GROUPS AND THEIR ACTIONS

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ABSTRACTS

Bicrossed descent theory of exact factorizations and the number of types of groups of finite order

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Let $A \leq G$ be a subgroup of a group G. A factorization A-form of G is a subgroup H of G such that $G = AH$ and $A \cap H = \{1\}$. Let $\mathcal{F}(A, G)$ be the category of all factorization A-forms of G and $\mathcal{F}^{sk}(A, G)$ its skeleton. The *bicrossed descent* problem asks for the description and classification of all factorization A-forms of G. We shall give the full answer to this problem in three steps. Let H be a given factorization A-form of G and $(\triangleright, \triangleleft)$ the canonical left/right actions associated to the factorization $G = AH$. In the first step H is deformed to a new A-form of G, denoted by H_r, using a certain map $r : H \to A$ called a descent map of the matched pair $(A, H, \triangleright, \triangleleft)$. Then the description of all forms is given: $\mathbb{H}$ is an A-form of G if and only if $\mathbb{H}$ is isomorphic to H_r, for some descent map $r : H \to A$. Finally, the classification of forms proves that there exists a bijection between $\mathcal{F}^{sk}(A, G)$ and a combinatorial object $\mathcal{D}\,(H, A \,|\, (\triangleright, \triangleleft))$. Let S_n be the symmetric group and C_n the cyclic group of order n. By applying the bicrossed descent theory for the factorization $S_n = S_{n-1}C_n$ we obtain the following: (1) any group H of order n is isomorphic to $(C_n)_r$, the r-deformation of the cyclic group C_n for some descent map $r : C_n \to S_{n-1}$ of the canonical matched pair $(S_{n-1}, C_n, \triangleright, \triangleleft)$ and (2) the number of types of isomorphisms of all groups of order n is equal to $|\,\mathcal{D}(C_n, S_{n-1} \,|\, (\triangleright, \triangleleft))\,|$.

Joint work with G. Militaru.

On the Commutativity Degree in Finite Moufang Loops

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Let A be a finite algebraic structure with at least one binary operation like as $''\cdot''$. Then, one may ask: *What is the probability that two (randomly chosen) elements of A commute (with respect to the operation $''\cdot''$)?* A formal answer is $Pr(A) = \frac{|\{(x,y)\in A^2 | xy=yx\}|}{|A^2|}$. For a finite group A it is proved that $Pr(A) = \frac{k(A)}{|A|}$, where, $k(A)$ is the number of conjugacy classes of A (see [3, 5, 4] for example). The computational results on $Pr(A)$ are mainly due to Gustafson [3] who shows that $Pr(A) \leqslant \frac{5}{8}$ for a finite non-abelian group A, and MacHale [5] who proves this inequality for a finite non-abelian ring. Also, the speaker of this talk and his colleagues have shown in [1] that the $\frac{5}{8}$ is not an upper bound for $Pr(A)$, where A is a finite non-abelian semigroup and/or monoid.

Now, let M be a finite non-abelian Moufang loop. In this talk, I will ask the same question for M and try to give a best upper bound for $Pr(M)$. Also, I will obtain some results related to the $Pr(M)$ and ask the similar questions raised in group theory about the relations of nilpotency of a finite group and its commutativity degree in finite Moufang loops.
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On simplicity of Lie ring of derivations in associative rings

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An additive map $d : R \to R$ is called a derivation of an associative ring R if $d(ab) = d(a)b + ad(b)$ for all $a, b \in R$. The set Der$R$ of all derivations of R is a Lie ring under the operations of pointwise addition and pointwise Lie multiplication.

The various aspects of a simplicity of derivations has been studied many times: N. Jacobson (1937), I.N. Herstein (1955), S.A. Amitsur (1957), R.E. Block (1969), D.A. Jordan (2000) and others.

Our aim is to present results on associative rings R with simple Lie rings Der$R$ of derivations.

