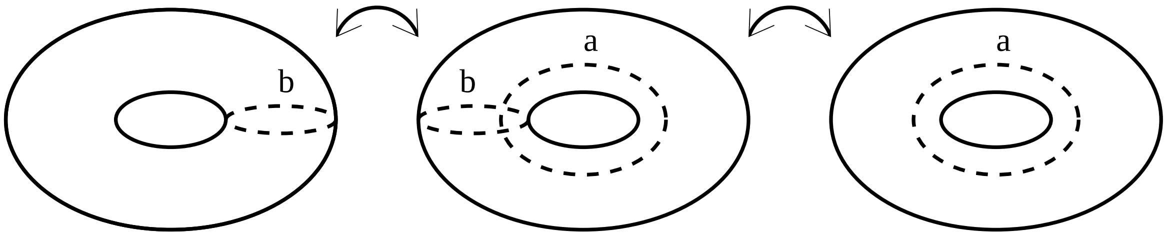

Young Geometric Group Theory Meeting

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Będlewo, Poland

Research Statements

Uniwersytet Warszawski
Uniwersytet Wrocławski
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Margarita Amchislavska

Cornell University

A recent major milestone in Group Theory was the realization that groups are often best thought of as geometric objects. Geometric Group Theory emerged as a result of the transformative work of Gromov and his ensuing program of understanding discrete groups up to quasi-isometry. A major theme in this development is the study of properties invariant under quasi-isometries. The Dehn function of a finitely presented group, the number of ends of a group, and the solvability of the word problem are some invariants that have received much attention. The filling length function of a group, which is a natural "space" analog of the Dehn function, has not been studied to the same extent.

The filling length of a word w, representing the identity in a finitely presented group, is the minimal integer L such that w can be converted to the empty word through words of length at most L by applying relators and freely reducing/expanding. The filling length function $\mathrm{FL}\colon \mathbb{N} \to \mathbb{N}$, for a finitely presented group Γ, is defined by

$$\mathrm{FL}(n) = \max\{\mathrm{FL}(w) : w = 1 \text{ in } \Gamma \text{ and } |w| \leq n\}.$$

It is known that groups having at most quadratic Dehn function (e.g. CAT(0) groups) necessarily have linear filling length. All combable groups and nilpotent groups have a linear filling length. In a recent paper [7], Olshanskii considered the notion of free filling length, introduced by Bridson and Riley [2]. This invariant is defined like filling length, except we allow an extra operation of cyclic conjugation. Olshanskii showed that the space complexity of an arbitrary deterministic Turing machine is equivalent to the free filling length function of some finitely presented group. So, there exist examples of groups with a wide variety of filling length functions. However, an understanding of filling length for standard classes of groups, such as finitely presentable metabelian, polycyclic, or solvable groups, remains elusive. This last class includes wild examples, as can be seen due to a result of Kharlampovich. In 1981, she constructed an example of a finitely presented solvable group with an unsolvable word problem [6]. The filling length function of this group must be huge; it is not bounded above by any recursive function, since a recursive bound on the filling length would give rise to a solution to the word problem.

An example that seems promising in this context is Baumslag's metabelian group, which, roughly speaking, is a 2-dimensional version of the lamplighter group. This was the first example of a finitely presented metabelian group with a free abelian normal subgroup of infinite rank, namely its commutator subgroup [1]. Baumslag's group with an extra torsion relation (that is, where the lamps have finitely many brightness levels) is known to have quadratic Dehn function, and so it has linear filling length [4]. The Dehn function for the non-torsion version of Baumslag's group was recently proved to be exponential by Kassabov and Riley [5]. Our conjecture is that the filling length function for Baumslag's group is quadratic.

Concretely, Baumslag's group has the presentation

$$\Gamma = \langle a, s, t \mid [a, a^t] = 1, a^s = aa^t, [s, t] = 1 \rangle.$$

The subgroup $L = \langle a, t \rangle = \mathbb{Z} \wr \mathbb{Z}$ is the well-known lamplighter group with countably infinite brightness levels for the lamps. The standard model for L, given by the real line with lamps located at the integer points and lamplighter starting at the origin, can be generalized to a two-dimensional model for Γ involving the combinatorics of Pascal's triangle [3].

