

A Note on Weighted Premium Calculation Principles

Marek Kałuszka, Andrzej Okolewski

Ogólnopolska Konferencja Naukowa
Zagadnienia Aktuarialne - teoria i praktyka

Warszawa, 15 maja 2012 r.

Let $X \geq 0$ be a loss random variable defined on a probability space (Ω, \mathcal{A}, P) . Goovaerts et al. (1984) considered the problem of determining a premium of the form

$$H(X) = E(Xz(X)),$$

where $z : [0, \infty) \rightarrow (0, \infty)$ is strictly increasing, continuous and such that $Ez(X) = 1$, maximizing the insurer's expected utility. More specifically, denoting the set of all such z 's by \mathcal{Z} , the objective of the insurer can be written as

$$\max_{z \in \mathcal{Z}} Eu(w - X + E(Xz(X))), \quad (1)$$

in which w is the insurer's initial wealth and u is the insurer's utility function such that the expected utility is finite.

Goovaerts et al. (1984, p. 84) claim that for $u(x) = (1 - e^{-\lambda x})/\lambda$ with $\lambda > 0$, the solution to (1) is

$$z(x) = e^{\lambda x}/Ee^{\lambda X},$$

and consequently an optimal premium is the Esscher premium (see also Kaas et al., 2008, Theorem 5.4.3, and Denuit et al., 2005, p. 83).

Denuit, M., Dhaene, J., Goovaerts, M. J., Kaas, R., 2005. Actuarial Theory for Dependent Risks. Wiley, Chichester.

Goovaerts, M. J., De Vylder, F., Haezendonck, J., 1984. Insurance Premiums: Theory and Applications. North-Holland, Amsterdam.

Kaas, R., Goovaerts, M. J., Dhaene, J., Denuit, M., 2008. Modern Actuarial Risk Theory Using R. Kluwer Academic Publishers. Second edition, Springer.

Unfortunately, their result is not true. In Proposition 1 below, we prove for general u the optimality of the maximal loss premium.

Proposition

Assume u is an arbitrary continuous and nondecreasing (not necessarily concave or differentiable) function such that $|Eu(W - X)| < \infty$ with (possibly random) W being the insurer's initial wealth. Then

$$\sup_{z \in \mathcal{Z}} Eu(W - X + E(Xz(X))) = Eu(W - X + \sup X), \quad (2)$$

where $\sup X$ stands for the essential supremum of X with respect to measure P and it is understood that $u(\infty) = \lim_{x \rightarrow \infty} u(x)$.

We propose a modification of problem (1), the solution to which is the Esscher premium.

Proposition

Let $Z = z(X)$ be an arbitrary nonnegative random variable such that $EZ = 1$. Let R , $0 \leq R \leq X$, be the insurer's part of X , w be the insurer's initial wealth and $u(x) = (1 - e^{-\lambda x})/\lambda$, $\lambda > 0$, be the insurer's utility function. If $Z = e^{\lambda r(X)} / Ee^{\lambda r(X)}$, then the contract $R = r(X)$ is a solution to the problem

$$\max_{0 \leq R \leq X} Eu(w - R + E(RZ)), \quad (3)$$

i.e. the premium for risk R is the Esscher premium

$H(R) = E(Re^{\lambda R}) / Ee^{\lambda R}$. Moreover, if the contract $R = r(X)$ in which $0 < r(x) < x$ for all $x > 0$, is a solution to problem (3), then $Z = e^{\lambda r(X)} / Ee^{\lambda r(X)}$.