A construction of almost morally self-similar groups

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I will give a construction of automorphism groups acting on rooted trees that are almost morally self-similar. This means that they contain a subgroup that is morally self-similar, i.e. they contain a direct product of a group that is generated using the same pattern but with different constants. It will be shown that these groups are not just-infinite but every proper normal subgroup is finitely generated. Using that they are branch groups it can be seen that every proper quotient is soluble. This will yield that groups of this type are not large. An argument using the non-solubility of the Grigorchuk group in fact shows that they do not contain any subgroups that map onto the free group of rank 2. Quoting technical results about the abelianisation of these groups I will prove that they have infinite virtual first Betti number. This addresses a conjecture coming from 3-manifolds stating that for finitely presented groups largeness is equivalent to infinite virtual first Betti number.

Branch Groups

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Branch groups are a class of infinite groups that are becoming increasingly important. Although they were only defined in the 1990's, they made their first appearance in the 1970's and 1980's as groups providing answers to a variety of questions such as Burnside's problem, questions on growth of groups and largeness of groups. Branch groups have remarkable properties and are related to many other areas of mathematics. For instance, many of them possess self-similarity or fractal properties linking them with fractal geometry, dynamical systems and probability. Branch groups are also of great importance in the study of infinite groups because they are one of the three classes (two in the profinite case) in which just infinite groups split.

This talk will provide an introduction to branch groups, motivating their study and showing some interesting results obtained in different areas of the subject.

Commutator width in infinite dimensional linear groups

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We describe a commutator subgroup of Vershik-Kerov group over an infinite field and the bound for its commutator width. This gives a partial solution of the problem posed by V. Sushchanskii in 2010. We also describe the lower central series of the group of infinite upper triangular matrices over an infinite field and the bound for its commutator width. We give a survey of similar results for other infinite dimensional groups.

Unit Group of Group Algebra

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Let F_2 be the fnite field with 2 elements and D_{2p} be the dihedral group of order $2p$, where p is an odd prime such that order of 2 mod p is $\Phi(p)$. Here, the structure of the unit group $U(F_2D_{2p})$ and the unitary subgroup $U_*(F_2D_{2p})$ with respect to classical involution of the group algebra F_2D_{2p} have been obtained. We also prove that the group generated by bicyclic units of the group algebra F_2D_{2p} is isomorphic to $SL_2(F_2l)$, where $p = 2l + 1$.

On lattices of radicals in the class of all finite groups

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Only radicals in the sense of Kurosh and Amistur will be considered here. The investigation of lattices of radicals in some classes of rings was initiated by R.L. Snider and was continued by many authors. However, there were no papers on lattices of radicals in classes of groups.

During the talk I'm going to consider the lattice $\mathbf{L}$ of all radicals in the class $\mathfrak{F}$ of all finite groups, the lattice $\mathbf{L}_h$ of all hereditary radicals in $\mathfrak{F}$ and the lattice $\mathbf{L}_{ch}$ of all cohereditary radicals in $\mathfrak{F}$. They are true, complete, algebraic lattices in the sense of algebra, being sets of cardinality $2^{\aleph_0}$.

Theorem 1. *For the lattice* $\mathbf{L}$ *we have:*

(1) $\mathbf{L}$ *is atomic and coatomic.*

(2) $\mathbf{L}$ *is neither atomistic nor coatomistic.*

(3) *Hereditary radicals and cohereditary radicals are distributive elements in* $\mathbf{L}$.

(4) $\mathbf{L}$ *is complemented and strongly balanced, but is not modular.*

Theorem 2. *The lattices* $\mathbf{L}_h$ *and* $\mathbf{L}_{ch}$ *are isomorphic to the Boolean algebra* $\mathbf{B}$ *of all classes of simple groups. Moreover,* $\mathbf{L}_h \cap \mathbf{L}_{ch} = \{(1), \mathfrak{F}\}$.

For needed terminology and results one can consult the references below. The above mentioned and further results of this talk are taken from the paper [4], prepared with Izabela A. Malinowska.

