Abstracts

Brasilian-Polish Topology Workshop

Toruń: July 09-13, 2012

Warsaw: July 16-20, 2012

Toruń: July 09-13, 2012

Weitzenboeck formula on Lie algebroids for pairs of connections

Bogdan Balcerzak (Technical University, Łódź)

The Weitzenböck formula states in general a relation between two operators: the Bochner Laplacian and the Beltrami Laplacian (both acting on exterior differential forms). In the talk we will discuss the Weitzenböck formula on Lie algebroids. The presented formula generalizes some recent result of [1] to the case of exterior differential forms on Lie algebroids with values in an arbitrary vector bundle. It generalizes and unifies versions for operators acting on differential forms in the case of Riemannian manifolds. The formula contains here a new summand: a zero order operator acting on forms and reflecting non-compatibility of some connection with the metric structure.

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Short exact sequences and transformation groups of homotopy surfaces

Zbigniew Błaszczyk (N. Copernicus University, Toruń, Poland)

Let M be a closed surface other than the 2-sphere and the projective plane. Kensô Fujii observed that a finite group G acts freely on M if and only if there exists a short exact sequence of groups

$$1 \to \pi_1(M) \to \pi_1(N) \to G \to 1,$$

where N is a closed surface such that $\chi(M) = |G| \cdot \chi(N)$. We will discuss an extension of this result to the case of properly discontinuous actions of arbitrary groups on homotopy surfaces, i.e., on finite-dimensional CW complexes homotopy equivalent to surfaces.

Higher track categories & derived functors and the Adams spectral sequence

David Blanc (University of Haifa, Israel) **(speaker)**
Hans Baues and Simona Paoli

The E^2-terms of many spectral sequences commonly used in homotopy theory can be identified as derived functors - for example, *Ext* over the Steenord algebra, for the Adams spectral sequence. The advantage of such an identification is that it allows one to use a convenient (not necessarily canonical) algebraic resolution to to calculate this E^2 term.

We explain how the higher terms in such spectral sequences can be identified as certain “higher order derived functors”. These provide an additional benefit: one can actually vary the category in which the resolutions are constructed. For this purpose, we introduce the notion of a higher track category, and explain how it can be applied in our context.

On the topological Kolmogorov property

Wojciech Bułatek (N. Copernicus University, Toruń)

The object of interest of topological dynamics is a topological dynamical system i.e., a pair (X, T), where X is a compact metric space and $T : X \longrightarrow X$ is a homeomprphism. Among other properties, there is investigated so called K-property - the topological analogue of the classical Kolmogorov property of a measure-theoretical dynamical system which implies extrimely chaotic behaviour of a system. There is presented an equivalent condition for K-property in the case of subshift i.e., T is shift on $\mathcal{A}^{\mathcal{Z}}$, $\mathcal{A}$ - finite, and $X \subset \mathcal{A}^{\mathcal{Z}}$ a T-invariant subset. By means of this condition the K-property is examined for two classical subhifts, the Chacon system and the Petersen system.

Large scale absolute extensors

Jerzy Dydak (University of Tennessee, Knoxville, TN 37996, USA)

We define the concept of a bounded metric space K to be large scale extensor of a metric space X if for every $a > 0$ there is $b > 0$ with the property that any (a, a)- Lipschitz map $f : A \to K$, A a subset of X, extends to a (b, b)-Lipschitz function $F : X \to K$.

Our main result is: *If the asymptotic dimension of X is finite then it equals the smallest n such that the n-sphere is a large scale extensor of X.*

Homotopy dominations of polyhedra- new results and open problems

Danuta Kołodziejczyk (Technical University, Warsaw)

According to my previous work (1996, 2003), there exist finite polyhedra homotopy dominating infnitely many different homotopy types (even polyhedral), which answers a question of K. Borsuk (1968). There are some other natural, but still unsolved problems of the similar nature. For example, an analog to the above for decompositions of finite polyhedra into Cartesian factors. The first examples of finite polyhedra with different homotopy decompositions into a product of two factors (even polyhedral) were published in the sixties and seventies by P. Hilton, J. Roitberg, A. Sieradski, L. Charlap, E. Conner and F. Raymond. The question, if there exists a finite polyhedron P such that $P \simeq X_i \times S^1$, for infinitely many X_i of different homotopy types, is also unanswered. We will prove that for some classes of polyhedra P, including those with nilpotent fundamental groups, the answer to this question is negative. (It should be noted that in 1987 P. Hilton published the example of a finite polyhedron with nilpotent fundamental group and two different homotopy decompositions into Cartesian product with factor S^1). Similar problems on "cancellation" in various categories were studied by many authors. We will present many open problems related to the subject. The talk will be continued in Warsaw on the Workshop on Algebraic and Geometric Topology.

