

Idiosyncromatic Poetry

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Abstract

For α, β, n ordinals $\alpha \rightarrow (\beta, n)$ means that any graph on the ordinal α either contains an independent set of order-type β or a complete subgraph of order-type n . We only consider cases where n is a natural number, hence the naming.

The partition relation $\alpha \rightarrow (\beta, n)$ for α, β finite multiples of κ^2 where $\kappa = \omega$ or κ is weakly compact is characterized by a finitary problem as is the same for α, β finite multiples of $\kappa\lambda$ where κ is weakly compact and λ is an infinite cardinal less than κ . If the continuum hypothesis fails yet Martin's Axiom holds true the latter characterization also applies to the case where κ is not weakly compact but ω_1 . Upper bounds are given, in the first case for $n = 3$ and in the second generally. The minimal α for which the partition relation holds true is given for $n = 3$ and $\beta = \kappa^2 2$ in the first case and for $n = 3$ and $\beta \in \{\kappa\lambda 2, \kappa\lambda 3\}$ in the second.

Many questions arise, for example one concerning the growth rate of the function assigning the minimal α satisfying the partition relation for given β and n or one asking whether and if yes how Martin's axiom can be weakened while still implying the truth of the given characterization.

This builds on work by Ramsey, Erdős, Rado, Specker, Larson, Mitchell but above all, Baumgartner.