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On twisted group rings of OTP representation type

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We use the following notations: K is a field of characteristic p, K^* is the multiplicative group of K; $S = K[[X]]$ is the ring of formal power series in the indeterminate X with coefficients in K, S^* is the unit group of S, $(S^*)^l = \{a^l \colon a \in S^*\}$, T is the quotient field of S; $G = G_p \times B$ is a finite group, where G_p is a Sylow p-subgroup and $|G_p| > 1$, $|B| > 1$.

Every cocycle $\lambda \in Z^2(G, S^*)$ is cohomologous to $\mu \times \nu$, where μ is the restriction of λ to $G_p \times G_p$ and ν is the restriction of λ to $B \times B$. Each cocycle $\tau \in Z^2(B, S^*)$ is cohomologous to a cocycle $\nu \in Z^2(B, K^*)$. We assume that if G_p is non-abelian, then $[K(\xi) : K]$ is not divisible by p, where ξ is a primitive $(\exp B)^{th}$ root of 1. A twisted group ring $S^\lambda G$ is of *OTP representation type*, if any indecomposable $S^\lambda G$-module is isomorphic to the outer tensor product $V\#W$ of an indecomposable $S^\mu G_p$-module V and an irreducible $S^\nu B$-module W. We prove the following theorems.

Theorem 1. *Let $p \neq 2$, $G = G_p \times B$, Ω be the subgroup of S^* generated by K^* and $(S^*)^p$, $\mu \in Z^2(G_p, \Omega)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. The ring $S^\lambda G$ is of OTP representation type if and only if one of the following conditions is satisfied:*

(i) G_p is abelian and $T^\mu G_p$ is a field;

(ii) K is a splitting field for $K^\nu B$.

Theorem 2. *Let $p = 2$, $G = G_2 \times B$, $|G_2'| \neq 2$; Ω be the subgroup of S^* generated by K^* and $(S^*)^4$; $\mu \in Z^2(G_2, \Omega)$, $\nu \in Z^2(B, K^*)$ and $\lambda = \mu \times \nu$. The ring $S^\lambda G$ is of OTP representation type if and only if one of the following conditions is satisfied:*

(i) G_2 *is abelian and* $\dim_T(T^\mu G_2/\operatorname{rad} T^\mu G_2) \geq \frac{|G_2|}{2}$*;*
(ii) K *is a splitting field for* $K^\nu B$*.*

Similar results we obtain also in the case when Ω is a subgroup of S^* generated by K^* and $f(X)$, where $f(X) \equiv 1(\operatorname{mod} X)$ and $f(X) \not\equiv 1(\operatorname{mod} X^2)$.

We note that derived theorems are generalizations of corresponding results in [1], where $\mu \in Z^2(G_p, K^*)$.

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Automorphisms of a metacyclic minimal nonabelian p-group, p odd

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We describe the automorphism group of a metacyclic minimal nonabelian finite p-group with $p > 2$. This is part of the solution to a problem posed by Yakov Berkovich and Zvonimir Janko (2009).

(Outer) automorphism groups of crystallographic groups

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A crystallographic group of dimension n is a discrete and cocomapct subgroup of the group of isometries of the euclidean space $\mathbb{R}^n$. In 1973 Charlap and Vasquez proved, that for every crystallographic group Γ there exists so called basic diagram, which gives us information about automorphism and outer automorphism group of the group Γ ($\operatorname{Aut}(\Gamma)$ and $\operatorname{Out}(\Gamma)$ respectively).

In the talk an algorithmic approach for constructing basic diagrams will be presented, with restriction that we are dealing with crystallographic groups Γ for which $\mathrm{Out}(\Gamma)$ is finite.

Groups with finite number of conjugacy classes

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Many authors considered groups with some restrictions on conjugacy classes. Groups with conjugacy classes of finite size, known as FC-groups, are well described. We consider the groups in some sense dual to FC-groups, that is groups with a Finite Number of Conjugacy Classes ($FNCC$-groups). In 1949 G. Higman, B. H. Neumann and H. Neumann proved that each torsion-free group can be embedded into a group with two conjugacy classes. By D. V. Osin (2010), any countable group with only finitely many orders of elements can be embedded into a 2-generator group where many two elements of the same order are conjugate. This allows to construct finitely generated infinite $FNCC$-groups with quite arbitrary number of conjugacy classes. By S. V. Ivanov, some of the Olshanskii-Tarski monsters of exponent p have p conjugacy classes. The class of the $FNCC$-groups is closed for honomorphic images and finite direct products. Every finitely generated locally graded $FNCC$-group is finite. Some problems concerning the $FNCC$-groups will be discussed.