Koszul complexes and Chevalley's theorems for Lie algebroids

Jan Kubarski (Technical University, Łódź)

The Chevalley theorems anable (for principal fibre bundles with compact connected structural Lie groups) to calculate the cohomology of the total space of a principal fibre bundle via cohomology of the base manifold and the Chern-Weil homomorphism. To aim this, the so-called Koszul complex and the Chevalley homomorphism is constructed. The Lie algebroid nature of this notions will be showed and next the Lie algebroid version (under some assumptions) of the Koszul complex and the Chevalley homomorphism into the algebra of Lie algebroids differential forms will be presented. Passing to the cohomology we obtain our isomorphism.

Fractal homeomorphisms

Krzysztof Leśniak (N. Copernicus University, Toruń)

An absolute retract (AR), introduced by K. Borsuk, generalizes the notion of polyhedron. In the same vein one can look at attractors of iterated function systems (IFS) as AR-s modeled on the spaces with a tree structure instead of the convex one. Homeomorphisms between attractors defined via the access to a tree structure are the fractal homeomorphisms appearing in the title. The idea is due to M. Barnsley.

Homotopy type of path components of certain map spaces

Thiago de Melo (Unesp Rio Claro, Brazil)
Marek Golasiński (Nicolaus Copernicus University, Poland)

Let $M(\mathbb{S}^m, \mathbb{S}^n)$ be the space of maps from the m-sphere $\mathbb{S}^m$ into the n-sphere $\mathbb{S}^n$ with $m, n \geq 1$. We estimate the number of homotopy types of path-components $M_\alpha(\mathbb{S}^{n+k}, \mathbb{S}^n)$ for $\alpha \in \pi_{n+k}(\mathbb{S}^n)$ and fibre homotopy types of evaluation fibrations $\omega_\alpha : M_\alpha(\mathbb{S}^{n+k}, \mathbb{S}^n) \to \mathbb{S}^n$ for $8 \leq k \leq 13$, extending the results of [1]. Further, the number of strongly homotopy types of $\omega_\alpha : M_\alpha(\mathbb{S}^{n+k}, \mathbb{S}^n) \to \mathbb{S}^n$ for $8 \leq k \leq 13$ is determined and some improvements for the results of [1] are obtained.

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Fixed Points on trivial Surface Bundles

Alice Kimie Miwa Libardi (Unesp Rio Claro, Brazil) **(speaker)**
Daciberg Lima Gonçalves (IME-USP São Paulo, Brazil)
Dirceu Penteado (UFSCar São Carlos, Brazil)
João Peres Vieira (Unesp Rio Claro, Brazil) **(speaker)**

Given a fibration $E \to B$ and $f : E \to E$ a fiber-preserving map over B, the question if f can be deformed over B (by a fiberwise homotopy) to a fixed point free map has been considered for several years by many authors. Among others, see for example [Do-74], [F-H-81], [Go-87], [G-P-V-04], [G-P-V-09I], [G-P-V-09II] and [G-P-V-10]. In [F-H-81], Fadell, E. and Husseini, S. showed that the above problem can be stated in terms of obstructions (including higher ones). This is done under the hypothesis that the base space, the total space and the fiber F are manifolds, and dimension of F is greater or equal to 3. The case where the fiber has dimension 2 the classical obstruction theory can not be used. This case, even when the base space is a point, is still a main open problem at least if the total space (which is a surface) has negative Euler characteristic.

Let us consider a fiber-preserving map $h : M \to M$, where M is the trivial bundle $B \times S$ over the base B. When the fiber is either RP^2 or S^2 we can apply the obstruction theory. For S the torus and $B = S^1$ the result had been obtained by other methods in [G-P-V-04]. For S the Klein bottle and $B = S^1$ the results also were obtained by other methods in [G-P-V-09II].

In general S fiber bundles over S^1 are obtained from the space $S \times [0, 1]$ by identifying the points $(x, 0)$ with the points $(\phi(x), 1)$, where ϕ is a self-homeomorphism of the surface S. In [G-P-V-04] and [G-P-V-09II] few generalities has been developed for such bundles and the problem of deforming a fiber map by a fiberwise homotopy to a fixed point free map has been completely solved.

The cases where the fiber is either the sphere S^2 or the projective plane RP^2 have different features. For S^2-bundles over S^1 see [G-P-V-10].

