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Introduction

Results by other people

Now in colour

Another context

Idiosynchromatic Poetry 03E02, 03E10, 05C15, 05C20, 05C63

Thilo Weinert Hausdorff Research Centre for Mathematics, Bonn, Germany

Trends in set theory, Sunday, 8^{th} of July 2012, 11:45-12:00

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Results by other people

Now in colour

Another context

Introduction The context Results by other people $\alpha = \beta$ $\alpha > \beta$ Ramsey numbers Now in colour An eclectical definition An analogue theorem Two counterexamples A new Ramsey number An upper bound Another context A variation Yet another definition Yet again some upper bounds Two other counterexamples Now we know more Open questions Gratitude

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Outline	Introduction •	Results by other people	Now in colour	Another context
The context				

Definition $\alpha \to (\beta, \gamma)$ means $\forall E \subset [\alpha]^2 (\exists X \in [\alpha]^\beta : [X]^2 \subset E \lor \exists X \in [\alpha]^\gamma : E \cap [X]^2 = \emptyset).$

Remark

Here we are always referring to the order-type, i.e. $[\gamma]^{\delta}$ is the set of all subsets of γ whose order-type is δ .

Fact (AC) For any linear order φ we have $\varphi \not\rightarrow (\overline{\varphi} + 1, \omega)$. Definition

 $r(\beta, \gamma)$ is the least ordinal α such that $\alpha \to (\beta, \gamma)$.

Outline

 $\alpha = \beta$

Introduction

Results by other people 0000

Now in colour

Another context

Theorem (Specker, 1957) $r(\omega^2, m) = \omega^2$ for all $m < \omega$.

Theorem (Specker, 1957) $r(\omega^l, 3) > \omega^l$ for all $l \in \omega \setminus 3$.

Theorem (Baumgartner, 1989)

 MA_{\aleph_1} implies $r(\omega_1\omega, m) = \omega_1\omega$ for all $m < \omega$

Theorem (Baumgartner, 1989) $\overline{\mathrm{MA}_{\aleph_1}}$ implies $r(\omega_1\omega^2, m) = \omega_1\omega^2$ for all $m < \omega$.

Outline	Introduction O	Results by other people	Now in colour	Another context
$\alpha > \beta$				

Notation

 $r(I_l, L_m)$ is the least n such that any digraph on n vertices contains an independent set of size l or a transitive induced subtournament of size m.

Theorem (Erdős & Rado, 1956)

 $r(\omega l, m) = \omega r(I_l, L_m).$

Theorem (Baumgartner, 1974) $r(\lambda l, m) = \lambda r(I_l, L_m)$ for all infinite cardinals λ . Outline $\alpha > \beta$

Introduction

Results by other people

Now in colour

Another context

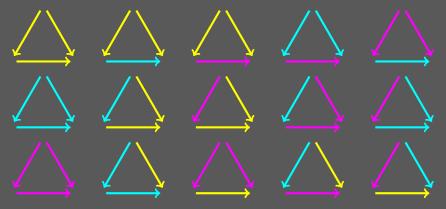
Theorem (Larson, Mitchell, 1997) $\forall n \in \omega \setminus 2 : r(I_n, L_3) \leq n^2.$ Theorem (Erdős, Moser, 1964) $\overline{\forall n \in \omega} \setminus 3: r(I_2, L_n) \leq 2^{n-1}.$ Theorem (Larson, Mitchell, 1997) $\forall m \in \omega \setminus 3, n \in \omega \setminus 4 : r(I_2, L_n) \leq u(m, n)$ with $u(m,n) := \frac{1}{2} \left(2^{n-3} \left(4 \binom{m+n-4}{n-1} + 6 \binom{m+n-5}{n-2} \right) \right)$ $\left(+9\binom{m+n-6}{n-3}\right) + 2^{n-4} \cdot 17\binom{m+n-6}{m-2} - 1$

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Out	utline Introduction Results by other people ○ ○ ○ ○ ○ ●		Now in colour		r Another cont				
Rar	nsey numbe	ers							
		3	4	5	6	7	8	9	m
	3	6	9	14	18	23	28	36	
	4	9	18	25					
	λ	λ	λ	λ	λ	λ	λ	λ	λ
	$\lambda 2$	$\lambda 4$	$\lambda 8$	$\lambda 14$	$\lambda 28$				
	$\lambda 3$	$\lambda 9$							
	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2
	$\omega^2 2$								
	ω^3	ω^4	ω^4	ω^5	ω^5	ω^5	ω^5	ω^6	$\omega^{2+\lceil \operatorname{ld}(m)\rceil}$
	ω^4	ω^7	ω^7	ω^{10}	ω^{10}	ω^{10}	ω^{10}		
	ω^{5+l}	ω^{9+2l}	ω^{9+2l}	ω^{13+3l}					$\omega^{1+(4+l)\lceil \operatorname{Id}(m)\rceil}$
	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}
	ω^{ω^2}	ω^{ω^2}	ω^{ω^2}						
	$\kappa\lambda 2$								
	$\kappa\lambda3$								

Outline	Introduction O	Results by other people	Now in colour ●○○○○○	Anoth
An eclectical	definition			

Definition A triple is called *agreeable* if and only if it is one of the following.