On the influence of subgroups on structure of finite groups

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A number of authors studied the structure of a finite group G under the assumption that some of its subgroups are well located in G (see [1, 2, 3]).

We are going to remind some history of this topic. Let G be a finite group. Recall that subgroups A and B of G *permute* if $AB = BA$. A subgroup H is said to be an *s-permutable* subgroup of G if H permutes with every Sylow subgroup of G; a subgroup H of G is *s-permutably embedded in* G if for each prime p dividing $|H|$, a Sylow p-subgroup of H is also a Sylow p-subgroup of some s-permutable subgroup of G.

We will generalize the notion of s-permutable and s-permutably embedded subgroups and we will show some new criterions of p-nilpotency and supersolubility of groups. We also are going to generalize some other known results. For more deatils see [4, 5] and references there.

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Extending structures: the level of groups

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Let H be a group and E a set such that $H \subseteq E$. We shall describe and classify up to an isomorphism of groups that stabilizes H the set of all group structures $\cdot$ that can be defined on E such that H is a subgroup of $(E, \cdot)$. Thus we solve at the level of groups what we have called the *extending structures problem*. A general product, which we call the unified product, is constructed such that both the crossed product and the Takeuchi's bicrossed product of two groups are special cases of it. It is associated to H and to a system $\big((S, 1_S, *), \triangleleft, \triangleright, f\big)$ called a group extending structure and we denote it by $H \ltimes S$. There exists a group structure $\cdot$ on E containing H as a subgroup if and only if there exists an isomorphism of groups $(E, \cdot) \cong H \ltimes S$, for some group extending structure $\big((S, 1_S, *), \triangleleft, \triangleright, f\big)$. All such group structures $\cdot$ on E are classified up to an isomorphism of groups that stabilizes H by a cohomological type set $\mathcal{K}^2_{\ltimes}(H, (S, 1_S))$. A general Schreier theorem is proved and an answer to a question of Kuperberg is given, both being special cases of our classification result.

Joint work with A.L. Agore.

On isometry groups of finitary wreath products of Hamming spaces

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We consider finitary wreath products of metric spaces and characterize their isometry groups. Especially the properties of isometry groups of finitary wreath products of Hamming spaces are studied. In particular, we prove the following result. Let $(m_1, m_2, \ldots)$ be an infinite increasing sequence of

natural numbers. It is shown that any countable residually finite group G is isomorphic to some subgroup of the isometry group of finitary wreath product of Hamming spaces $H_{m_1}, H_{m_2}, \ldots$.

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The talk will be based on a recent joint work with D.V.Osin. We show that every group H of at most exponential growth with respect to a length function can be embedded into a finitely generated group G such that (1) the length in H becomes, up to equivalence, the restriction of the word length in G and (2) G is solvable (respectively, satisfies a non-trivial identity, amenable, elementary amenable, of finite decomposition complexity, etc.) whenever H is. Some applications to asymptotic group theory will be discussed (distortion functions, Folner functions, compression functions, decomposition complexity of elementary amenable groups).

Polynomial sequences applied to unsolved problems in linear group theory

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A general question "when a set of given matrices generates a free group" appears in many problems, for example *15.83* and *15.84* in the Kourovka Notebook.

