Σ^1_1-definability at uncountable regular cardinals

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We call a subset of $({}^{\kappa}\kappa)^n$ a $\boldsymbol{\Sigma}^1_1$-subset if it is the projection of a closed subset of $({}^{\kappa}\kappa)^{n+1}$. Given $0 < 1 < \omega$, we define $\boldsymbol{\Sigma}^1_n$-, $\boldsymbol{\Pi}^1_n$- and $\boldsymbol{\Delta}^1_n$-subsets in the usual way.

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- *A is definable in the structure $\langle \mathrm{H}(\kappa^+), \in \rangle$ by a Σ_1-formula with parameters.*

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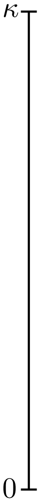
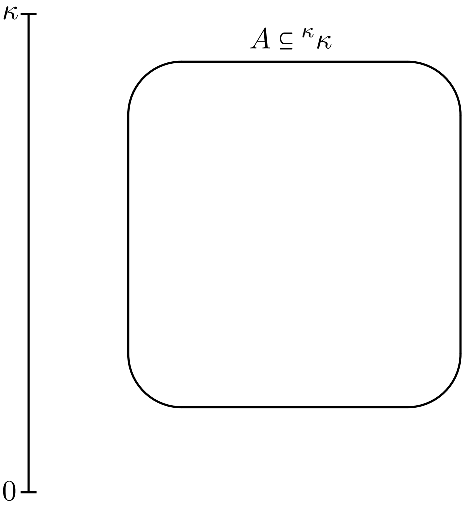

κ
0


κ
$A \subseteq {}^{\kappa}\kappa$
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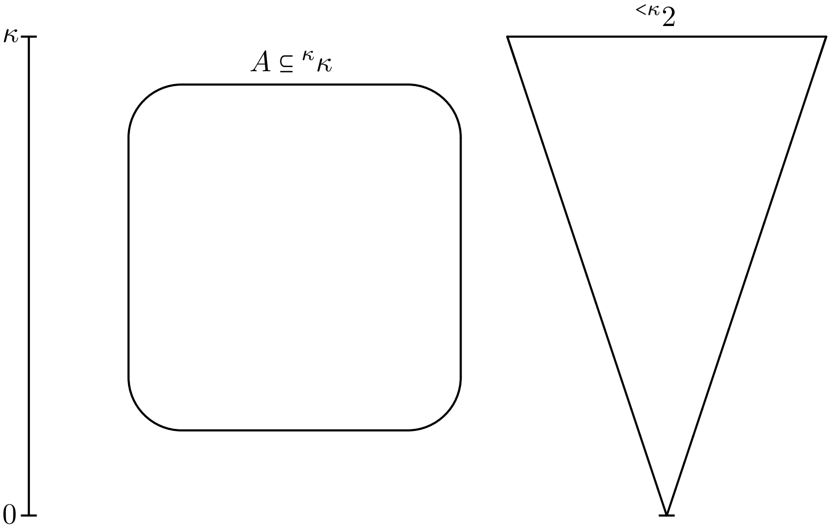
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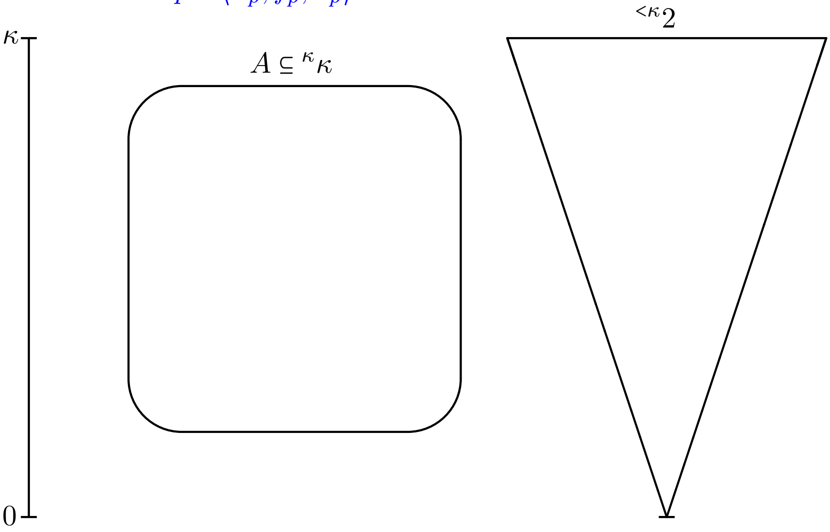

