n \in L^2(\mathbb{R}^d)$

$$T_t \varphi_n = e^{-\lambda_n t} \varphi_n, \qquad \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots \to \infty$$

- all φ_n are bounded and continuous, $\varphi_0 > 0$

Feynman-Kac semigroup of X_t

Assumption 4

For a given Lévy process X_t let

(a) $V \in \mathcal{K}_{\pm}^{X}$

(b) $V(x) \to \infty$ as $|x| \to \infty$

Under Assumptions 1-4:

- (T_t) is a strongly continuous semigroup of symmetric operators in $L^2(\mathbb{R}^d)$
- generator: $-H$, where $H = L + V$ (generalized Schrödinger operator)
- all $T_t : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$ are compact $\Rightarrow$ there exists an orthonormal basis consisting of $\varphi_n \in L^2(\mathbb{R}^d)$

$$T_t \varphi_n = e^{-\lambda_n t} \varphi_n, \qquad \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots \to \infty$$

- all φ_n are bounded and continuous, $\varphi_0 > 0$
- $(-\Delta)^{\alpha/2} + V$, $(-\Delta + m^{2/\alpha})^{\alpha/2} - m + V$, $(-\Delta)^{\alpha/2} + (-\Delta)^{\beta/2} + V$, $(-\Delta)^{\alpha/2} - \Delta + V$, $\log(1 + (-\Delta)^{\alpha/2}) + V$

Bounds on eigenfunctions

For an open and bounded set $D \subset \mathbb{R}^d$ and a non-negative or bounded Borel function φ we define the *V-Green operator* for (T_t) and D

$$G_D^V \varphi(x) = \mathbf{E}^x \left[\int_0^{\tau_D} e^{-\int_0^t V(X_s)ds} \varphi(X_t) dt \right], \quad x \in D$$

(Below $f(x) \stackrel{C}{\asymp} g(x)$ means $C^{-1} g(x) \leq f(x) \leq C g(x)$, and $\mathbf{1}(x) := \mathbf{1}_{\mathbb{R}^d}(x)$)

Bounds on eigenfunctions

For an open and bounded set $D \subset \mathbb{R}^d$ and a non-negative or bounded Borel function φ we define the *V-Green operator* for (T_t) and D

$$G_D^V \varphi(x) = \mathbf{E}^x \left[\int_0^{\tau_D} e^{-\int_0^t V(X_s)ds} \varphi(X_t)dt \right], \quad x \in D$$

(Below $f(x) \stackrel{C}{\asymp} g(x)$ means $C^{-1}g(x) \leq f(x) \leq Cg(x)$, and $\mathbf{1}(x) := \mathbf{1}_{\mathbb{R}^d}(x)$)

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x, 1)$. Then for some $R > 0$

$$|\varphi_n(x)| \leq C_{X,V,\eta} \, \|\varphi_n\|_\infty \, G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R, \quad n \geq 0$$

Bounds on eigenfunctions

For an open and bounded set $D \subset \mathbb{R}^d$ and a non-negative or bounded Borel function φ we define the *V-Green operator* for (T_t) and D

$$G_D^V \varphi(x) = \mathbf{E}^x \left[\int_0^{\tau_D} e^{-\int_0^t V(X_s)ds} \varphi(X_t) dt \right], \quad x \in D$$

(Below $f(x) \stackrel{C}{\asymp} g(x)$ means $C^{-1}g(x) \leq f(x) \leq Cg(x)$, and $\mathbf{1}(x) := \mathbf{1}_{\mathbb{R}^d}(x)$)

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x, 1)$. Then for some $R > 0$

$$|\varphi_n(x)| \leq C_{X,V,\eta} \, \|\varphi_n\|_\infty \, G_D^{V+\eta} \mathbf{1}(x) \nu(x), \quad |x| > R, \quad n \geq 0$$

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x, 1)$. Then for some $R > 0$

$$\varphi_0(x) \stackrel{C_{X,V,\eta}}{\asymp} G_D^{V+\eta} \mathbf{1}(x) \nu(x), \quad |x| > R$$

Bounds on eigenfunctions

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x, 1)$. Then for some $R > 0$

$$|\varphi_n(x)| \leq C_{X,V,\eta} \|\varphi_n\|_\infty G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R, \quad n \geq 0$$