In this work we consider the study of this question in the case when the fiber of the fibration is a surface S with $\chi(S) < 0$.

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Displacement sequence of an orientation preserving circle homeomorphism

W. Marzantowicz (Adam Mickiewicz University of Poznań, Poland)
J. Signerska (Gdańsk University of Technology, Poland)

We describe the behaviour of the sequence of displacements $\eta_n(z) = \Phi^n(x) - \Phi^{n-1}(x) \bmod 1$, $z = \exp(2\pi i x)$, along a trajectory $\{\varphi_n(z)\}$, where φ is an orientation preserving circle homeomorphism and $\Phi : R \to R$ its lift. If the rotation number $\varrho(\varphi) = \frac{p}{q}$ is rational then $\eta_n(z)$ is asymptotically periodic with frequency q. This convergence to a periodic sequence is uniform in z if we admit iterations forward or backward. If $\rho(\varphi) \notin \mathbb{Q}$ then the values of $\eta_n(z)$ are dense in a set which depends on the map γ (semi-)conjugating φ with the rotation by $\varrho(\varphi)$ and which is the support of the displacements distribution, analyzed further in the paper.

Remarks on Lefschetz coincidence class

Carlos Biasi (ICMC-USP São Carlos, Brazil)
Thaís Fernanda Mendes Monis (Unesp Rio Claro, Brazil) **(speaker)**

Given two maps $f, g : X \to Y$, it is defined the set of coincidences between f and g by

$$\mathrm{Coin}(f, g) = \{x \in X \mid f(x) = g(x)\}.$$

If $\mathrm{Coin}(f, g) = \emptyset$ then the induced homomorphism

$$[i \circ (f, g)]^* : H^*(Y \times Y, Y \times Y - \Delta) \to H^*(X)$$

is null, where $i : Y \times Y \to (Y \times Y, Y \times Y - \Delta)$ is the inclusion map.

Let Y be a connected, closed and oriented manifold of dimension n, and let $\mu \in \mathrm{H}^n(Y \times Y, Y \times Y - \Delta)$ be the Thom class of Y. The Lefschetz coincidence class for (f, g) is defined by

$$L(f, g) = [i \circ (f, g)]^*(\mu) \in H^n(X).$$

If X is also a connected closed oriented n-manifold, denoting by $\zeta \in \mathrm{H}_n(X)$ its fundamental class, it is well known that

$$\Lambda(f, g) = \sum_i (-1)^i \mathrm{trace}_i(f^* g^!) = [\zeta, L(f, g)] \tag{1}$$

where [ ] is the Kronecker product.

In this work, even if X is not a manifold, we obtain an expression to the class $L(f, g)$, similar to equation (1), under the hypothesis that, for each $0 < q < n$, there exists a homomorphism $\varphi_q : \mathrm{H}^q(Y) \to \mathrm{H}^q(Y)$ such that $f_q^* = g_q^* \circ \varphi_q$.

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On the existence of G-equivariant maps

Denise de Mattos (University of São Paulo - ICMC - USP - São Carlos - SP - Brazil) **(speaker)**
Edivaldo L. dos Santos and Francielle R. C. Coelho, SP - Brazil)

Let X, Y be spaces equipped with free G-actions, where G is a compact Lie group. Then, it makes sense to ask for the existence of equivariant maps $f : X \to Y$. For example, one formulation of the Borsuk-Ulam theorem is that there is no map from $S^m \to S^n$ equivariant with respect to the antipodal action, when $m > n$. In this work, by using a numerical index $i(X)$, defined in terms of the Leray Serre spectral sequence of the Borel fibration $p : (EG \times X)/G \to BG$, we obtain some Borsuk-Ulam results concerning the existence of G-equivariant maps $X \to Y$.

Algebraic models of mapping spaces

Aniceto Murillo (Malaga University, Spain)

I will describe precise and explicit objects in different algebraic categories describing the rational homotopy type of mapping spaces. Applications will be also presented.

On the Lusternik-Schnirelmann category approach to topological complexity

Petar Pavesic, Ljubliana University, Slovenia

The navigation problem (or motion planning problem) is a term used in robotics to describe the problem of determining the movement of a mechanical system (e.g. a mobile robot) needed to perform a prescribed task. In topological robotics one treats the internal states of the robot as points in certain configuration space. Then the problem is to find rules that take as input the pair of points corresponding to the the initial and the final state of the system, and give as the output a path in the configuration space connecting the given points. It turns out that, only the simplest navigational problems can be solved by just one rule that depend continuously on the input, while in general one needs several rules to handle all possible inputs.