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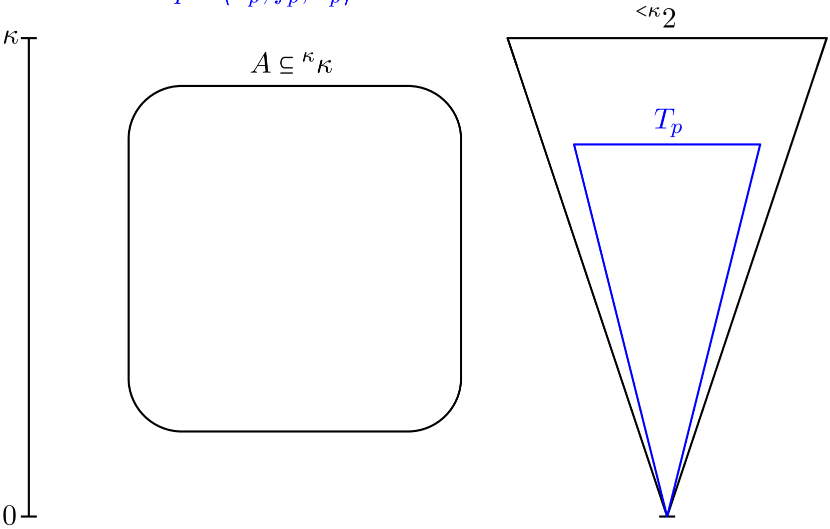

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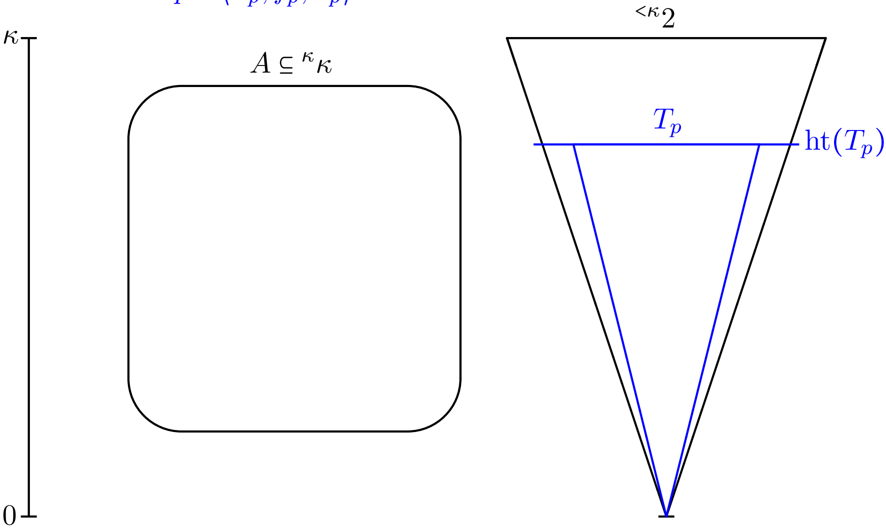

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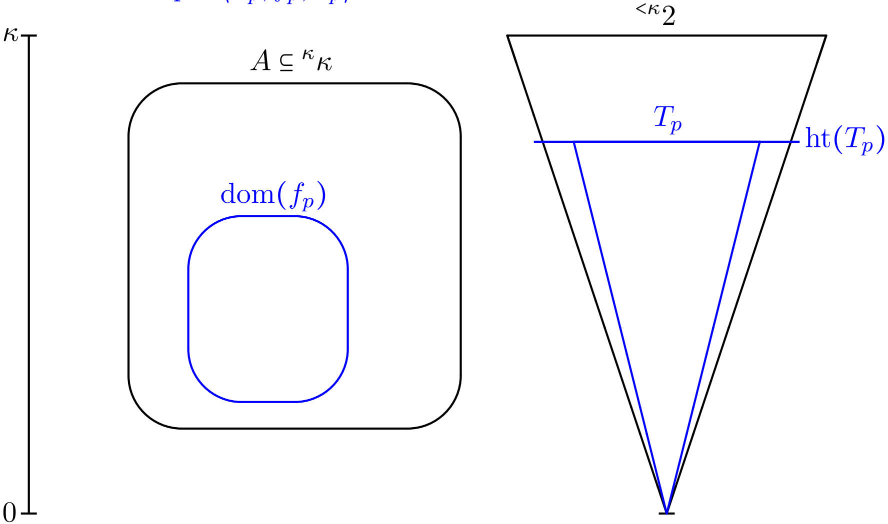

