THE BASIS PROBLEM FOR COMPACTA SATISFYING HIGH SEPARATION AXIOMS

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We inspect the topological aspect of Todorcevic's seminal program to study basis problems in combinatorial set theory, outlined in [1]. In this context, the stage for a basis conjecture had already been set by Gruenhage (in a work on open problems on perfectly normal compacta). The "3-element basis conjecture" for uncountable first-countable regular spaces states that, consistently, each such space contains a set of reals of cardinality $\aleph_1$ with either the metric, the Sorgenfrey, or the discrete topology. We have previously found out that under Souslin's hypothesis this conjecture is true for uncountable subspaces of monotonically normal compacta. For uncountable subspaces of perfectly normal compacta, the existence of a 3-element basis is equivalent, assuming PFA, to Fremlin's conjecture that each such space admits an at most 2-to-1 continuous map onto a compact metric space. We improve on a result of Fremlin and obtain other partial solutions, particularly exhibiting strong evidence (or rather confirming the suspicion) that OCA is the main ingredient in a positive answer to Fremlin's conjecture. It is worth noting the pleasant synchronism in this work of Aronszajn compacta, and a question on perfectly normal compacta "attributable" to Alexandroff.

References

[1] S. Todorcevic, *Basis problems in combinatorial set theory*, Proceedings of the International Congress of Mathematicians, Vol. II (Berlin, 1998), Doc. Math. 1998, Extra Vol. II, 43–52.

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