We define some families of polynomials which are generalizations of well known recursive polynomials (for example, Fibonacci or Lucas polynomials) and apply them to the problems above. One of the presented results is

Theorem 1. *Let x be a nonzero complex number,*

$$W_0(x) = 1, \qquad W_1[a_1](x) = a_1 x, \qquad W_2[a_1, a_2](x) = a_2 a_1 x^2 + 1,$$

$$W_{n+1}[a_1, ..., a_n, a_{n+1}](x) = a_{n+1} x W_n[a_1, ..., a_n](x) + W_{n-1}[a_1, ..., a_{n-1}](x),$$

$$A_a = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}, B_b = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}.$$

Then

(i) A group $gp(A_x, B_x)$ is the free group if and only if for every natural k and nonzero integers $n_1, ..., n_k$,

$$W[n_1, ..., n_k](x) \neq 0.$$

(ii) A semigroup $sgp(A_x, B_x)$ is free if and only if for every naturals k, l, $n_1, ..., n_l$ and $k \leq l$,

$$W[n_1, ..., n_k, -n_{k+1}, ..., -n_l](x) \neq 0.$$

Structure of Chevalley groups over rings via universal localization

Alexei Stepanov

Beginning from works of Suslin and Quillen on Serre's conjecture, localization methods proved their importance in the theory of algebraic groups over rings. I shall talk on a new version of a localization mehtod. The idea is to use localization in a "universal" ring U, e. g. the affine algebra of a groups G, get a result in $G(U)$, and then project it to $G(R)$ for an arbitrary ring R. Clearly, the results obtained in this way does not depend on R. For example, we obtain the following theorem.

Theorem 1. *Let G be a Chevalley–Demazure group scheme with a root system Φ of rank ≥ 2, and let E be its elemetary subgroup subfunctor. Then there exists a constant $L = L(G)$ such that for any ring R and any elements $a \in G(R)$ and $b \in E(R)$ the commutator $[a, b]$ is a product of at most L elementary root unipotent elements.*

Let me explain the importance of this result. Width of a group H with respect to a generating set S is the smallest integer L (or infinity) such that any element of H decomposes in a product of at most L generators. The width of the linear elementary group $E_n(R)$ with repect to elementary generators or the set of all commutators was studied by Carter, Keller, Dennis, Vaserstein, van der Kallen and others. It is concerned with computing of the Kazhdan constatant. For example, the width of $E(R)$ is finite if R is semilocal (by Gauss decomposition) or $R = \mathbb{Z}$ (Carter, Keller), but is infinite for $R = \mathbb{C}[x]$ (van der Kallen). The answer is unknown already for $R = F[x]$, where F is a finite field.

Van der Kallen noticed that the group $E_n(R)^\infty/E_n(R^\infty)$ is an obstruction for the finitness of width of $E_n(R)$, where infinite power means the direct product of countably many copies of a ring or a group. The theorem above is equivalent to the fact that this group is central in $K_1(R^\infty)$, so one can study it using homological algebra.

During the proof we obtain the standard commutator formulas with an estimate of width of conjugates with elementary root unipotents and commutators. The proof can be applied to establish the nilpotent structure of K_1.

The proof bases on Gauss decomposition, elementary calculations and easy splitting arguments. This gives a hope to extend main structure theorems for Chevalley groups over rings (commutator formulas, normal structure, niloptent structure of K_1, width of commutators) to nonsplit isotropic reductive groups and generalized congruence subgroups, e. g. generalized unitary groups of A. Bak.

On subnormal subgroups and series of normalizators in groups

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Let H be a subgroup of a group G. We shall investigate a necessary and sufficient conditions for H to be a subnormal subgroup. With a pair $H \subset G$ we can connect two sequences of intermediate groups:

The lower sequence defined by induction $G_0 = G$

and$G_i = < g^{-1}hg \,|\, g \in G_{i-1},\, h \in H >$ a subgroup generated by conjugacy for $i > 0$.

The upper sequence $N_0 = H$ and $N_i = N_G(N_{i-1})$ for $i > 0$.

The main results are:

Theorem 1. *There exist subnormal groups, that has sequences of normalizers stabilizing on proper subgroup.*

To prove it we need the following description on selfnormalizing subgroups.

Theorem 2. *Let $A_1 \cup A_2 \cup ... \cup A_t$ be partition of a set $\{1, 2, ..., n\}$. Let $H = S(A_1) \times S(A_2) \times ... \times S(A_t)$ ba a subgroup S_n.*

Then H is selfnormalizing if and only if every subsets A_i has different number of elements.

