

M. Elekes proved that any infinite-fold cover of a σ -finite measure space by a sequence of measurable sets has a subsequence with the same property such that the set of indices of this subsequence has density zero. Applying this theorem he gave a new proof for the random-indestructibility of the density zero ideal. He asked about other variants of this theorem concerning I -almost everywhere infinite-fold covers of Polish spaces where I is a σ -ideal on the space and the set of indices of the required subsequence should be in a fixed ideal \mathcal{J} on ω .

We introduce the notion of the \mathcal{J} -covering property of a pair (\mathcal{A}, I) where \mathcal{A} is a σ -algebra on a set X and $I \subseteq \mathcal{P}(X)$ is an ideal. We present some counterexamples, discuss the category case and the Fubini product of the null ideal and the meager ideal. We investigate connections between this property and forcing-indestructibility of ideals. Furthermore, we prove a general result about the cases when the covering property “strongly” fails.

This is a joint work with Marek Balcerzak and Barnabas Farkas.