In 2003 M. Farber defined the topological complexity of a configuration space as the minimal number of continuous rules needed to solve a given navigation problem.

We will start our talk with an introductory exposition of the topological complexity and give some basic results and computations, due to M. Farber and others. In the second part we will present some recent work (joint with A. Franc) on topological complexity which uses methods from fibrewise topology and Lusternik-Schnirelmann category.

Čech cohomology of attractors of discrete dynamical systems

Francisco Romero Ruiz del Portal (Universidad Complutense de Madrid, Spain) **(speaker)**
Jaime Jorge Snchez-Gabites

Let K be a compact attractor of a flow and $A(K)$ its basin of attraction. There are many papers in the literature relating the homotopy properties that $A(K)$ inherits in K. Since K may have a very complicate topological structure, the homotopy theory which is adequate to study this problem is shape theory. If the flow is defined in a nice space, the main conclusions are that the inclusion of K in $A(K)$ is a shape equivalence and that K has the shape of a finite polyhedron. The proofs use consistently the homotopies that a flow provides for free.

In the case of discrete dynamical systems very few results are known about the topological relationships between K and $A(K)$ and in the known results the absence of homotopies which flows provide, forced the authors to impose strong conditions on the homeomorphism, say f, or in the attractor. This conditions are not useful in practice because *a priori* one does not know how strange our attractor can be. One of the properties which allows to obtain results, but difficult to check in advance, is some kind of movability of K. Our main idea is to consider Čech (co)-homology with coefficients in $\mathbb{Q}$ or $\mathbb{Z}/p$ for all primes p to by pass the absence of movability. and then we will be able to prove under very mild conditions that the inclusion of K in $A(K)$ induces isomorphisms in Čech (co)-homology. On the other hand, in the 3-dimensional case, if f contracts volume and $Per(f) \cap K = \emptyset$ we compute completely these groups and we apply them to the study of attractors of periodic differential equations in $\mathbb{R}^3$.

Ważewski retract method and its time scale version

Sebastian Ruszkowski (N. Copernicus University in Toruń)

We will start with a statement of the Ważewski theorem, in which by checking the behavior of the flow (or the process) at the boundary of a set we can confirm existence of trajectory, that is remaining in this set. After a brief introduction to time scales (a unification of the theory of difference equations with that of differential equations), we will deal with Ważewski retract method for processes on time scales.

Coincidence theorems for maps of free $\mathbb{Z}_p$-spaces

Edivaldo L. dos Santos (Federal University of São Carlos - UFSCAR - SP - Brazil) **(speaker)**
Francielle R. C. Coelho (Federal University of Uberlândia - UFU - SP - Brazil)

The classic Borsuk-Ulam theorem says that every map from the sphere S^n into the euclidean k-dimensional space $\mathbb{R}^k$ has an antipodal coincidence, if $n \geq k$. This result can be generalized in many ways: S^n and $\mathbb{R}^k$ can be replaced by more general spaces X and Y, and the antipodal action $\mathbb{Z}_2$ on S^n can be replaced by actions of others groups. In one of these generalizations Aarts, Fokkink and Vermeer [1, Theorem 1] proved that if $i : X \to X$ is a fixed-point free involution of a normal space X with color number $n+2$ and k is a natural number, then for every k-dimensional cone CW-complex Y and every continuous map $\varphi : X \to Y$ there is a $\mathbb{Z}_2$-coincidence, whenever $n \geq 2k$; and this result is the best possible. Let us observe that for $X = S^n$, the result was obtained independently by Shchepin in [5].

In this work, requiring that X is a Hausdorff paracompact space, we generalized the Aarts, Fokkink and Vermeer's result for free $\mathbb{Z}_p$-actions, with p prime. Specifically, we prove the following

Theorem. *Let X be a Hausdorff paracompact space equipped with a free $\mathbb{Z}_p$-action generated by $\alpha : X \to X$ such that* $\mathrm{gen}(X, \mathbb{Z}_p) \geq n+1$ *and let k be a natural number. Then the following hold:*

(a) *If $n > p\,k$, then for every k-dimensional metrizable space Y and every continuous map $f : X \to Y$ there is a $\mathbb{Z}_p$-coincidence point, i.e., there is $x \in X$ such that $f(x) = f(\alpha^i(x))$, $\forall\ i \in \{1, 2, \ldots, p-1\}$.*

(b) *If $n = p\,k$, then for every k-dimensional cone* CW*-complex Y and for every continuous map $f : X \to Y$ there is a $\mathbb{Z}_p$-coincidence point.*

(c) *If $n < p\,k$ and $\mathrm{gen}\,(X, \mathbb{Z}_p) = n+1$, then there exists a k-dimensional cone* CW*-complex Y and a continuous map $f : X \to Y$ such that f has no $\mathbb{Z}_p$-coincidence points.*

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S^1-invariant vector fields on manifolds with semi-free circle action

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Let W^{2n} be a compact closed manifold of dimension with a semi-free circle action which has finitely many fixed points. We study the S^1-invariant Morse-Smale vector fields on W^{2n}. The aim of this report is to describe exact values of minimal numbers of closed orbits of some indices of S^1-invariant Morse-Smale vector field on W^{2n}.