I am working towards a proof of a quadratic upper bound for the filling length of Baumslag's group. The problem of investigating the filling length of this group reduces to investigating certain loops in the model. There is a way to interpret the Cayley graph of Baumslag's group as a generalized Distel-Leader graph – a horocyclic product of three countably infinite branching trees. Investigating this model appears promising for obtaining a lower bound on the filling length of Baumslag's group.

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Sylvain Arnt

Université d'Orléans

My field of working is in Geometric Group Theory and more specifically, I'm interested in the geometric action of Gromov hyperbolic groups (and some generalisations of these groups) on some metric spaces, and weak versions of the Haagerup property and strong versions of the property (T) for these groups. My PhD thesis subject is: δ-median spaces and Haagerup property. One of my goals is to generalize a result of G. Yu (2004) which says the following:

Theorem. Let Γ be a discrete group. If Γ is Gromov hyperbolic, then Γ has the weak Haagerup property i.e. there exists a geometric action of Γ on a ℓ^p space for some $p \geq 2$.

I study the same statement with weaker conditions than the hyperbolicity: the δ-median property. We say that a geodesic metric space is δ-median for some $\delta \geq 0$ if for all geodesic triangles in this space, there exists a point, called δ-median point of the triangle, in the intersection of the δ-geodesics between the three vertices of this triangle. Moreover, we assume that two δ-medians of a triangle are at a distance uniformly bounded over all the triangles. A group is called δ-median if it acts geometrically on a δ-median space.

A Gromov hyperbolic group is a δ-median group, and moreover a product of hyperbolic groups, which is not hyperbolic in general, is δ-median.

To study the Yu result on median groups, I wish to define a good structure of (quasi)wall space on hyperbolic groups so as to generalize it naturally on median spaces.

I'm also interested in the study of buildings structures: one of the question that I'm working on is: what kind of convex can induce a δ-median metric for the buildings structure defined by this convex.

Benjamin Beeker

Université Paris-Sud 11

My research concentrates on JSJ theory and decision problems.

I consider a specific class of groups: let Γ be a finite graph of groups with every vertex group finitely presented free abelian. Let G be the fundamental group of Γ. If the rank of every edge and vertex group is equal to a fixed integer n, then Γ is called a GBS_n decomposition and G a GBS_n group, where GBS stands for Generalized Baumslag-Solitar groups of rank n. When the rank is variable, such a group is called a $vGBS$ group.

The JSJ theory is a way to describe canonical splittings of finitely generated groups in graph of groups. I am studying two different kinds of JSJ decompositions.

To define what a JSJ decomposition is, we need the notion of universally elliptic subgroups. Given a group G and a decomposition Γ, a subgroup $H \subset G$ is *elliptic* in Γ, if H is conjugate into a vertex group. Given a class of subgroups $\mathcal{A}$ of G, a subgroup $H \subset G$ is *universally elliptic* if for every decomposition of G as a graph of groups with edge groups in $\mathcal{A}$, the group H is elliptic. A decomposition is universally elliptic if every edge group is universally elliptic.

A decomposition Γ *dominates* another decomposition Γ', if every elliptic group of Γ is elliptic in Γ'. A decomposition is a JSJ decomposition if it is universally elliptic, and it dominates every other universally elliptic decomposition.

The most classical example is the one given by Sela, and then Bowditch who built the JSJ decomposition of 1-ended hyperbolic groups over 2-ended subgroups.

I give the construction of a JSJ decomposition of $vGBS$ groups.