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x, 1)$. Then for some $R > 0$

$$\varphi_0(x) \overset{C_{X,V,\eta}}{\asymp} G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R$$

Bounds on eigenfunctions

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x,1)$. Then for some $R > 0$

$$|\varphi_n(x)| \leq C_{X,V,\eta} \left\|\varphi_n\right\|_\infty G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R, \quad n \geq 0$$

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x,1)$. Then for some $R > 0$

$$\varphi_0(x) \overset{C_{X,V,\eta}}{\asymp} G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R$$

Corollary (K–Lőrinczi 2012)

$$|\varphi_n(x)| \leq C_{X,V} \left\|\varphi_n\right\|_\infty \varphi_0(x), \quad x \in \mathbb{R}^d$$

Bounds on eigenfunctions

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x,1)$. Then for some $R > 0$

$$|\varphi_n(x)| \leq C_{X,V,\eta} \left\|\varphi_n\right\|_\infty G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R, \quad n \geq 0$$

Theorem (K–Lőrinczi 2012)

Let $\eta \geq 0$ be such that $\lambda_0 + \eta > 0$ and let $D := B(x,1)$. Then for some $R > 0$

$$\varphi_0(x) \stackrel{C_{X,V,\eta}}{\asymp} G_D^{V+\eta}\mathbf{1}(x)\nu(x), \quad |x| > R$$

Corollary (K–Lőrinczi 2012)

$$|\varphi_n(x)| \leq C_{X,V} \left\|\varphi_n\right\|_\infty \varphi_0(x), \quad x \in \mathbb{R}^d$$

For $-\Delta + |x|^2$ (Harmonic Oscillator) such domination property does not hold!

Bounds on eigenfunctions

Corollary (K–Lőrinczi 2012)

There is $R > 0$ such that for all $|x| > R$

$$|\varphi_n(x)| \leq C_{X,V} \, \|\varphi_n\|_\infty \frac{\nu(x)}{\inf_{y \in B(x,1)} V(y)}, \quad n = 0, 1, 2, \ldots,$$

and

$$C_{X,V} \frac{\nu(x)}{\sup_{y \in B(x,1)} V(y)} \leq \varphi_0(x) \leq C'_{X,V} \frac{\nu(x)}{\inf_{y \in B(x,1)} V(y)}.$$

Bounds on eigenfunctions

Corollary (K–Lőrinczi 2012)

There is $R > 0$ such that for all $|x| > R$

$$|\varphi_n(x)| \leq C_{X,V} \|\varphi_n\|_\infty \frac{\nu(x)}{\inf_{y \in B(x,1)} V(y)}, \quad n = 0, 1, 2, \ldots,$$

and

$$C_{X,V} \frac{\nu(x)}{\sup_{y \in B(x,1)} V(y)} \leq \varphi_0(x) \leq C'_{X,V} \frac{\nu(x)}{\inf_{y \in B(x,1)} V(y)}.$$

If for all unit balls $B \subset B(0,R)^c$ it holds that $\sup_{y \in B} V(y) \leq C \inf_{y \in B} V(y)$, then

$$|\varphi_n(x)| \leq C_{X,V} \|\varphi_n\|_\infty \frac{\nu(x)}{V(x)}, \quad n = 0, 1, 2, \ldots,$$

and

$$C_{X,V} \frac{\nu(x)}{V(x)} \leq \varphi_0(x) \leq C'_{X,V} \frac{\nu(x)}{V(x)}$$

for all $|x| \geq R$.

Bounds on eigenfunctions – previously known results

(1) **Exponentially localized Lévy measures**:

R. Carmona, W.C. Masters, B. Simon: *Relativistic Schrödinger operators: asymptotic behaviour of the eigenfunctions*, J. Funct. Anal. 91, 1990, 117-142.

