On Borel sets belonging to every invariant ccc σ-ideal on $2^{\mathbb{N}}$

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Abstract

Let I_{ccc} be the σ-ideal of subsets of the Cantor group $2^{\mathbb{N}}$ generated by Borel sets which belong to every translation invariant ccc σ-ideal on $2^{\mathbb{N}}$. It turns out that I_{ccc} strongly violates ccc. This generalizes a theorem of Balcerzak-Rosłanowski-Shelah stating the same for the σ-ideal on $2^{\mathbb{N}}$ generated by Borel sets $B \subseteq 2^{\mathbb{N}}$ which have perfectly many pairwise disjoint translates. The last condition does not follow from $B \in I_{ccc}$ even if B is assumed to be compact.

The search for other conditions which for a Borel set B imply that $B \in I_{ccc}$ has interesting links with topics like the existence of perfect homogeneous sets for infinite dimensional relations (Shelah, Kubiś, Matrai) or of coverings of $2^{\mathbb{N}}$ by less than $\mathfrak{c}$ many translates of its compact subset (Darji, Keleti, Elekes, Steprans, Miller and others). A simple but nicely formulated by-product is the equivalence of CH to the statement that $2^{\mathbb{N}}$ can be partitioned into $\aleph_1$ many pairwise disjoint translates of its compact subset.