Theorem 1. Let G be a $vGBS$ group and Γ be a $vGBS$ decomposition of G. Given a vertex v of Γ with vertex group G_v, the subgroup of G_v generated by the groups of edges adjacent to v is denoted by $\tilde{G}_v$. Let $\bar{G}_v$ be the set of elements of G_v with a power in $\tilde{G}_v$. A JSJ decomposition of G over abelian groups is obtained from Γ by expanding the groups G_v such that $G_v/\tilde{G}_v$ is virtually cyclic into an HNN extension $\bar{G}_v *_{\bar{G}_v}$, and collapsing edges which carry non-universally elliptic edge groups.

Furthermore, I give an explicit description of the non-universally elliptic edge groups. I thus prove that the JSJ is algorithmically constructible.

The second kind of JSJ decomposition is the compatibility JSJ decomposition. Given a group G, a *G-tree is* a simplicial tree on which G acts cocompactly, without inversion. Given two G-trees $\mathcal{T}$ and $\mathcal{T}'$, we say that $\mathcal{T}$ *refines* $\mathcal{T}'$ if $\mathcal{T}'$ is obtained from $\mathcal{T}$ by collapsing some edge orbits. We say that $\mathcal{T}$ and $\mathcal{T}'$ are *compatible* if there exists a third G-tree $\mathcal{A}$ which refines both $\mathcal{T}$ and $\mathcal{T}'$.

Given a class of subgroups $\mathcal{A}$ of G, a *compatibility JSJ tree* over $\mathcal{A}$, is a G-tree which is compatible with every G-tree with edge stabilizers in $\mathcal{A}$, and maximal for refinement under this assumption. This definition has been given by Guirardel and Levitt. They prove the existence of a compatibility JSJ tree. Unlike usual JSJ decomposition, the compatibility JSJ tree is canonical and thus preserved by the automorphisms of the group.

I describe a compatibility JSJ decomposition of GBS_n groups over free abelian groups of rank $\leq n$.

Under some restrictions, I describe the compatibility JSJ tree of $vGBS$ groups over free abelian groups with no hypothesis on the rank.

I give some algorithmic results on the computation of these compatibility JSJ trees:

Theorem 2.

- There is no algorithm computing the abelian compatibility JSJ tree of a $vGBS$ group.

- Let G be a GBS_1 group and $\mathcal{T}$ be a G-tree. If no edge stabilizer of $\mathcal{T}$ is conjugate to one of its proper subgroups, then both the abelian and the cyclic compatibility JSJ trees are computable.

I use the descriptions of JSJ decompositions of $vGBS$ groups to study algorithmic problems on this class of groups. I prove that given a $vGBS$ decomposition, the conjugacy problem is solvable for hyperbolic elements. More precisely, I show the following theorem.

Theorem 3. Let G be a $vGBS$ group with $vGBS$ decomposition Γ. Let $A = (a_0, \ldots, a_n)$ and $B = (b_0, \ldots, b_n)$ be two n-tuples of elements of G. If the group $\langle a_0, \ldots, a_n \rangle$ is not elliptic in Γ, then we can decide whether A and B are conjugate in G or not.

Antoine Beljean

Universität Münster

I am currently starting the second year of my PhD in the University of Münster. This PhD project is concerned with the interplay of certain combinatorial structures, called buildings, and continuous groups acting on them.

A building is a very particular simplicial complex with certain properties that resemble loosely the definition of a manifold. It admits a covering by a family of subcomplexes which are called apartments. These apartments are in turn isomorphic to a fixed Coxeter complex. The buildings this project is concerned with are spherical buildings, where the Coxeter complex is a triangulated sphere, and affine (or Euclidean) buildings, where the Coxeter complex is a triangulated Euclidean space. Typical examples of spherical buildings arise from isotropic reductive groups $\underline{G}$ over a field F. The group of F-rational points $\underline{G}(F)$ can then be identified with the automorphism group of a building $\Delta(\underline{G}, F)$. In group theoretic terms, $\Delta(\underline{G}, F)$ is the set of all F-parabolics in $\underline{\mathrm{G}}$, partially ordered by the reversed inclusion. If F is a valued field, then a second building can be constructed : an affine building, the Bruhat-Tits building of $\underline{G}$. Its vertices are the maximal bounded subgroups of $\underline{G}(F)$, the group of F-rational points of $\underline{G}$.