Definition 1. A smooth S^1-invariant vector field V on $W^n$is called an S^1-invariant Morse-Smale if:

i) each connected component of the nonwandering set of V is either a non degenerate fixed point or a non degenerate circle and

ii) the stable and unstable manifolds of critical elements of V are S^1-transversal.

Definition 2. Let V be an S^1-invariant Morse-Smale vector field on W^{2n} for smooth semi-free circle action with isolated fixed points $p_1, \ldots, p_{2k}$ on W^{2n}. Suppose that the index of a singular point p_i of f is λ_i. **The state of** V is the collection of numbers $\lambda_1, \lambda_2, \ldots, \lambda_{2k}$, which we will be denoted by $St_V(\lambda_i)$.

Theorem 1. *For every smooth semi-free circle action on W^{2n} with fixed points $p_1, \ldots, p_{2k}$ and any collection even numbers $\lambda_1, \lambda_2, \ldots, \lambda_{2k}$, such that $0 \leq \lambda_i \leq 2n$ there exists an S^1-invariant Morse-Smale vector field V on W^{2n} with state $St_V(\lambda_i)$.*

Definition 3. Let W^{2n} be a closed smooth manifold with smooth semi-free circle action which has finitely many fixed points. The S^1**-equivariant Morse number** $\mathcal{M}^{\nu}_{S^1}(W^{2n}, St(\lambda_i))$ **of index** ν **of a state** $St(\lambda_i)$ of W^{2n} is the minimum number of circles of index ν taken over all S^1-invariant Morse-Smale vector fields on W^{2n} with state $St(\lambda_i)$.

Theorem 2. *Let W^{2n} $(2n > 6)$ be a closed smooth manifold admits a smooth semi-free circle action with isolated fixed points $p_1, \dots, p_{2k}$. Then for W^{2n} with the state $St(\underbrace{0, \dots, 0}_{r}, \underbrace{2n, \dots, 2n}_{2k-r}) = St(0, r, 2n)$*

$$\mathcal{M}^{\lambda}_{S^1}(W^{2n}, St(0, r, 2n)) = \mathbb{D}^{\lambda}(W^{2n}/S^1, p_1, \dots, p_r) + \widehat{S}^{\lambda}_{(2)}(W^{2n}/S^1 p_1, \dots, p_r) +$$
$$+ \widehat{S}^{\lambda+1}_{(2)}(W^{2n}/S^1, p_1, \dots, p_r) + \dim_{N(Z[\pi])}(H^{\lambda}_{(2)}(W^{2n}/S^1, p_1, \dots, p_r, \mathbb{Z})),$$

for $3 \leq \lambda \leq 2n - 4$.

Homotopy invariants for set-valued maps

Robert Skiba (N. Copernicus University in Toruń)

In my talk I am going to present two functors from algebraic topology which are not very well known. More exactly, I will introduce the weighted homotopy functor and the Darbo homology functor. Some applications of the above functors in the fixed point theory will be discussed.

Warsaw: July 16-20, 2012

On the non-existence of free actions of alternating groups on products of spheres

Zbigniew Błaszczyk (N. Copernicus University, Toruń, Poland)

We will discuss the background and recent developments of the problem of existence of free actions of alternating groups on products of equidimensional spheres. Most notably, we will see that the alternating group $\mathcal{A}_d$ cannot act freely on any finite-dimensional CW-complex with the integral cohomology ring isomorphic to that of $(S^n)^{d-2}$ for any positive integer n, which extends previous results due to B. Oliver and L.P. Plakhta.

