Set theory and Hausdorff measures

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Hausdorff measures are the most fundamental concept in fractal geometry. They naturally generalise Lebesgue measure, for example, a subset H of a metric space X is of *d-dimensional Hausdorff measure zero* if for every $\varepsilon > 0$ there is a sequence of balls $B_i(x_i, r_i)$ covering H such that $\sum_i r_i^d < \varepsilon$. For integer d we get back the classical notions of length, area, volume, etc, but the case of non-integer d is also very important.

First we discuss the natural set theoretical properties of these measures, e.g. how their cardinal invariants fit into the Cichoń diagram, are these measures isomorphic, etc.

We also take up some problems concerning Hausdorff measures that do not a priori look set theoretical, but turn out to be independent of ZFC. More surprisingly, we also present purely set theoretical problems where Hausdorff measures are the key technical tool in the proof.