Theorem 3. *Let F be a field, and H be a subgroup of block matrices* $\begin{bmatrix} K_1 & 0 & \cdots & 0 \\ 0 & K_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_s \end{bmatrix} \in Gl(n, F)$, *where consecutive blocks has following sizes $n_1, n_2, ...n_s$. Then H is selfnormalizing if and only if every blocks has different sizes.*

Simplicial nonpositive curvature and some exotic infinite discrete groups

Jacek Swiatkowski

Simplicial nonpositive curvature (shortly SNPC) is an easily checkable purely combinatorial condition for a simplicial complex that succesfully mimicks the geometric concept of nonpositive curvature. Unlike the small cancellation concept, SNPC works for complexes of arbitrary dimension. A *systolic grup* is a group that acts geometrically (i.e. properly, cocompactly, by simplicial automorphisms) on an SNPC simplicial complex. Examples of such groups do exist in arbitrary cohomological dimension. In the talk I will explain the concept of SNPC, relate it to more classical curvature concepts, sketch the construction of systolic groups in arbitrary dimension, describe various exotic properties of high dimensional systolic groups, and comment on other consequences or applications of the developed concepts and ideas.

Symmetric operations in groups and Marczewski-Płonka problem

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Let G be an arbitrary group. For a word $w(x_1, \ldots, x_n)$ from the free group on n variables, we define an operation $f : G^n \to G$, $f(g_1, \ldots, g_n) = w(g_1, \ldots, g_n)$. We say that this operation is symmetric if for every permutation $\sigma \in S_n$ and every n-tuple $(g_1, \ldots, g_n) \in G^n$ we have $f(g_{\sigma(1)}, \ldots, g_{\sigma(n)}) = f(g_1, \ldots, g_n)$. We consider a class $\mathcal{K}$ of groups in which group operation xy can be represented as a composition of symmetric operations (such composition need not be symmetric). It is clear that abelian groups belong to $\mathcal{K}$. In 1967 Edward Marczewski asked whether $\mathcal{K}$ consists only of abelian groups. In 1970 Ernest Płonka show the example of nonabelian group in $\mathcal{K}$ and posed a question: which groups are in $\mathcal{K}$? This problem is still open. We will call it the Marczewski-Płonka problem. Many authors has showed, that different classes of groups do not belong to $\mathcal{K}$. In this talk we will present the history and prospects of the Marczewski-Płonka problem.

Finitely presented soluble groups, polynomials and polyhedra

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Finitely presented soluble groups have been studied seriously for about 40 years. In this lecture old and very new results will be described, together with some of the remarkably diverse methods from algebra, number theory and geometry used in establishing them.

On some topological generation of infinitely iterated wreath products of Abelian groups

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We provide some construction of a topological generating set for profinite groups which are infinitely iterated wreath products $\mathcal{W} = \ldots \wr A_2 \wr A_1$ of finite Abelian groups. By using the language of automorphisms of a rooted tree we introduce the notions of a homogeneous automorphism, a mutually coprime automorphism and a crack of an automorphism. We show the following

Theorem 1. *The union of any set S of homogeneous mutually coprime automorphisms with transitive Abelian vertex-groups and a set $\check{S}$ of arbitrary cracks of elements from S topologically generates an infinitely iterated wreath product of finite Abelian groups.*

Conversely, a given sequence $(A_i)_{i\geq 1}$ of finite Abelian groups we construct a topological generating set $S \cup \check{S}$ for the group $\mathcal{W}$ which is a disjoint union of some ρ-element set S of homogeneous mutually coprime automorphisms and a ρ-element set $\check{S}$ of the corresponding cracks such that both the group generated by S and the group generated by $\check{S}$ are isomorphic with a free Abelian group of rank ρ, where ρ is the topological rank of the profinite Abelian group $A_1 \times A_2 \times \ldots$. We present some properties of the group $G = \langle S \cup \check{S} \rangle$.

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