Reidemiester number of nilpotent groups finitely and infinitely generated

Daciberg Lima Gonçalves (São Paulo University, Brasil)

Invariants on coincidences of fiberwise maps

Alice Kimie Miwa Libardi (Unesp Rio Claro, Brazil)
Joint work with Daciberg L.Gonçalves, Ulrich Koschorke and Oziride M. Neto

Let $M \to S^1$ and $N \to S^1$ be fibrations and let $f_1, f_2 : M \to N$ be a pair of fiber preserving maps. The question which arises is when f_1 and f_2 can be deformed in a fiberwise fashion until they are coincidence free? If this can be done we say that the pair (f_1, f_2) is *loose over* S^1. This generalizes a problem which arises very naturally in fixed point theory, where $M = N$ and f_2 is the identity map id.

In a very general setting the looseness obstruction

$$\omega_B(f_1, f_2) \in \Omega_{m-n+1}(M; \varphi)$$

was introduced by Gonçalves and Koschorke. This normal bordism class depends only on the fiberwise homotopy classes of f_1 and f_2 and vanishes for loose pairs. It reflects important geometric aspects of a generic coincidence submanifold $C(f_1; f_2)$ of M.

We consider this invariant and the map

$$deg_B : \mathcal{F} \to \Omega_{m-n+1}(M; \varphi)$$

which sends $[f]$ to $\omega_B(f, a \circ f)$ (here a denotes the antipodal map) and we will show some conditions on deg_B to be equivalent to the pair (f_1, f_2) be loose over S^1.

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Local Nash equilibrium and the Lefschetz-Hopf fixed point theorem

Carlos Biasi (ICMC-USP São Carlos, Brazil)
Thaís Fernanda Mendes Monis (Unesp Rio Claro, Brazil) **(speaker)**

Based on the definition of Nash equilibrium, we established the following concept of local equilibrium.

Definition 4. *Let $(S_1, d_1), \ldots, (S_n, d_n)$ be metric spaces and $p_1, \ldots, p_n : S_1 \times \cdots \times S_n \to \mathbb{R}$ real functions. We say that $\tilde{s} = (\tilde{s}_1, \ldots, \tilde{s}_n) \in S$ is a* **weak local equilibrium (abbrev., w.l.e.)** *for $p_1, \ldots, p_n$ if for all $\varepsilon > 0$ there exists $\delta > 0$ such that*

$$p_i(\tilde{s}_1, \ldots, \tilde{s}_{i-1}, s_i, \tilde{s}_{i+1}, \ldots, \tilde{s}_n) \leq p_i(\tilde{s}) + \varepsilon d_i(s_i, \tilde{s}_i),$$

for every $s_i \in B(\tilde{s}_i, \delta)$, $1 \leq i \leq n$, where $B(\tilde{s}_i, \varepsilon)$ is the open ball in S_i with center in $\tilde{s}_i$ and radius ε.

The classical theorem on existence of equilibrium points is an application of the Brouwer fixed point theorem and it asserts that if each space of strategies $S_i \subset \mathbb{R}^{m_i}$ is compact and convex and each payoff function $p_i : S \to \mathbb{R}$ is continuous as a function of n variables, and $p_i(s_1, \ldots, s_n)$ is linear as a function of s_i when the other variables are kept fixed, then there exists at least one Nash equilibrium point.

In this work, applying the Lefschetz-Hopf fixed point theorem, we prove the existence of the weak local equilibrium.

References

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Path components of map spaces

Thiago de Melo (Unesp Rio Claro, Brazil) **(speaker)**
Marek Golasiński (Nicolaus Copernicus University, Poland)

Let $M(\mathbb{S}^m, \mathbb{S}^n)$ be the space of maps from the m-sphere $\mathbb{S}^m$ into the n-sphere $\mathbb{S}^n$ with $m, n \geq 1$. We estimate the number of homotopy types of path-components $M_\alpha(\mathbb{S}^{n+k}, \mathbb{S}^n)$ for $\alpha \in \pi_{n+k}(\mathbb{S}^n)$ and fibre homotopy types of evaluation fibrations $\omega_\alpha : M_\alpha(\mathbb{S}^{n+k}, \mathbb{S}^n) \to \mathbb{S}^n$ for $8 \leq k \leq 13$, extending the results of [1]. Further, the number of strongly homotopy types of $\omega_\alpha : M_\alpha(\mathbb{S}^{n+k}, \mathbb{S}^n) \to \mathbb{S}^n$ for $8 \leq k \leq 13$ is determined and some improvements for the results of [1] are obtained.

References

[1] M. Golasiński, *Evaluation fibrations and path-components of the map space $M(\mathbb{S}^{n+k}, \mathbb{S}^n)$ for $0 \leq k \leq 7$*, Mat. Stud. **31 (2)** (2009), 189–194.