In other words, buildings are interesting geometric objects which allow one to obtain information about the structure of these groups. Knowing more about the geometry of buildings certainly means knowing more about these groups. I am particularly interested in the study of affine buildings. Indeed, in the case, one can very naturally define a metric on such a building. One then obtains a $CAT(0)$-metric space. $CAT(0)$-metric spaces could be described as the generalisation for metric spaces of Riemannian manifolds with non-positive sectional curvature. Yet, for a $CAT(0)$-space, one loses the notion of tangent bundle. The first step of my work was to define a generalisation of the tangent bundle for $CAT(0)$-spaces. This generalisation is the '*Direction Bundle*'. For each point p of a $CAT(0)$-space X, there is construction called the *space of directions of X at p*, S_pX: this is the set of all geodesics starting from p, quotiented by a certain equivalence relation (2 geodesics are then equivalent if they define an angle at p of value zero). Each S_pX is naturally given a metric (the angle metric). I defined the direction bundle SX as the disjoint union of all these S_pX. The main difficulty was to find a topology on SX which unifies all the metric spaces S_pX. I checked that the topology I chose on SX is indeed such a unification for the following types of $CAT(0)$-spaces : simply connected Riemannian manifolds with non-positive sectional curvature, metric trees, certain M_κ-simplicial complexes (simplicial complexes obtained by gluing simplices taken from a fixed simply connected Riemannian manifold of constant sectional curvature), and Euclidean buildings. This Direction bundle coincides with the notion of tangent bundle in the case of a Riemannian manifold (simply connected, with non-positive sectional curvature).

I am currently studying the many uses than can be made of this direction bundle : for example, there should be an analogue of the exponential map from SX towards X, and isometries of X should have an interesting interplay with it. I am also trying to see until what point the direction bundle can be defined for general $CAT(0)$ spaces, and also for spaces which are only locally $CAT(0)$.

My PhD project is now also focused on the study of sheaves and coefficient systems on buildings. Indeed, there are various natural sheaves on a building. One of them is the orientation sheaf (which is defined in exactly the same way as for a manifold). The orientation sheaf is important since it is on the one hand a purely topological invariant of the space. On the other hand, it has combinatorial significance, because appartments can be encoded in it. The connection with the Direction bundle is that it should be the 'natural habitat' for the orientation sheaf.

Michael Björklund

Eidgenössische Technische Hochschule Zürich

My research is mostly concerned with interplay between ergodic theory/topological dynamics, combinatorics and number theory. Quite a lot of surprising connections between these seemingly disconnected branches of mathematics have been made, and new and exciting interactions emerge on a regular basis. My primary focus is on ergodic Ramsey theory, which today is a very rich field of research. A fundamental example of the power of the theory is Furstenberg's celebrated proof [1] of a deep combinatorial theorem, originally due to E. Szemeredi [4], using ergodic techniques, which spurred (and keeps producing) a plethora of far-reaching generalizations and opened up plenty of new directions, not only within ergodic theory, but also in combinatorial number theory and graph theory. Furstenberg's proof dates back to the '70'ies, but the ideas are still implemented in modern results, such as the Green-Tao's proof [2] on the existence of arbitrarily large arithmetic progressions in the primes. In joint collaborations with A. Fish (Madison) I have developed very general techniques to transfer product set theorems for Haar measurable subsets of positive Haar measures in compact groups, and product set theorems for large subsets with respect to the upper Banach density on countable amenable groups. The proofs are very much inspired by Furstenberg's ideas.

