

Σ_1^1 -DEFINABILITY AT UNCOUNTABLE REGULAR CARDINALS

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ABSTRACT. Let κ be an uncountable regular cardinal with $\kappa = \kappa^{<\kappa}$ and let ${}^\kappa\kappa$ denote the set of all functions $f : \kappa \rightarrow \kappa$. A subset of $({}^\kappa\kappa)^n$ is a Σ_1^1 -subset if it is definable in the structure $\langle H(\kappa^+), \in \rangle$ by a Σ_1 -formula with parameters. We define Σ_n^1 -, Π_n^1 - and Δ_n^1 -subsets as usual.

In my talk, I want to present the main result of [1]: *given an arbitrary subset A of ${}^\kappa\kappa$, there is a $<\kappa$ -closed partial order \mathbb{P} of cardinality 2^κ that satisfies the κ^+ -chain condition and forces A to be a Δ_1^1 -subset in every \mathbb{P} -generic extension of the ground model.* This result has several implications, e.g. it can be used to force the existence of a Δ_2^1 -definable well-ordering of ${}^\kappa\kappa$ while preserving the value of 2^κ or to show that generic absoluteness for Σ_3^1 -formulas over ${}^\kappa\kappa$ under $<\kappa$ -closed forcings that satisfy the κ^+ -chain condition always fails.

Finally, I want to present joint work with Sy-David Friedman [2] showing that it is possible to make arbitrary subsets of ${}^\kappa\kappa$ Σ_1^1 -definable while preserving large cardinal properties of κ , like supercompactness, by forcing with a class-sized partial order.

REFERENCES

- [1] Philipp Lücke. Σ_1^1 -definability at uncountable regular cardinals. Accepted for publication in the *Journal of Symbolic Logic*.
- [2] Sy-David Friedman and Philipp Lücke. *Large cardinals and definable well-orders, without the GCH*. Submitted.

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