Coincidence of maps on Torus Fiber Bundles over the Circle

João Peres Vieira (Unesp Rio Claro, Brazil)

Let $T \to M \xrightarrow{p} S^1$ be a torus bundle over S^1. In this work we study the following question: given a pair of fiber-preserving maps over S^1 when can it be deformed by a fiberwise homotopy over S^1 into a pair of fiber-preserving maps over S^1, coincidence free? Answering this question is equivalent to study the existence of a section in a geometric diagram or equivalently, to study the existence of a lifting involving the fundamental groups of the spaces M, $M \times_{S^1} M$ and $M \times_{S^1} M - \Delta$, where $M \times_{S^1} M$ is the pullback of $p : M \to S^1$ by $p : M \to S^1$ and Δ is the diagonal in $M \times_{S^1} M$. The set of the homotopy classes of pairs (f, g) over S^1 such that (f, g) restricted to the fiber T can be deformed into a pair of maps coincidence free has been determined. We present a formulation for the problem using a system of equations involving presentations of the given groups and then we decide if we can deform or not (f, g) to a pair of maps coincidence free.

Parametrized Borsuk-Ulam theorems for spaces of type (a, b)

Denise de Mattos (University of São Paulo - ICMC - USP - São Carlos - SP -Brazil) **(speaker)**
Pedro Luiz Queiroz Pergher and Edivaldo L. dos Santos (Federal University of São Carlos - UFSCAR - São Carlos - SP - Brazil)

Let X be a simply connected finite CW complex, whose cohomology groups satisfy $H^j(X; \mathbb{Z}) = \mathbb{Z}$, if $j = 0$, n, $2n$ or $3n$, with $n > 1$, and $H^j(X; \mathbb{Z}) = 0$, otherwise. Let u_i generator of $H^{in}(X; \mathbb{Z})$, for $i = 0, 1, 2$ and 3. We say that X has type (a, b), for integers a and b, if $u_1^2 = au_2$ and $u_1 u_2 = bu_3$.

In this work, using recent results of [1] on the structure of the G-cohomology ring of the orbit space of a free action of G on a space of type (a, b), where $G = \mathbb{Z}_2$ or $G = S^1$, we prove parametrized Borsuk-Ulam theorems for bundles whose fibre is a space of type (a, b) with a free G-action.

References

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[2] De Mattos, D. and Dos Santos, E. L. *A parametrized Borsuk-Ulam theorem for a product of spheres with free $\mathbb{Z}_p$-action and free S^1-action* Algebr. Geom. Topol. 7 (2007) 1791-1804.

Bourgin-Yang version of the Borsuk-Ulam theorem for Z_{p^k}

Edivaldo L. dos Santos (Federal University of São Carlos - UFSCAR - SP - Brazil) **(speaker)**
Wacław Marzantowicz and Denise de Mattos

Let $G = Z_{p^k}$ be a cyclic group of prime power order. We give an estimate of the dimension of counter-image of $\{0\}$ for a Z_{p^k}-equivariant mapping from a sphere $S(V)$ of a orthogonal representation V into another orthogonal representation W of G in terms of V and W. It extends the Bourgin-Yang version of the Borsuk-Ulam theorem onto this class of groups. As a consequence, we estimate the size of the Z_{p^k}-coincidences set of a continuous map from $S(V)$ into W.

Non-commutative Geometry and Morse theory

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Let W^n be a smooth manifold. By definition the i-th Morse number $\mathcal{M}_i(W^n)$ of W^n is the minimal number of critical points of index i taken over all Morse functions on W^n.

It is known that for closed smooth manifolds of dimension greater than 6 the i-th Morse numbers are invariants of the homotopy type. There is unsolved problem: **find exact values of Morse numbers for every** i.

Using new homotopy invariants $\mathbb{D}^i(W^n)$ of free cochain complexes and Hilbert complexes of non simply-connected manifolds W^n we proved the following theorem.

Theorem 3. *Let W^n $(n \geq 6)$ be a smooth closed manifold with $\pi = \pi_1(W^n)$. Then for $2 \leq i \leq n-2$ the following equality holds true:*

$$\mathcal{M}_i(W^n) = \mathbb{D}^i(W^n) + \widehat{S}^i_{(2)}(W^n) + \widehat{S}^{i+1}_{(2)}(W^n) + \dim_{N(Z[\pi])}(H^i_{(2)}(W^n, \mathbb{Z}))\,.$$

For cobordism $(W^n, V_0^{n-1}, V_1^{n-1})$ $(n \geq 6)$ situation is more complicated [1] and new results in this direction shall be describe in the talk.

