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ASP vs AASP

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- A joint work with:
 - Dominik Kwietniak (Jagiellonian University)
 - Piotr Oprocha (AGH University of Science and Technology)

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Let (X, d) be a metric space. Let $f: X \mapsto X$ be a continuous function.



he a	verage shadowing property (ASP)
	Let $\varepsilon, \delta > 0$ and $y \in X$. Let $\{x_n\}_{n=0}^{n}$ be a δ -average pseudo-orbit
	of t.
	The ε -shadowing in average
	The provide orbit $\int x \right]^{\infty}$ is a shadowed in average by x if
	The pseudo-orbit $\chi_{n_{n=0}}$ is <i>c-shadowed</i> in average by y if
	$\limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} d(f'(y), x_i) < \varepsilon.$
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Example 2

A Few Remarks

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Example 1

ASP vs AASP

Introduction

Introd	luction
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Example 1

Example 2

A Few Remarks

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The average shadowing property (ASP)

ASP

The map f has the average shadowing property (ASP) if for every $\varepsilon > 0$ there exists $\delta > 0$ such that every δ -average pseudo-orbit of f is ε -shadowed in average by some point in X.

Introduction

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The asymptotic average shadowing property (AASP)

Introduced by R. Gu in 2007.

Articles on AASP

2011 Dyn. Syst. Flows with the (asymptotic) average...
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Introduction	Example 1	Example 2	A Few Remark
The asymptotic average sh	adowing property (AASP)		
Let $\{x_n\}_{n=0}^{\infty}$	be a sequence of p	oints from X.	
An asympto	tic-average pseudo-«	orbit	
The sequence $\lim_{n\to\infty}\frac{1}{n}\sum_{n\to\infty}$	$\{x_n\}_{n=0}^{\infty} \text{ is an } asy \\ \sum_{i=0}^{n-1} d(f(x_i), x_{i+1}) = 0$	<i>mptotic-average pseudo</i> = 0.	o-orbit of f if
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Intro 0000	luction ○○●○○	Example 1	Example 2	A Few Remar
The a	symptotic average shadowing	property (AASP)		
	Let $y \in X$ and le of f .	t $\{x_n\}_{n=0}^\infty$ be an asym	1ptotic-average pseudo	-orbit
	The asymptotic s	shadowing in average		
	The pseudo-orbit y if $\limsup_{n\to\infty} \frac{1}{n}$	$\{x_n\}_{n=0}^{\infty}$ is asymptotic $\frac{1}{n}\sum_{i=0}^{n-1}d(f^i(y), x_i) =$	<i>cally shadowed in aver</i> 0.	age by
	Super-super-super-			



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Example 1

Example 2

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The asymptotic average shadowing property (AASP)

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The map f has the asymptotic average shadowing property (AASP) if every asymptotic-average pseudo-orbit of f is asymptotically shadowed in average by some point in X.

Introduction	Example 1	Example 2	A Few Remarks		
Statement of the problem					

So far, all maps analyzed in publications either have both the ASP and the AASP or have neither...

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ASP = AASP?

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ASP = AASP?

As it turns out, there exist counterexamples in noncompact spaces.

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The space X_1 and the map f_1

heorem 1

The map f_1 has the ASP, but does not have the AASP.

(in preparation)

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The space X_1 and the map f_1

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Theorem 1

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关系语 新闻语言

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(Also in preparation)

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Theorem 3

If X is compact, then AASP implies ASP.

In preparation, too)

Question A

Does ASP imply AASP when X is compact? think so...

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re there any implications in the class of bounded metric spaces?

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The End

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