Abstract

SET THEORETICAL METHODS IN ALGEBRAIC CONSTRUCTIONS

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The motivation for the talk comes from the paper *Algebrability, non-linear properties, and special functions* (by A. Bartoszewicz, S. Głąb, D. Pellegrino and J. B. Seoane-Sepúlveda, to appear in Proc. Amer. Math. Soc.). There was proved that the set of Zygmund-Siepiński functions is a strongly κ-algebrable subset of linear algebra $\mathbb{R}^{\mathbb{R}}$ where κ is the cardinality of a family of almost disjoint subsets of $\mathbb{R}$. It was the first time when **strong** κ-algebrability for $\kappa > \mathfrak{c}$ was proved, since in ZFC there is a family of almost disjoint subsets of $\mathbb{R}$ with cardinality $\mathfrak{c}^+$. The use of a family of almost disjoint subsets of $\mathbb{R}$ was the only known method which gave **strong** $2^{\mathfrak{c}}$-algebrability of $\mathbb{R}^{\mathbb{R}}$. However, such a result was independent of ZFC, so the question arose if one can prove in ZFC that there is a free subalgebra with $2^{\mathfrak{c}}$ generators of $\mathbb{R}^{\mathbb{R}}$. We plan to answer this question during the talk. Moreover, we give a nice method of proving strong $2^{\mathfrak{c}}$-algebrability of families of strange functions from $\mathbb{R}$ to $\mathbb{R}$.

We will prove also that if X is a set of cardinality κ where $\kappa^\omega = \kappa$, then $\mathbb{R}^X$ and $\mathbb{C}^X$ contain free linear algebras of 2^κ generators, that is of the largest possible size. We show also that the set of all strongly everywhere surjective functions from the remainder $\beta\kappa \setminus \kappa$ of Čech-Stone compactification of κ is strongly $2^{2^{2^\kappa}}$-algebrable. We will focus on families of some strange function in $\mathbb{R}^{\mathbb{R}}$ and $\mathbb{C}^{\mathbb{C}}$, among then strongly everywhere surjective and perfectly everywhere surjective functions from $\mathbb{C}$ to $\mathbb{C}$, everywhere discontinuous Darboux functions from $\mathbb{R}$ to $\mathbb{R}$, real functions whose sets of continuity points equals K for some fixed closed set $K \subsetneq \mathbb{R}$.

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