Another area of interest is random walks on groups; in particular quantitative limit results of random walks on automorphism groups of geometric objects, e.g. isometry groups of metric spaces. This is a very active and fertile research field, with many interesting connections to number theory and group theory. The investigation of random walks on semisimple Lie groups motivated, among other things, the Oseledet's theorem, which is today an indispensable tool in hyperbolic dynamics, and has recently been extended to very general situations by A. Karlsson and F. Ledrappier [3], with many interesting applications to random walks on discrete groups. Recently I developed a very general approach to central limit theorems on metric groups, reducing the problem to mean asymptotic questions, which in a large variety of cases can be reduced to potential theoretic questions on associated boundaries, which can be solved using geometric techniques.

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Henry Bradford

University of Oxford

The dominant goal of my present study is to understand the geometric properties of groups through their profinite completions. For G a group, we shall denote by $\hat{G}$ the profinite completion of G. Finitely-generated, residually finite groups G and H (as all groups with which we concern ourselves shall be) are said to be profinitely equivalent if $\hat{G} \cong \hat{H}$. A related criterion is that of strong profinite equivalence: a homomorphism $\phi : G \to H$ is said to be a strong profinite equivalence from G to H if the induced map $\hat{\phi} : \hat{G} \to \hat{H}$ is an isomorphism. For P a property of groups, we shall therefore be interested in answering the following questions: If $P(G)$, must we have $P(H)$ for any H which is profinitely equivalent to G (that is, is P a "profinite property"of groups)? If $\phi : G \to H$ is a strong profinite equivalence, does $P(G)$ imply $P(H)$? Does $P(H)$ imply $P(G)$?

One set of problems which may be fruitfully illuminated by this approach concern the spectral properties of groups. It is already known, for instance, that neither Kazhdan's property (T) nor property (τ) are preserved by profinite equivalence in general. The latter is a surprising result, since property (τ) depends only on data from the finite quotients of a group. Nevertheless, if $\phi : G \to H$ is a strong profinite equivalence, and G has property (τ), then so does H. It is not yet known whether the same is true for property (T), and indeed a positive answer to this question would also prove the existence of a non-residually finite hyperbolic group.

Another area which has much seen recent interest is the study of the extent to which the complexity of groups (as measured by the difficulty of their fundamental decision problems) is preserved by profinite equivalence. There are for instance many open questions concerning which properties of the Dehn functions of groups are profinite properties. Unfortunately, thus far, most of the known results in this direction are negative: Cotton-Barratt and Wilton [1] have recently shown that there exist finitely presented groups G, H and $\phi : G \to H$ a strong profinite equivalence, such that H is conjugacy separable but G does not even have solvable conjugacy problem. It is also known that word-hyperbolicity (that is, the property of possessing a linear Dehn function) is not a profinite property. However, all known witnessing examples of the previous result are "trivial"in the sense that whenever groups G, H, such that $\hat{G} \cong \hat{H}$, with G hyperbolic and H non-hyperbolic have been produced, H has not been finitely presented. It is therefore still legitimate to ask whether hyperbolicity is a profinite property in the case of finitely presented groups.

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Ayala Byron

Hebrew University

I am now starting my PhD degree, in which I wish to study geometric group theory in general, and more particularly, relations between algebraic structures and logic, using geometric group theoretic tools.

For my master's thesis I have studied the rank gradient of finitely generated groups. For a f.g group G, the rank gradient of G relative to a decreasing chain of finite-index subgroups (G_n) is the following limit:

$$\mathrm{RG}\left(G,(G_n)\right) = \lim_{n\to\infty} \frac{d\left(G_n\right)-1}{[G:G_n]}$$

The main motivation for this definition is geometric - it comes from the study of the structure of finite covers of manifolds. For example, if the chain of fundamental groups of a tower of finite-sheeted covers of a manifold M has positive rank gradient and does not have property τ with respect to $\pi_1(M)$, then there is a group in the chain that decomposes into a non-trivial amalgamated product, and the corresponding cover is Haken.

