

Recent developments in the Fraïssé-Jónsson theory

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Abstract

We shall discuss some recent results concerning universal homogeneous structures. Given a class \mathcal{F} of “small” structures, a structure U is \mathcal{F} -homogeneous if every isomorphism between \mathcal{F} -structures contained in U extends to an automorphism of U . Fraïssé-Jónsson theory deals with constructions and properties of such structures. In the most general setting, “small” structures are objects of a fixed category \mathcal{K} , while bigger structures are constructed as limits of (possibly transfinite) sequences in \mathcal{K} .

We shall discuss, in particular, the existence of arbitrarily large universal homogeneous structures, without any cardinal arithmetic assumptions. We show that, under some natural assumptions, such objects exist and can be described as Fraïssé-Jónsson limits. (This part is a joint work with Antonio Avilés).

Another topic concerns retracts of Fraïssé limits. These objects can be characterized in terms of injectivity properties, exhibiting connections with homomorphism-homogeneous structures, recently introduced and studied by Cameron & Nešetřil, Mašulović, and others.

We shall finally discuss *approximate Fraïssé limits*, which can be axiomatized using a metric on the given category, that allows us to define isometries and almost isometries. Homogeneity is replaced by *almost homogeneity* with the obvious meaning. The main problem here is uniqueness of the universal almost homogeneous structure with respect to an isometry.