

Definable graphs of low complexity

Stefan Geschke

Abstract: In the uncountable, there are some examples of rather pathological graphs. These pathologies vanish if we restrict our attention to definable graphs on Polish spaces. In the past there have been very interesting results on open graphs and on analytic graphs. Many interesting examples are of rather low complexity, for example closed or F_σ . We study the definable graphs on Polish spaces of very low complexity, namely clopen graphs. It turns out that these graphs allow for an interesting structure theory.

We discuss cardinal invariants, whether there are universal objects in various classes of clopen graphs, and embeddability of countable graphs into clopen graphs. Among other things, we show that there is no clopen graph on a compact metric space such that all clopen graphs on the Cantor space embed into it. Also, there are countable graphs that do not embed into any clopen graph on a compact space. Finally, we discuss the question whether such results can be extended to graphs of higher complexity.