If: (i) $e^{-t\psi(\cdot)} \in L^1(\mathbb{R}^d)$, $t > 0$,
(ii) $\int_{\mathbb{R}^d} e^{b|x|}\nu(dx) < \infty$ for some $b > 0$,

then $|\varphi_n(x)| \leq Ce^{-C'|x|}$, $x \in \mathbb{R}^d$, with $C = C(X, V, n)$, $C' = C'(X, V, n)$

Bounds on eigenfunctions – previously known results

(1) **Exponentially localized Lévy measures**:

R. Carmona, W.C. Masters, B. Simon: *Relativistic Schrödinger operators: asymptotic behaviour of the eigenfunctions*, J. Funct. Anal. 91, 1990, 117-142.

If: (i) $e^{-t\psi(\cdot)} \in L^1(\mathbb{R}^d)$, $t > 0$,
(ii) $\int_{\mathbb{R}^d} e^{b|x|}\nu(dx) < \infty$ for some $b > 0$,

then $|\varphi_n(x)| \leq Ce^{-C'|x|}$, $x \in \mathbb{R}^d$, with $C = C(X, V, n)$, $C' = C'(X, V, n)$

(2) **Relativistic α-stable process**, $\alpha \in (0, 2)$: Kulczycki–Siudeja,TAMS 2006
If V is a non-negative, locally bounded potential comparable to a rotationally symmetric function, radially non-decreasing and comparable on unit balls with $\lim_{|x|\to\infty} V(x)/|x| = \infty$, then

$$\varphi_0(x) \asymp \frac{e^{-m^{1/\alpha}|x|}}{(1+|x|)^{(d+\alpha+1)/2}(1+V(x))}, \quad x \in \mathbb{R}^d.$$

Bounds on eigenfunctions – previously known results

(1) **Exponentially localized Lévy measures**:

R. Carmona, W.C. Masters, B. Simon: *Relativistic Schrödinger operators: asymptotic behaviour of the eigenfunctions*, J. Funct. Anal. 91, 1990, 117-142.

If: (i) $e^{-t\psi(\cdot)} \in L^1(\mathbb{R}^d)$, $t > 0$,
(ii) $\int_{\mathbb{R}^d} e^{b|x|}\nu(dx) < \infty$ for some $b > 0$,

then $|\varphi_n(x)| \leq Ce^{-C'|x|}$, $x \in \mathbb{R}^d$, with $C = C(X, V, n)$, $C' = C'(X, V, n)$

(2) **Relativistic α-stable process**, $\alpha \in (0, 2)$: Kulczycki–Siudeja,TAMS 2006
If V is a non-negative, locally bounded potential comparable to a rotationally symmetric function, radially non-decreasing and comparable on unit balls with $\lim_{|x|\to\infty} V(x)/|x| = \infty$, then

$$\varphi_0(x) \asymp \frac{e^{-m^{1/\alpha}|x|}}{(1 + |x|)^{(d+\alpha+1)/2}(1 + V(x))}, \quad x \in \mathbb{R}^d.$$

(3) **Symmetric α-stable process**, $\alpha \in (0, 2)$: we recover results of Kaleta–Kulczycki PA 2010 and Kaleta–Lőrinczi SPA 2012.

Intrinsic ultracontractivity-type properties

Recall $\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x)$

Definition

(1) (T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$ $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

Intrinsic ultracontractivity-type properties

Recall $\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x)$

Definition

(1) (T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$ $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(2) (T_t) is asymptotically intrinsically ultracontractive (AIUC) if there is $t_0 > 0$ such that $\widetilde{T}_{t_0}$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

Intrinsic ultracontractivity-type properties

Recall $\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x)$

Definition

(1) (T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$ $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(2) (T_t) is asymptotically intrinsically ultracontractive (AIUC) if there is $t_0 > 0$ such that $\widetilde{T}_{t_0}$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(3) (T_t) is ground state dominated (GSD) if for every $t > 0$ $T_t\mathbf{1}(x) \leq C_t\varphi_0(x)$, $x \in \mathbb{R}^d$.

(4) (T_t) is asymptotically ground state dominated (AGSD) if the domination property in (3) holds for some $t_0 > 0$. This bound extends to all $t \geq t_0$ with constant $C_{t_0} e^{-\lambda_0(t-t_0)}$.

