Workshop on the differential and difference equations in the complex domain

Warsaw, September 10–13, 2012

Abstracts of lectures and talks

Werner Balser

Okubo's hypergeometric system

In my lecture I shall describe some recent results on the Stokes multipliers of the confluent form of Okubo's hypergeometric systems. Some, but not all, of these results can be found in my recent article *Non-linear difference equations and Stokes matrices* which can be downloaded via my internet site.

Yuliya Bibilo

On the isomonodromic deformations of linear systems of ODEs with resonant irregular singularities

The talk is devoted to the isomonodromic deformations of linear meromorphic differential systems with Fuchsian and irregular singularities. These systems are allowed to have resonant irregular singularities. In such case deformation form may have some additional members and these deformations are similar to non-Schlesinger deformations of Fuchsian systems. We discuss the general view of the deformation differential form and provide an estimation of the form poles orders in the case of irregular resonant singularities of a special type. In the talk we also give corresponding examples.

Boele Braaksma

Local Stokes phenomenon of a system of ODE's with 2 levels

We consider a system of ODE's with 2 levels. A formal solution of this system is multisummable. We consider the Stokes phenomenon for multisums in neighbouring sectors. It can be described by means of convergent transseries.

Laura Desideri

Describing minimal surfaces using isomonodromic deformations

I will present a correspondence due to R. Garnier between minimal surfaces with a polygonal boundary curve and a certain class of Fuchsian equations. In this correspondence, the monodromy of an equation is prescribed by the edge directions of the polygonal boundary curve of the associated minimal surface. We will see how isomonodromic deformations can then provide us with an explicit description of minimal disks, that can be used to solve the Plateau problem.

Galina Filipuk

Time scales and orthogonal polynomials

In the talk I shall briefly explain the basics of time scales calculus and show how this theory can be applied to find ladder operators for orthogonal polynomials. This is a joint work with S. Hilger (KU-Eichstatt, Germany).

Renat Gontsov

Holomorphic vector boundles with meromorphic connections and triangular linear differential systems

We consider linear differential systems on Riemann sphere and use approach of holomorphic vector boundles endowed with meromorphic connections. We study the question concerning integration in finite terms of systems with bounded (Levelt's) exponents and some inverse problems for systems with triangular monodromy.

Irina Goryuchkina

Nonformal solutions to an ODE

We consider a nonlinear ordinary differential equation $f(x, y, y', \dots, y^{(n)}) = 0$, where the function $f(x, y, y', \dots, y^{(n)})$ is a polynomial of its variables $x, y, y', \dots, y^{(n)}$. Let this equation has a formal solution in the form of Laurent series with finite main part. We discuss sufficient conditions of convergence of such series.

Marc Huttner

Riemann P-scheme, monodromy and diophantine approximations

The aim of this article is devoted to a better understanding of many constructions of effective rational approximations to solutions of some linear hypergeometric functions from the perspective of the monodromy theory. In particular, we give concrete examples of Riemanns ideas redis- covered by G-V Chudnovsky in of the notion of Riemanns module. (In a modern language, this notion is known as "Fuchsian local system"). We apply this theory to generalize results concerning the simultaneous ra- tional approximations of polylogarithmic functions given by many authors and we effectively construct the system of Padé or Padé type approximants of first kind at $z = \infty$ of the family $\mathcal{S}$ defined for $1 \leq k \leq q$ by the Lerchs functions

$$\Phi_k(x, z) = \sum_{n=1}^{\infty} \frac{(1/z)^n}{(n+x)^k}.$$

(defined here at $z = \infty$) and the Hurwitz zeta-function $\zeta(k, x) = \Phi_k(x, 1)$.

To obtain, almost without calculations such explicit rational approximations, we solve in this particular case a "Riemann-Hilbert problem".

In addition we give some applications to diophantine approximations.

Alberto Lastra (joint work with Stéphane Malek)

On singularly perturbed $q-$difference-differential problems with an irregular singularity

A $q-$analog of a singularly perturbed Cauchy problem with irregular singularity in the complex domain is studied. Our result generalizes a previous result by S. Malek in [1]. First, we construct solutions defined in open $q-$spirals to the origin. Afterwards, we obtain the existence of a formal power series in the perturbation parameter which represents the solution and is the $q-$Gevrey asymptotic expansion of the actual solutions. This is achieved by means of a $q-$Gevrey version of Malgrange-Sibuya theorem.

References

[1] S. Malek, *Singularly perturbed $q-$difference-differential equations with irregular singularity*, J. Dynam. Control. Syst. 17 (2011), no. 2.

Grzegorz Łysik

Analytic solutions of heat equations with variable coefficients

We give an extension of the mean-value property and its converse to the case of real analytic functions and to functions of Laplacian growth. As an application we give characterizations of analytic and Borel summable solutions in time variable of the initial value problem to the heat equation $\partial_t u = \Delta u$ in terms of holomorphic properties of the solid and/or spherical means of the initial data. Finally we shall generalize the results to the case when the Laplace operator Δ is replaced by a sum of squares of commuting analytic vector fields.

