An axiomatic theory of classes for physics

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1900, Hilbert's 6th problem: "Mathematical treatment of the axioms of physics" briefly explained by others as a challenge for mathematicians and physicists to axiomatize all of physics with mathematics as a tool.

Some of my basic questions:

1) What should the axioms of physics be?
2) What are the objects of physics?
3) What do set-theorists intend to use mathematics to?
4) Which one of the collections of axioms for foundations of mathematics and logic should we choose to go closer to perfectness in understanding physics?

The choice of the best available logic.

The classical two-valued logic is not sufficient for deeper investigations of such problems.

Recommended book: G. Priest, An Introduction to Non-Classical Logic, Cambridge University Press, 2001, 2008.

My choice: (3+U)VL described in brief in my talk at the conference “Non-classical logics. Theory and Applications” in September 2011.

Not obtainable theories of everything (T0E).

A quotation from Stephen Hawking: “Some people will be very disappointed if there is not an ultimate theory, that can be formulated as a finite number of principles”.

Highly recommended books relevant to TOE:

1. Philosophy: N. Rescher, *Studies in Metaphilosophy,* Germany 2006.
2. Physics: S. Weinberg, *Dreams of a Final Theory: The Search for the Fundamental Laws of Nature,* New York 1993.

3. K. Kunen, *The Foundations of Mathematics*, London 2009.

An almost quotation from K. Kunen: Set theory is the **theory of everything**, it is all-important because all abstract mathematical concepts are set-theoretic and all concrete mathematical objects are specific sets.

Prof. Kenneth Kunen (b. 1943)

Kunen's TOE:

- Based on ZFC.
- All mathematical objects are sets, in particular, all elements of sets are sets.
- Proper classes do not exist, in consequence, the totality of all elements does not exist.
- That elements exist follows from hypothetical axioms of the theory but there are not visible examples of elements.
- Objects of physics (for instance, elementary particles of the Standard Model (SM)) are not objects of Kunen's TOE.

My conclusion: ZFC and its subtheories are not what physicists and the best philosophers would call TOE.

ZFA-like theories.

Recommended book:

4.T. Jech, *The Axiom of Choice,* New York 1973.

- Adaptation of ZF to the extra assumption that there is a set of atoms, atoms and sets are elements, atoms are not sets, atoms do not belong to themselves.
- There are not visible examples of atoms of ZF.

NBG and MK-like theories.

- Proper classes exist.
- A class is a set if and only if it is an element of a class.
- Every set is a class.
- Classes are mathematical objects.
- Modifications of ZFC to NBG or MK.
- It seems that it has not been written clearly whether all elements are sets.
- The totality of all elements can exist.

NBGA and MKA-like theories.

- Deformations of NBG and MK to theories with atoms.

Are the set theories appropriate for axiomatic physics?

To check whether the axiomatic set theories suit well to physics, one should learn more about physics.

Very good ideas are:

- To read the excellent books:

5. A. K. Wróblewski, *Historia Fizyki ("The History of Physics")*, PWN, Warszawa 2006.

Prof. Andrzej Kajetan Wróblewski (b. 1933)

6. L. M. Brown, A. Pais, B. Pippard, *Twentieth Century Physics,* New York 1995.

- To take into consideration Feynman's lectures on physics and the most important written works of other physicists.

My axiomatic theory Z[W] of classes.

Z- Ernst Zermelo [1871-1953], W- Wajch

Special attention paid to: elements, sets, classes, totalities, basic relations, units for measurement.

Remarks about axioms of Z[W]:

- Most axioms of Z[W] are modifications of the axioms of Z^- - Inf.
- It is not assumed that all elements are sets.
- Proper classes exist.
- Axiom on a relationship between sets and classes: every set is a class.
- Axiom of non-elements: proper classes are not elements of classes.

- Comprehension: for every property P of elements of sets, there exists the class of all elements that have the property P.
- It is not assumed that it is certainly true that infinite sets exist.
- The class ω of all finite ordinal numbers of Zermelo-von Neumann exists; however, it is neither true nor false that ω is a set.
- The system contains neither AC nor replacement.

Basic true numbers in Z[W].

- Non-negative integers: elements of ω.
- Negative integers: ordered pairs $-n = (n, 0)$ where $n \in \omega \setminus \{0\}$.
- $\mathbb{Z}$- the class of all integers (negative and non-negative).
- Rational numbers: ordered pairs (n, m) where n is an integer, $m \in \omega \setminus \{0\}$ and n, m are co-prime.
- $\mathbb{Q}$ - the class of all rational numbers.

- Irrational numbers can be obtained as classes contained in $\mathbb{Q}$ by using Dedekind's cuts of $\mathbb{Q}$ corresponding to irrationals in the usual Dedekind's construction of $\mathbb{R}$.

Warning: All integers and all rational numbers are sets, while it is neither true nor false that irrationals are sets.

One of possible applications: measurement, non-measurable magnitudes.

Recommended book:

7. K. Berka, *Measurement. Its Concepts, Theories and Problems*, Holland 1983.

Polynomials as finite objects.

A quotation from K. Kunen (2010)**:** "Polynomials should be finite objects".

If R is a ring and $n \in \omega$, a polynomial of degree n with all coefficients in R should be a mapping $f: n+1 \to R$ such that $f(n)$ is not the zero of R.

Finiteness and countability in Z[W].

Recommended article on independence results about notions of finiteness:

8. Omar De la Cruz, *Finiteness and Choice,* Fund. Math. 173 (2002), 57-76.

A set is Q-infinite if and only if it is not Q-finite (cf. [8]). Let me assign to each notion of Q-infiniteness the notion of an at most Q-countable set.

Definition. A set X is called at most Q-countable if every Q-infinite subset of X is equipollent with X.

Definition. A set X is called Q-uncountable if X is not at most Q-countable.

Problem 1. Investigate without lack of precision possible implications between the notions of at most Q-countability for distinct Q in Z[W] or at least in ZF.

Problem 2. Give an appropriate definition of an infinite proper class to prove in Z[W], if this is possible, that if the totality of all electrons in this room is a class, then it is not infinite.

Mathematics without AC.

Highly recommended books and a thesis, not mentioned above:

9. G. H. Moore, *Zermelo's Axiom of Choice, Its Origins, Development and Influence,* New York, Heidelberg, Berlin 1982.
10. H. Herrlich, *Axiom of Choice,* Berlin, Heidelberg, New York 2006.
11. G. Gutierres, *The Axiom of Countable Choice in Topology,* Portugal 2004.

Problem 3. Is Gutierres right when he believes that it can be proven in ZF that the Hausdorff's completion of the space $\mathbb{Q}$ of rationals is a complete metric space? I doubt it.

Thank you for your attention!