Intrinsic ultracontractivity-type properties

Recall $\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x)$

Definition

(1) (T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$ $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(2) (T_t) is asymptotically intrinsically ultracontractive (AIUC) if there is $t_0 > 0$ such that $\widetilde{T}_{t_0}$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(3) (T_t) is ground state dominated (GSD) if for every $t > 0$ $T_t\mathbf{1}(x) \leq C_t\varphi_0(x)$, $x \in \mathbb{R}^d$.

(4) (T_t) is asymptotically ground state dominated (AGSD) if the domination property in (3) holds for some $t_0 > 0$. This bound extends to all $t \geq t_0$ with constant $C_{t_0}e^{-\lambda_0(t-t_0)}$.

IUC $\Rightarrow$ AIUC $* * *$ GSD $\Rightarrow$ AGSD $* * *$ AIUC $\Leftrightarrow$ AGSD $* * *$ IUC $\Rightarrow$ GSD

Intrinsic ultracontractivity-type properties

Recall $\widetilde{T}_t f(x) = \frac{e^{\lambda_0 t}}{\varphi_0(x)} T_t(f\varphi_0)(x)$

Definition

(1) (T_t) is intrinsically ultracontractive (IUC) if for every $t > 0$ $\widetilde{T}_t$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(2) (T_t) is asymptotically intrinsically ultracontractive (AIUC) if there is $t_0 > 0$ such that $\widetilde{T}_{t_0}$ is a bounded operator from $L^2(\mathbb{R}^d, \varphi_0^2(x)dx)$ to $L^\infty(\mathbb{R}^d)$

(3) (T_t) is ground state dominated (GSD) if for every $t > 0$ $T_t\mathbf{1}(x) \leq C_t\varphi_0(x)$, $x \in \mathbb{R}^d$.

(4) (T_t) is asymptotically ground state dominated (AGSD) if the domination property in (3) holds for some $t_0 > 0$. This bound extends to all $t \geq t_0$ with constant $C_{t_0}e^{-\lambda_0(t-t_0)}$.

IUC $\Rightarrow$ AIUC $***$ GSD $\Rightarrow$ AGSD $***$ AIUC $\Leftrightarrow$ AGSD $***$ IUC $\Rightarrow$ GSD

When $p(t,\cdot) \leq C_t$ for every $t > 0$, then also GSD $\Rightarrow$ IUC

Intrinsic ultracontractivity-type properties

Theorem (K–Lőrinczi 2012)

(1) $\lim_{|x|\to\infty} \frac{V(x)}{|\log \nu(x)|} = \infty \;\Rightarrow\; (T_t)$ is GSD

Intrinsic ultracontractivity-type properties

Theorem (K–Lőrinczi 2012)

(1) $\lim_{|x|\to\infty} \frac{V(x)}{|\log \nu(x)|} = \infty \;\Rightarrow\; (T_t)$ is GSD

(2) (T_t) is GSD $\Rightarrow\; \forall \varepsilon \in (0,1]\;\; \lim_{|x|\to\infty} \frac{\sup_{y\in B(x,\varepsilon)} V(y)}{|\log \nu(x)|} = \infty$

Intrinsic ultracontractivity-type properties

Theorem (K–Lőrinczi 2012)

(1) $\lim_{|x|\to\infty} \frac{V(x)}{|\log \nu(x)|} = \infty \;\Rightarrow\; (T_t)$ is GSD

(2) (T_t) is GSD $\;\Rightarrow\; \forall \varepsilon \in (0,1]\;\; \lim_{|x|\to\infty} \frac{\sup_{y\in B(x,\varepsilon)} V(y)}{|\log \nu(x)|} = \infty$

Theorem (K–Lőrinczi 2012)

(1) $\exists R > 0, C_V > 0\; \forall |x| > R\;\; \frac{V(x)}{|\log \nu(x)|} \geq C_V \;\Rightarrow\; (T_t)$ is AGSD

Intrinsic ultracontractivity-type properties

Theorem (K–Lőrinczi 2012)

(1) $\lim_{|x|\to\infty} \frac{V(x)}{|\log \nu(x)|} = \infty \;\Rightarrow\; (T_t)$ is GSD

(2) (T_t) is GSD $\;\Rightarrow\; \forall \varepsilon \in (0,1]\;\; \lim_{|x|\to\infty} \frac{\sup_{y\in B(x,\varepsilon)} V(y)}{|\log \nu(x)|} = \infty$

