

Recent results on splitting and almost disjointness

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A classical theorem of Mathias says that a maximal almost disjoint (mad) family of subsets of the natural numbers cannot be analytic. Thus, if we let $\mathfrak{a}_{\text{closed}}$ denote the least size of a family of closed a.d. sets whose union is a mad family, then $\mathfrak{a} \geq \mathfrak{a}_{\text{closed}} \geq \aleph_1$. A number of interesting results have been proved recently on $\mathfrak{a}_{\text{closed}}$. For example, Raghavan and Shelah showed that $\mathfrak{d} = \aleph_1$ implies $\mathfrak{a}_{\text{closed}} = \aleph_1$. (For \mathfrak{a} instead of $\mathfrak{a}_{\text{closed}}$, this is a famous open problem of Roitman.) In joint work with Khomskii we proved the consistency of $\mathfrak{a}_{\text{closed}} < \mathfrak{b}$, and in the joint work with Raghavan, the consistency of $\mathfrak{a}_{\text{closed}} > \mathfrak{b}$. In this talk I will present some of the main ideas behind results on $\mathfrak{a}_{\text{closed}}$ and on the related phenomenon of splitting.