Stéphane Malek

On the parametric Stokes phenomenon for singularly perturbed linear PDEs

We study a family of singularly perturbed linear partial differential equations with irregular type in the complex domain. In a previous work, we have given sufficient conditions under which the Borel transform of a formal solution with respect to the perturbation parameter converges near the origin in the complex domain and can be extended on a finite number of unbounded sectors with small aperture. The proof rests on the construction of neighboring sectorial holomorphic solutions whose difference have exponentially small bounds in the perturbation parameter (Stokes phenomenon) for which the classical Ramis-Sibuya theorem can be applied. In this paper, we introduce new conditions for the Borel transform to be analytically continued in larger sectors where it develops isolated singularities of logarithmic type lying on some half lattice. In the proof, we use a criterion of analytic continuation of the Borel transform introduced recently by A. Fruchard and R. Schaefke and is based on a more accurate description of the Stokes phenomenon for the sectorial solutions mentioned above.

Sławomir Michalik

Summability of formal solutions of linear partial differential equations with divergent initial data

We study the Cauchy problem for a general homogeneous linear partial differential equation in two complex variables (t, z) with constant coefficients and with divergent initial data.

We state necessary and sufficient conditions for the summability of formal solutions in terms of properties of divergent Cauchy data.

In the talk we consider both the summability in one variable t (with coefficients belonging to the Banach space of Gevrey series with respect to the second variable z) and the summability in two variables (t, z) in the sense of Balser.

Jorge Mozo Fernández

On polynomial asymptotics and Ramis-Sibuya theorem

In a joint paper with M. Canalis-Durand and R. Schäfke, a notion of monomial asymptotics was introduced, in order to study doubly singular differential equations, i.e., singularly perturbed differential equations with a singularity in the parameter, and an irregular singularity in the variable.

In this talk, we propose a generalization of this notion, defining polynomial asymptotics in two variables. For this aim we use the reduction of the singularities of plane curves in order to reduce the polynomial to a normal crossing situation, where monomial asymptotics in applicable. Using reduction of singularities and cohomological arguments, we can prove a theorem of Ramis-Sibuya type suitable for this class of asymptotics.

It is a joint work with Reinhard Schäfke (Université de Strasbourg).

Anastasia Parusnikova

Asymptotic forms and asymptotic expansions of solutions to the fifth Painlevé equation

We consider the fifth Painlevé equation. For all values of its complex parameters we are looking for the expansions of its solutions of the form $\sum_{\mathbf{K}} c_s(z)z^s$, where $\mathbf{K}$ is a countable set, $c_s(z)$ are either complex constants, or polynomials, or series in $\log z$, or elliptic functions of z^a, $a \in \mathbb{R}$.

Javier Sanz

Summability in ultraholomorphic classes

A new construction of linear continuous right inverses for the asymptotic Borel map is provided in the framework of general Carleman ultraholomorphic classes in narrow sectors. This was already achieved by V. Thilliez by means of Whitney extension results, but our procedure closely resembles the classical one in the case of Gevrey classes, since it makes use of a suitable truncated integral, Laplace-like operator with a flat kernel. This indicates the way for the introduction of a concept of summability which generalizes $k-$summability theory as developed by J. P. Ramis, and which is inspired by the study of general summability methods by W. Balser. Some applications to the analysis of formal power series solutions of some classes of partial differential equations will be discussed. Joint work with A. Lastra and S. Malek.

Ilya Vyugin

Isomonodromic confluence of singularities and solutions of fifth and sixth Painlevé equations

We study an isomonodromic confluence of Fuchsian singular points and irregular singular points of Poincaré rank one. Using this approach, we find the forms of local expansions of solutions of fifth and sixth Painlevé equations.

Michał Zakrzewski

GKZ, GG and GK systems and multiple zeta values

In papers [1], [2], [4] and [5] Israel Gelfand, his co-workers and his School developed general theory of Hypergeometric functions. It generalizes the theory of Euler-Gauss, General, Appell, Lauricella, Aomoto and many other hypergeometric functions.

Later Gelfand, Graev and Kapranov introduced generalizations of GKZ systems corresponding to the action of nonintegral latices [6], [3] (and also noncompact dual groups) and to noncommutative, reductive group [7].

I'm going to talk about the latest approach to multiple zeta functions, generalizing Rimeann Zeta function ζ, i.e. the functions of the form

$$\zeta(s_1, s_2, ..., s_p) := \sum_{n_1 > n_2 > ... > n_p > 0} n_1^{-s_1} n_2^{-s_2} ... n_p^{-s_p},$$

whenever the series converges, from the point of view of Gelfand's School theory of GKZ systems and their generalizations.

References

[1] Gelfand I., *General theory of hypergeometric functions.* Soviet Math. Dokl. 33 (1986), p. 573 - 577.

[2] Gelfand I., Gelfand S., *General theory of hypergeometric systems.* Soviet Math. Dokl. 33 (1986), p. 279 - 283.

[3] Gelfand I., Graev M., *GG Functions and their Relations to General Hypergeometric Functions.* Letters in Math. Phys. 50 no. 1 (1999), p. 1 - 28.

[4] Gelfand I., Kapranov M., Zalevinsky A. V., *Equations of hypergeometric type and Newton polytopes.* Soviet Math. Dokl., 37 (1988), p. 678 - 683.

[5] Gelfand I., Kapranov M., Zalevinsky A. V., *Hypergeometric functions and toric varieties.* Funct. Anal. and Appl. 23 no. 2 (1989), 12 - 26.

[6] Graev M., *General Hypergeometric Functions.* Funct. Anal. and Appl. 26 no. 2 (1992), 131 - 133.

[7] Kapranov M., *Hypergeometric functions on reductive groups.* In "Integrable systems and algebraic geometry" (M.H. Saito, Y. Shimizu, K. Yeno Eds.), p. 236 - 281, World Scientific Publ. 1998.