Theorem (K–Lőrinczi 2012)

(1) $\exists R > 0, C_V > 0\; \forall |x| > R\; \frac{V(x)}{|\log \nu(x)|} \geq C_V \;\Rightarrow\; (T_t)$ is AGSD

(2) (T_t) is AGSD $\;\Rightarrow\; \forall \varepsilon \in (0,1]\; \exists R > 0\; \frac{\sup_{y\in B(x,\varepsilon)} V(y)}{|\log \nu(x)|} \geq C_V$ for $|x| > R$

Intrinsic ultracontractivity-type properties

Theorem (K–Lőrinczi 2012)

(1) $\lim_{|x|\to\infty} \frac{V(x)}{|\log \nu(x)|} = \infty \;\Rightarrow\; (T_t)$ is GSD

(2) (T_t) is GSD $\;\Rightarrow\; \forall \varepsilon \in (0,1] \;\; \lim_{|x|\to\infty} \frac{\sup_{y\in B(x,\varepsilon)} V(y)}{|\log \nu(x)|} = \infty$

Theorem (K–Lőrinczi 2012)

(1) $\exists R > 0, C_V > 0 \; \forall |x| > R \; \frac{V(x)}{|\log \nu(x)|} \geq C_V \;\Rightarrow\; (T_t)$ is AGSD

(2) (T_t) is AGSD $\;\Rightarrow\; \forall \varepsilon \in (0,1] \; \exists R > 0 \; \frac{\sup_{y\in B(x,\varepsilon)} V(y)}{|\log \nu(x)|} \geq C_V$ for $|x| > R$

Corollary (K–Lőrinczi 2012)

If for some $R > 0$ and all unit balls $B \subset B(0,R)^c$ we have $\sup_{y\in B} V(y) \leq C \inf_{y\in B} V(y)$, then

$$(T_t) \text{ is GSD} \iff \lim_{|x|\to\infty} \frac{V(x)}{|\log \nu(x)|} = \infty$$
$$(T_t) \text{ is AGSD} \iff V(x) \geq C_V |\log \nu(x)|, \; |x| > R$$

Intrinsic ultracontractivity-type properties

Example (various types of borderline growths)

$|\log \nu(x)| \asymp \log |x| \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp |x|^{-d-\alpha}$, $\alpha > 0$

$\vdots$

$|\log \nu(x)| \asymp |x|^{\beta}$, $\beta \in (0,1) \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp e^{-a|x|^{\beta}} \times$ sth of smaller order

$\vdots$

$|\log \nu(x)| \asymp |x| \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp e^{-a|x|} \times$ sth of smaller order

Intrinsic ultracontractivity-type properties

Example (various types of borderline growths)

$|\log \nu(x)| \asymp \log |x| \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp |x|^{-d-\alpha}$, $\alpha > 0$

$\vdots$

$|\log \nu(x)| \asymp |x|^{\beta}$, $\beta \in (0,1) \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp e^{-a|x|^{\beta}} \times$ sth of smaller order

$\vdots$

$|\log \nu(x)| \asymp |x| \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp e^{-a|x|} \times$ sth of smaller order

Previous works:
Relativistic α-stable processes ($\alpha \in (0,2)$): Kulczycki–Siudeja TAMS 2006 (V–nonnegative, locally bounded, comparable to rotationally symmetric, radially nondecreasing function comparable on unit balls:
IUC $\Leftrightarrow \lim_{|x| \to \infty} \frac{V(x)}{|x|} = \infty$)
Symmetric α-stable processes ($\alpha \in (0,2)$): Kaleta–Kulczycki PA 2010, Kaleta–Lőrinczi SPA 2012

Intrinsic ultracontractivity-type properties

Example (various types of borderline growths)

$|\log \nu(x)| \asymp \log |x| \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp |x|^{-d-\alpha}$, $\alpha > 0$

$\vdots$

$|\log \nu(x)| \asymp |x|^{\beta}$, $\beta \in (0,1) \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp e^{-a|x|^{\beta}} \times$ sth of smaller order

$\vdots$

$|\log \nu(x)| \asymp |x| \quad \Leftarrow \quad$ e.g. $\nu(x) \asymp e^{-a|x|} \times$ sth of smaller order