References

[1] V. Sharko, Morse functions on cobordisms. Ukrainian Math. Journal, 2011, vol. 63, no. 1, pp. 282-294.

Movability in dynamics

Krystyna Kuperberg (Auburn University, Alabama, USA)

HNBH Menger-McCord solenoids

Krystyna Kuperberg (Auburn University, Alabama, USA)

Homotopy dominations of polyhedra - new results and open problems, part II

Danuta Kołodziejczyk (Technical University in Warsaw, Poland))

Large scale absolute extensors

Jerzy Dydak (University of Tennessee, USA))

We define the concept of a bounded metric space K to be large scale extensor of a metric space X if for every $a > 0$ there is $b > 0$ with the property that any (a,a)-Lipschitz map f:A to K, A a subset of X, extends to a (b,b)-Lipschitz function F:X to K. Ou main result is: If the asymptotic dimension of X is finite then it equals the smallest n such that the n-sphere is a large scale extensor of X.

As an application we get a simple proof of a result of Dranishnikov connecting the asymptotic dimension to the dimension of Higson corona.

Coarse homology theories

Piotr Nowak (Warsaw University, Poland)

Coarse homology was first defined by Roe in the early nineties and has found several applications in geometry and topology. We will survey the large scale geometric setting and early constructions of coarse homology theories of Roe, Block-Weinberger, as well define a more recent controlled coarse homology theory (N.-Spakula). We will also present several applications to group theory and geometry of manifolds.

The fixed point property of the simplest uniquely arcwise connected continua and their cylinders

Mirosław Sobolewski (Warsaw University, Poland)

A *continuum* is a metric compact connected space. A continuum is called *uniquely arcwise connected* if it is arcwise connected and does not contain any simple closed curve. A continuum is *chainable* if it can be represented as an inverse limit of arcs. A continuum is *weakly chainable* if it is a continuous image of a chainable continuum. There are two results of P.Minc connecting weak chainability with fixed point: on one hand he shows that every nonseparating plane continuum with weakly chainable boundary has the fixed point property; on the other hand he constructs a tree-like weakly chainable continuum without fixed point property. D. Bellamy asked whether uniquely arcwise connected continua when weakly chainable must have fixed point property We constructed a counterexample. (First example of a uniquely arcwise connected continuum without fixed point property belongs to G.S.Young). On the other hand we show that if a weakly chainable and uniquely arcwise connected continuum has only finite number of branching points (we call such continua *simplest*) then it has fixed point property. Using a theorem of R. Manka we can extend this result to products of such continua by compact ARs.

Mean dimension and and an sharp embedding result for extensions of aperiodic zero-dimensional systems

Yonatan Gutman, Institute of Mathematics, Polish Academy of Sciences)

Let (X, T) be a topological dynamical system (t.d.s). Denote by $\operatorname{edim}(X, T)$ the minimal $d \in \mathbb{N} \cup \{\infty\}$ such that $(X, T) \hookrightarrow (([0,1]^d)^{\mathbb{Z}}, shift)$. It is well known that in general $\operatorname{edim}(X, T) > 2\operatorname{mdim}(X, T)$, where $\operatorname{mdim}(X, T)$ denotes the mean-dimension of (X, T). E. Lindenstrauss conjectured that for aperiodic t.d.s this is the only obstruction. We establish this conjecture for extensions of aperiodic zero-dimensional t.d.s.Joint work with Masaki Tsukamoto.

Inverse problem for multiplicity; touch-search after a hidden set

Henryk Fast (Warsaw, Poland)

The problem consist in restoring a closed subset in R^n from the knowledge of numbers of its intersection points with half lines in space, perhaps not all, just having origins in a certain 'seed' set. Provided is an instrument for telling which points in space belong to the searched set and which do not.

Trees of manifolds with boundaries

Paweł Zawiślak (Warsaw University, Poland)

Trees of manifolds with boundaries are examples of trees of metric compacta, which were introduced by J. Swiatkowski. These spaces are analogues of trees of manifolds, spaces introduced and investigated by W. Jakobsche, which are inverse limits of certain inverse systems of closed manifolds. During my talk I will introduce the notion of trees of manifolds with boundaries and sketch proofs of its basic properties (uniquness, dimension etc.). I will also mention few conjectures concernig boundaries of non-posively curved groups.

Free and linear representations of $\mathrm{Out}(F_n)$

Dawid Kielak (University of Oxford)

The problem of understanding homomorphisms between outer automorphism groups of free groups of different ranks will be introduced. We will look into its connection with linear representation theory of $\mathrm{Out}(F_n)$ and survey known results in both areas.