Idiosynchromatic Poetry

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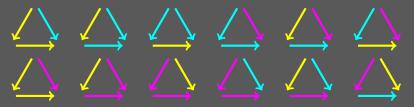
Outline	Introduction O	Results by other people	Now in colour ○●○○○○○	Another context
An eclectical de	efinition			

Fact

- A triple is disagreeable if and only if it is either...
 - ▶ ... a cyclic triple, regardless of the colouring, i.e.



▶ ... or one of the following transitive triples:



Outline	Introduction O	Results by other people	Now in colour	Another context
An analogue the	orem			

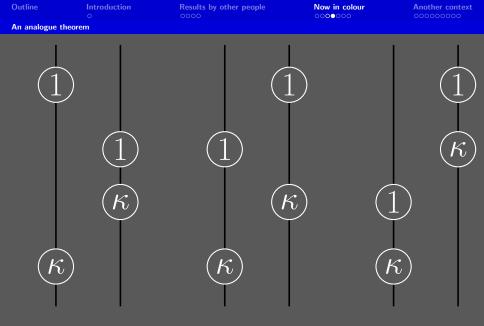
Notation

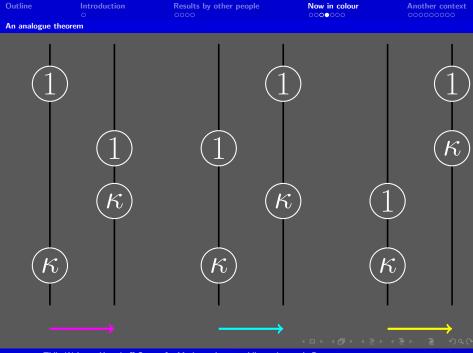
 $r(I_l, A_m)$ is the least n such that any arc-3-coloured digraph on n vertices contains an independent set of size l or an induced subtournament of size m all induced 3-person-subtournaments of which are agreeable.

Theorem (W.)

Let κ be such that $r(\kappa, \kappa) = \kappa$ and $\kappa \in \Omega \setminus 3$ —this means that $\kappa = \omega$ or κ is weakly compact. Then $r(\kappa^2 m, n) = \kappa^2 r(I_m, A_n)$.

Theorem (Erdős, Hajnal, 1971) $2^{\kappa} = \kappa^{+} \text{ implies that } \kappa^{+2} \neq (\kappa^{+2}, 3).$





Outline	Introduction O	Results by other people	Now in colour ○○○○●○○	Another conte
Two counteres	xamples			
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		X + A		

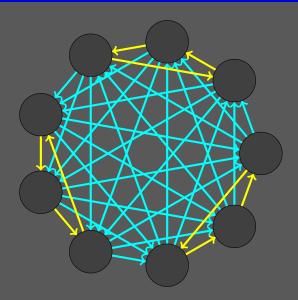
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Outline	Introduction	Results by othe
Two countered	examples	

Now in colour

Another context



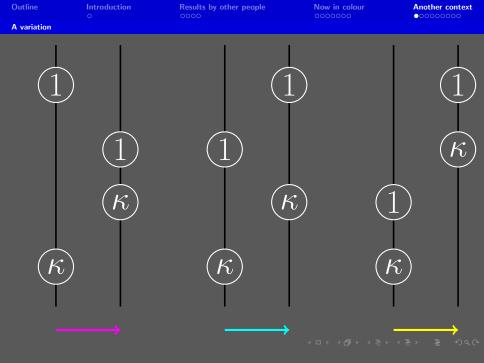
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Outline	Introduction	Results by other people	Now in colour	Anoth
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An upper boun	d			

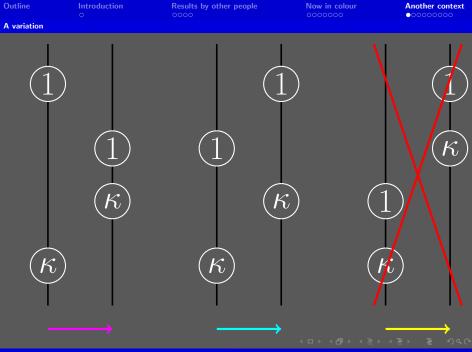
Theorem (W.)

$$\forall n \in \omega \setminus 2 : r(I_n, A_3) \leqslant \frac{(2n+1)(n^2+4n-6)}{3}$$

Corollary $r(I_2, A_3) = 10.$ Remark We have $r(n, 3), r(I_n, L_3) \in \mathcal{O}(n^2).$ her context







Yet another definition	Outline	Introduction O	Results by other people	Now in colour	Another context
	Yet another de	efinition			

Definition

A triple is called *strongly agreeable* if and only if it is agreeable and does not contain any yellow arrow. So it is strongly agreeable precisely if it is one of these:

 $\bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup$

Notation

 $r(I_l, S_m)$ is the least n such that any arc-2-coloured digraph on n vertices contains an independent set of size l or an induced subtournament of size m all induced 3-person-subtournaments of which are strongly agreeable.

