Turning Borel sets into Clopen sets effectively

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Abstract: It is well-known that for every Polish space $(X, \mathcal{T})$ and for every Borel set $A \subseteq X$ there exists a Polish topology $\mathcal{T}_\infty$ on X which extends $\mathcal{T}$ (and thus has the same Borel sets as $\mathcal{T}$) which turns A into a clopen set. We present the effective version of this result and we give conditions under which the Polish space $(X, \mathcal{T}_\infty)$ admits a presentation which is recursive in a hyperarithmetical parameter. The latter implies a uniformity result of classical type.