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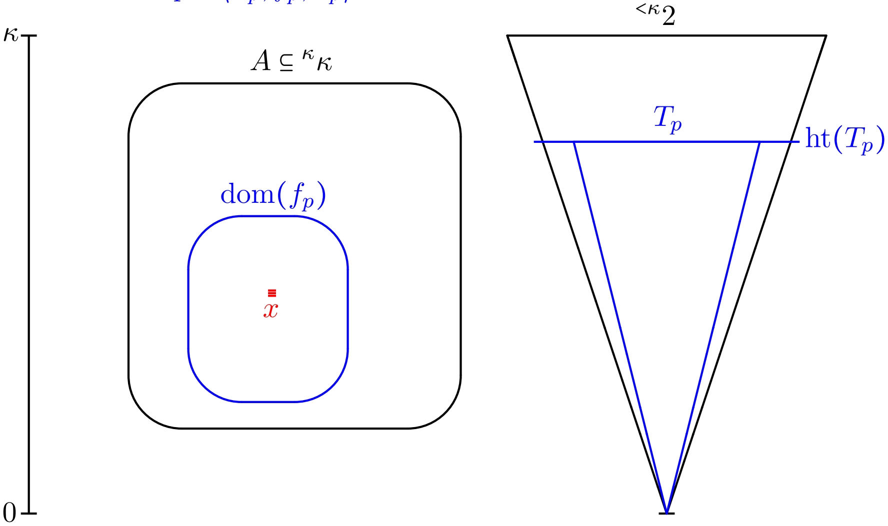

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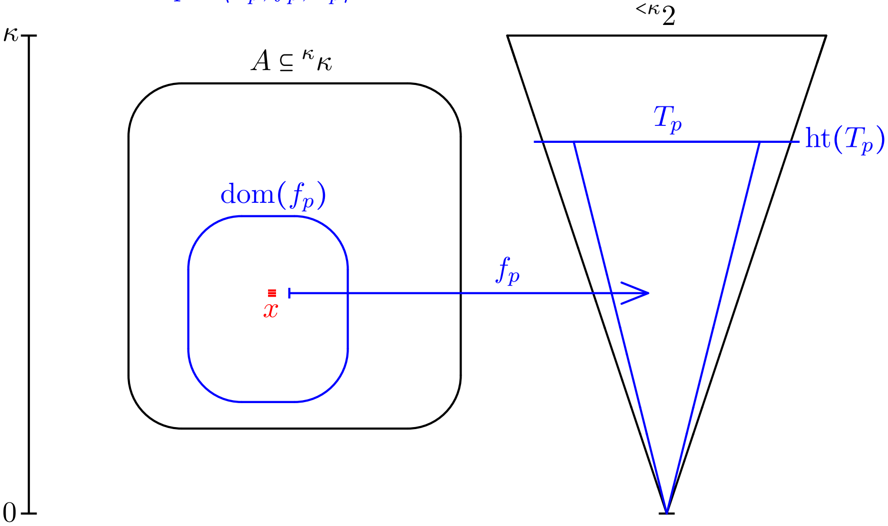

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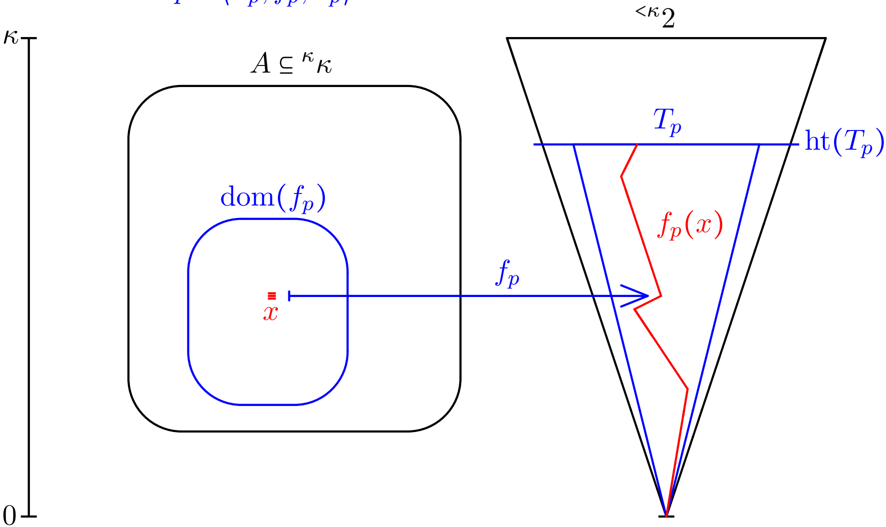