Theorem (W.)

Let κ be weakly compact, let $\lambda \in \kappa \setminus \omega$ be a cardinal and let $l, m < \omega$. Then $r(\kappa \lambda l, m) = \kappa \lambda r(I_l, S_m)$.

Theorem (W.)

Assume MA_{\aleph_1}. Then for all $l, m < \omega$ we have $r(\omega_1 \omega l, m) = \omega_1 \omega r(I_l, S_m)$.

Theorem (W.)

For all $m \in \omega \setminus 2$ and all $n \in \omega \setminus 3$ we have $r(I_m, S_n) \leq u(m, n)$ where

$$u(m,n) := \frac{1}{4} \left(3 + \sum_{i=0}^{n-1} \binom{i+m-2}{i} 4^i \right)$$

Results by other people

Now in colour

Another context

Yet again some upper bounds

Theorem (W.) For all $m \in \omega \setminus 2$ and all $n \in \omega \setminus 3$ we have $r(I_m, S_n) \leq u(m, n)$ where

$$u(m,n) := \frac{1}{4} \left(3 + \sum_{i=0}^{n-1} \binom{i+m-2}{i} 4^i \right)$$

Corollary For all $m \in \omega \setminus 3$ we have $r(I_m, S_3) \leq m(2m-1)$. Corollary

For any
$$n \in \omega \setminus 3$$
 we have $r(I_2, S_n) \leq \frac{4^{n-1}+2}{3}$.

Outline	Introduction O	Results by other people	Now in colour	Another context
Two other co	unterexamples			

Outline	

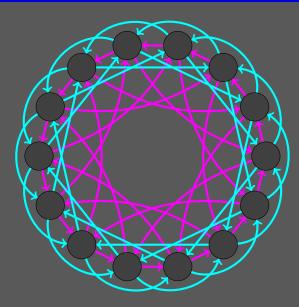
Introduction

Results by other people

Now in colour

Another context

Two other counterexamples



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Out	line	Introduction O		Results by other people				<mark>Now in coloι</mark> Σ000000	Ir Another context
Now we know more									
		3	4	5	6	7	8	9	m
	3	6	9	14	18	23	28	36	
	4	9	18	25					
	λ	λ	λ	λ	λ	λ	λ	λ	λ
	$\lambda 2$	$\lambda 4$	$\lambda 8$	$\lambda 14$	$\lambda 28$				
-	$\lambda 3$	$\lambda 9$							
	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2
_	$\omega^2 2$	$\omega^2 10$							
	ω^3	ω^4	ω^4	ω^5	ω^5	ω^5	ω^5	ω^6	$\omega^{2+\lceil \operatorname{Id}(m)\rceil}$
-	ω^4	ω^7	ω^7	ω^{10}	ω^{10}	ω^{10}	ω^{10}		
	ω^{5+l}	ω^{9+2l}	ω^{9+2l}	ω^{13+3l}	ω^{13+3l}	ω^{13+3l}	ω^{13+3l}	ω^{17+4l}	$\omega^{1+(4+l)\lceil \operatorname{ld}(m)\rceil}$
	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}	ω^{ω}
	ω^{ω^2}	ω^{ω^2}	ω^{ω^2}						
	$\kappa\lambda 2$	$-\kappa\lambda 6$							
	$\kappa\lambda3$	$\kappa\lambda15$							

Outline	Introduction	Results by other people	Now in colour	Another

Question What are $\mathcal{O}(r(I_n, A_3))$ and $\Omega(r(I_n, A_3))$?

Remark Proving lower bounds is often difficult.

Example (Jeong Han Kim, 1995) $r(n,3) \in \Theta(\frac{n^2}{\log n})$

Example (Noga Alon, 2005) $r(n,3,3) \in \Theta(\frac{n^3}{\operatorname{polylog} n}).$

Question

Assume that $r(\omega_1\omega, m) = \omega_1\omega$ for all $m < \omega$. Does it follow that $r(\omega_1\omega l, m) = \omega_1\omega r(I_l, S_m)$ for all $l, m < \omega$?

context

Outline	Introduction O	Results by other people	Now in colour	Another context
Gratitude				

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Outline	Introdu
Gratitude	

tion

Results by other people

Now in colour

Another context 00000000

Thank you very much for your attention!

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