Optimal and Critical Public Debt levels

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Abstract

This paper introduces the new model of public debt which allows to obtain analytically the optimal level and critical level of public debt as a share of GDP. The critical level of public debt means an amount of public debt which leads to default or reduction of GDP to zero.

The first part of this article describes the model of public debt, enabling the government to find an optimal policy of public debt management, when economic growth and interest rates on bonds are constant and independent of the amount of debt. With this simplifications obtained differential equations, describing public debt and budget deficit, are linear and possible to solve analytically. Equations for optimal public debt and budget deficit are obtained in optimal control framework. Unfortunately the simplification introduced to ensure linearity is too remote from reality to use this model in practice. However derived solution may be helpful in the next presented model.

In the second part linear model described above is used to build nonlinear model explaining behavior of some main macroeconomic variables related to public debt, such as GDP growth, interest rates on bonds and budget deficit. In this model both economic growth and interest rates on bonds depends on the debt to GDP rate. Presented system of nonlinear difference equations cannot be solved analytically, however optimal level and critical level of public debt may be obtained analytically.

Public debt model presented in this paper leads to some important conclusions. First of all, there is an optimal level of public debt. There also exists a critical level of debt. The last conclusion concerns influence of initial public debt on final GDP. Higher initial GDP leads to lower GDP in future.

Keywords: public debt model, optimal control theory, optimal public debt level, critical public debt level

JEL classification: C61, E62, H63

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