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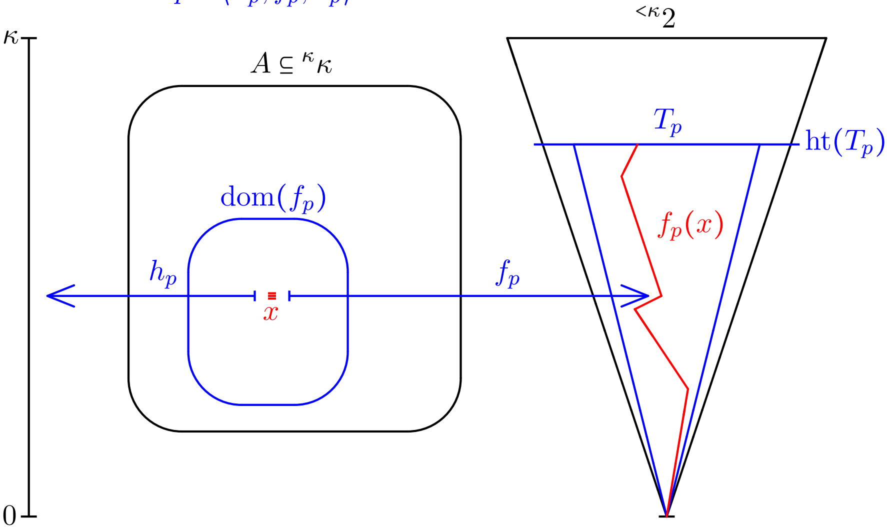

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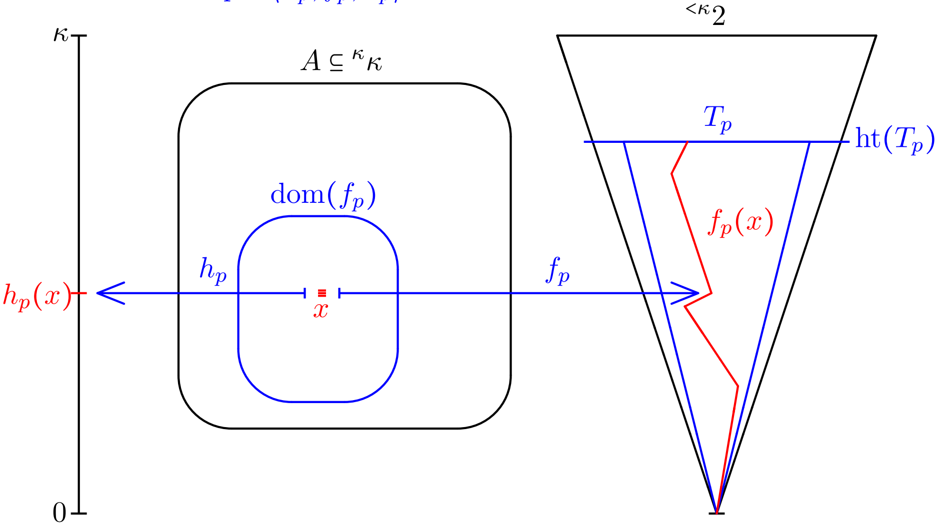

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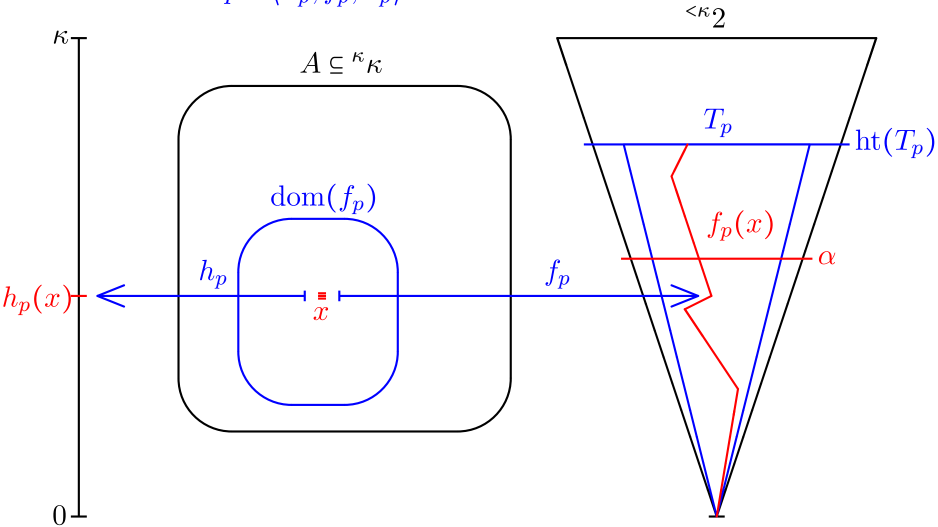

