Transfinite constructions in $V = L$

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Arnold Miller's classical result is that in L, Gödel's constructible universe, there exist a coanalytic 2-point set (a subset of the plain that intersects every line in exactly two points), a coanalytic maximal almost disjoint family of subsets of ω and a coanalytic Hamel-basis. Roughly speaking if we have a transfinite construction under CH which is nice enough and in every step the set of possible choices is large enough then in $V = L$ the induction can be realized so that it produces a coanalytic set.

We present a general theorem for such cases, which can be applied without the deep knowledge of Miller's method and we show some new consequences of this theorem.