

SELECTED RESULTS ON MEASURE-CATEGORY PRODUCTS OF IDEALS

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We consider measure-category product ideals $\mathcal{J} \in \{\mathcal{M} \otimes \mathcal{N}, \mathcal{N} \otimes \mathcal{M}\}$ on \mathbb{R}^2 and their properties. We include some recent results:

- \mathcal{J} is not invariant with respect to non-zero rotations [BG];
- the restriction of a Borel function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ to a set from the dual filter \mathcal{J}^* is Baire 1 [BG];
- consistently, there is no monotone Borel base of \mathcal{J} but, under CH, there is a monotone Borel hull with respect to \mathcal{J} on $\mathcal{P}(\mathbb{R}^2)$ [BF] (this continues the studies by M. Elekes and A. Máthé [EM]).

Also, we recall the construction of density-type topologies $\tau_{\mathcal{M} \otimes \mathcal{N}}$ and $\tau_{\mathcal{N} \otimes \mathcal{M}}$ on \mathbb{R}^2 . It is still not known whether they are homeomorphic.

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