BORELL EQUIVALENCE RELATIONS GIVEN BY F_σ P-IDEALS AND LAVER FORCING

MICHAL DOUCHA

Let $\mathcal{I}$ be an F_σ P-ideal on ω. It induces an F_σ equivalence relation $E_{\mathcal{I}}$ on 2^ω defined as: $xE_{\mathcal{I}}y \equiv \{n : x(n) \neq y(n)\} \in \mathcal{I}$. By E_{ℓ_p} we denote the equivalence relation on $\mathbb{R}^\omega$ defined as: $xE_{\ell_p}y \equiv x-y \in \ell_p$, for $p \in [1,\infty]$.

The main result is the following.

Theorem. *Let T be a Laver tree, $\mathcal{I}$ an F_σ P-ideal and E an equivalence relation on $[T]$ Borel reducible to $E_{\mathcal{I}}$. Then there exists a Laver subtree $S \subseteq T$ such that either $E \restriction [S] = \mathrm{id}([S])$ or $E \restriction [S] = [S] \times [S]$.*

In particular, if E on $[T]$ is Borel reducible to E_2 or E_{ℓ_p}, for $p \in [1,\infty)$, then there is a Laver subtree $S \subseteq T$ such that either $E \restriction [S] = \mathrm{id}([S])$ or $E \restriction [S] = [S] \times [S]$.

On the other hand, J. Zapletal found an F_σ relation E on ω^ω which is bireducible with E_{ℓ_∞} and such that for every Laver tree T $E \restriction [T]$ is bireducible with E.

References

[1] V. Kanovei, M. Sabok, J. Zapletal, *Canonical Ramsey Theory on Polish Spaces*, Cambridge university press,to appear in 2013

Prague, Czech Republic
E-mail address: m.doucha@post.cz