Previous works:
Relativistic α-stable processes ($\alpha \in (0,2)$): Kulczycki–Siudeja TAMS 2006 (V–nonnegative, locally bounded, comparable to rotationally symmetric, radially nondecreasing function comparable on unit balls:
IUC $\Leftrightarrow \lim_{|x| \to \infty} \frac{V(x)}{|x|} = \infty$)
Symmetric α-stable processes ($\alpha \in (0,2)$): Kaleta–Kulczycki PA 2010, Kaleta–Lőrinczi SPA 2012

Remark

Brownian component and small jump part have no effect on AGSD/AIUC!

Methods

Our arguments combine:

(1) bounds on local extrema of functions that are harmonic with respect to subprocess of X_t obtained under the Feynman-Kac functional: if $f(z) = \mathbf{E}^z[\exp(-\int_0^{\tau_{B(x,1)}} V(X_s)ds) f(X_{\tau_{B(x,1)}})]$ for $x \in B(x,1)$ and $f, V \geq 0$ on $B(x,1)$, then

$$f(z) \stackrel{C_X}{\asymp} G^V_{B(x,1)}\mathbf{1}(z) \int_{B(x,3/4)^c} f(y)\nu(y)dy, \quad z \in B(x,1/2)$$

K. Bogdan, T. Kumagai, M. Kwaśnicki: *Boundary Harnack inequality for Markov processes with jumps*, preprint, 2012

Methods

Our arguments combine:

(1) bounds on local extrema of functions that are harmonic with respect to subprocess of X_t obtained under the Feynman-Kac functional: if $f(z) = \mathbf{E}^z[\exp(-\int_0^{\tau_{B(x,1)}} V(X_s)ds) f(X_{\tau_{B(x,1)}})]$ for $x \in B(x,1)$ and $f, V \geq 0$ on $B(x,1)$, then

$$f(z) \stackrel{C_X}{\asymp} G^V_{B(x,1)}\mathbf{1}(z) \int_{B(x,3/4)^c} f(y)\nu(y)dy, \quad z \in B(x,1/2)$$

K. Bogdan, T. Kumagai, M. Kwaśnicki: *Boundary Harnack inequality for Markov processes with jumps*, preprint, 2012

(2) sufficiently powerful estimates of 'jumps of the process' between carefully chosen annular regions and a use of the estimation scheme devised in

K. Burdzy, T. Kulczycki: *Stable processes have thorns*, Ann. Probab. 31, 2003, 171-194.

T. Kulczycki, B. Siudeja: *Intrinsic ultracontractivity of the Feynman-Kac semigroup for the relativistic stable process*, Trans. Amer. Math. Soc. 358 (11), 2006, 5025-5057

Intrinsic ultracontractivity and ground state estimates

This talk is based on:

K. Kaleta, J. Lőrinczi: *Pointwise eigenfunction estimates and intrinsic ultracontractivity-type properties of Feynman-Kac semigroups for a class of Lévy processes*, preprint, 2012

Intrinsic ultracontractivity and ground state estimates

This talk is based on:

- K. Kaleta, J. Lőrinczi: *Pointwise eigenfunction estimates and intrinsic ultracontractivity-type properties of Feynman-Kac semigroups for a class of Lévy processes*, preprint, 2012

Previous works:

- K. Kaleta, J. Lőrinczi: *Fractional $P(\phi)_1$-processes and Gibbs measures*, Stochastic Process. Appl. 122, 2012, 3580-3617
- K. Kaleta, T. Kulczycki: *Intrinsic ultracontractivity for Schrödinger operators based on fractional Laplacians*, Potential Anal. 33 (4), 2010, 313-339
- M. Kwaśnicki: *Intrinsic ultracontractivity for stable semigroups on unbounded open sets*, Potential Anal. 31 (1), 2009, 57-77
- T. Kulczycki, B. Siudeja: *Intrinsic ultracontractivity of the Feynman-Kac semigroup for the relativistic stable process*, Trans. Amer. Math. Soc. 358 (11), 2006, 5025-5057

Thank you!

kamil.kaleta@pwr.wroc.pl