The *Borel Conjecture* (BC) is the statement that there are no uncountable strong measure zero sets (a set X is *strong measure zero* if for any sequence of ε_n's, X can be covered by intervals I_n of length ε_n, or, equivalently, if it can be translated away from each meager set). The *dual Borel Conjecture* (dBC) is the analogous statement about *strongly meager* sets (the sets which can be translated away from each measure zero set). Together with my advisor Martin Goldstern, Jakob Kellner and Saharon Shelah, I worked on the following theorem (see [1]):

> There is a model of ZFC in which both the Borel Conjecture and the dual Borel Conjecture hold, i.e., $\mathrm{Con}(\mathrm{BC} + \mathrm{dBC})$.

In my talk, however, I will concentrate on another variant of the Borel Conjecture, which I call the *Marczewski Borel Conjecture* (MBC). It is the assertion that there are no uncountable sets in s_0^*, where s_0^* is the collection of those sets which can be translated away from each set in the *Marczewski ideal* s_0. Is MBC consistent? I do not know, but I will present a connection of s_0^* to the family of Sacks dense ideals, which I introduced to investigate the status of MBC (under CH).

An $\mathcal{I} \subseteq \mathcal{P}(2^\omega)$ is a *Sacks dense ideal* if

- $\mathcal{I}$ is a (non-trivial) translation-invariant σ-ideal
- $\mathcal{I}$ is "dense in Sacks forcing": each perfect set P contains a perfect subset $Q \subseteq P$ which belongs to $\mathcal{I}$.

I will talk about a few results on Sacks dense ideals, but I will also mention the analogous situation (I started to look at very recently) when the Marczewski ideal s_0 (connected to Sacks forcing) is replaced by other ideals (e.g., v_0, which is connected to Silver forcing).

Website: http://www.wohofsky.eu/math/

References

[1] Martin Goldstern, Jakob Kellner, Saharon Shelah, and Wolfgang Wohofsky. Borel Conjecture and dual Borel Conjecture. *Transactions of the American Mathematical Society, to appear.* `http://arxiv.org/abs/1105.0823`.