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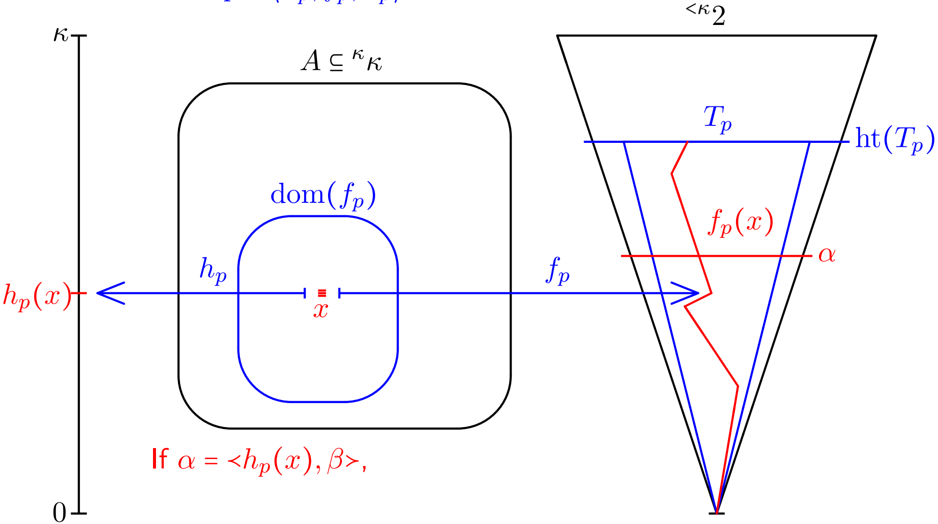

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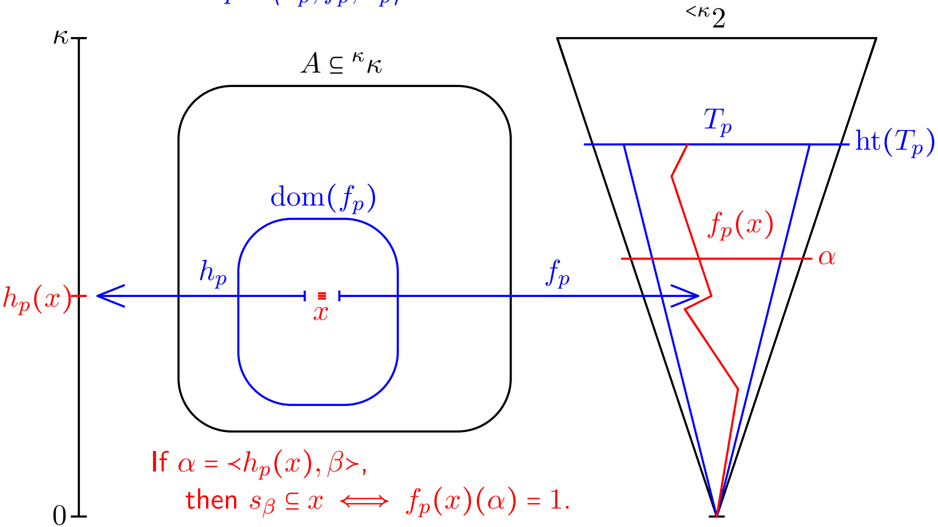

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Theorem (S. Friedman & P.L.)

There is a $\mathbf{ZFC}$*-preserving class forcing* $\mathbb{P}$ *definable without parameters that satisfies the following statements.*

- *Let* κ *be a cardinal with the property that there is no singular limit of inaccessible cardinals* ν *with* $\nu^+ < \kappa \leq 2^\nu$*. Then forcing with* $\mathbb{P}$ *does not collapse* κ *and, if* κ *is regular, then* $\mathbb{P}$ *preserves the regularity of* κ*.*
- $\mathbb{P}$ *preserves the inaccessibility of inaccessible cardinals and the supercompactness of supercompact cardinals.*
- *If* α *is an inaccessible cardinal and* G *is* $\mathbb{P}$ *generic over* V*, then* $(2^\alpha)^{\mathrm{V}} = (2^\alpha)^{\mathrm{V}[\mathrm{G}]}$*.*
- *If* κ *is an inaccessible cardinal and* A *is a subset of* ${}^{\kappa}\kappa$*, then there is a condition* p *in* $\mathbb{P}$ *with the property that* A *is a* $\mathbf{\Sigma}^1_1$*-subset of* ${}^{\kappa}\kappa$ *in* $\mathrm{V}[\mathrm{G}]$ *whenever* G *is* $\mathbb{P}$*-generic over* V *with* $p \in \mathrm{G}$*.*

Thank you for listening!