For my PhD research, I first wish to understand the structure of homomorphisms from a finitely generated group H to a free group F, using limit groups. A limit group is a quotient of a finitely generated group H by the kernel of a stable sequence of homomorphisms from H to F. A sequence of homomorphisms $\phi_n : H \to F$ is stable if for every $h \in H$, either $\phi_n(h) = 1$ for all but finitely many n's, or $\phi_n(h) \neq 1$ for all but finitely many n's. The kernel of the sequence is $\{h \in H \mid \phi_n(h) = 1 \text{ for all but finitely many } n'\text{s}\}$

Studying of the structure of such homomorphisms is studying sets of solutions for equations over a free group: let $F = \langle a_1, ..., a_k \rangle$ be a free group on k generators and Φ a set of equations in n variables over F:

$$\begin{gathered} w_1\left(a_1,...,a_k,x_1,...,x_n\right) \\ \vdots \\ w_l\left(a_1,...,a_k,x_1,...,x_n\right) \end{gathered}$$

A solution to Φ is an n-tuple of elements of F satisfying the equations, and so can be thought of as a homomorphism from a group H into F, with H having the following representation: $H = \langle a_1, ..., a_k, x_1, ..., x_n | w_1, ..., w_l \rangle$ such that for all i, a_i is sent to a_i, and every such homomorphism gives rise to a solution to Φ. On the other hand, sets of solutions to a finite set of equations over a free group are definable sets in the first order theory of the free group, and so understanding $\mathrm{Hom}(H, F)$ is a good way to start answering questions about the first-order theory of free groups.

Caterina Campagnolo

Université de Genève

After my master thesis on growth functions of right-angled Coxeter groups, I began in May 2011 a PhD under the supervision of Michelle Bucher.

My aim is to study characteristic classes of surface bundles, as defined by Morita (see [2]). In particular, Morita asked whether these classes are bounded, or in other words, if they can be represented by cocycles which are uniformly bounded. This is known to hold for the classes in degree $2(k+1)$ since these are pullbacks of primary classes on the symplectic group, which are bounded by a result of Gromov. The question for the remaining classes in degree $2k$ is open from degree 4 already.

One advantage of the theory of bounded cohomology, initiated by Gromov in the beginning of the 80's [1], is that good bounds for norms of cohomology classes naturally give rise to Milnor-Wood inequalities. I will thus try to compute the norms of the characteristic classes of surface bundles, with an aim to produce new inequalities between classical invariants of surface bundles.

At the moment, I am studying general and bounded cohomology of groups and singular cohomology, in order to use them in the setting of surface bundles. In parallel, I am learning about the mapping class group. I will then understand its group cohomology, since characteristic classes of surface bundles are, in the universal case, cohomology classes of the mapping class group.

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Mathieu Carette

Université catholique de Louvain

Let F_N be the free group of rank N. An essential tool in the study of $\mathrm{Out}(F_N)$ is its action on Culler-Vogtmann's Outer space CV_N consisting of certain F_N-trees (or equivalently F_N-marked metric graphs Γ). It was introduced in [7] as an analogue of the action of the mapping class group $\mathrm{MCG}(\Sigma)$ of a compact surface Σ of negative Euler characteristic on Teichmüller space $\mathcal{T}(\Sigma)$. This point of view has been very fruitful, with a lot of recent research focusing on finding analogies and differences between $\mathrm{Out}(F_N)$ and mapping class groups. In the following we focus on applications and interactions between the following tools (each developed using corresponding notions for surfaces and Teichmüller space): Lipschitz distortion on Outer Space, currents on free groups and train tracks for free groups.

Spectral rigidity

To any marked metric graph $\Gamma \in CV_N$ one associates a translation length function (or length spectrum) $||\cdot||_\Gamma : F_n \to \mathbb{R}$ from which it is possible to recover the marked metric graph Γ. A subset X of F_n is *spectrally rigid* if for any $\Gamma, \Gamma' \in CV_N$ the condition $||x||_\Gamma = ||x||_{\Gamma'}$ for all $x \in X$ implies that $\Gamma = \Gamma'$ in CV_n. A surprising result [11] asserts that there are no finite spectrally rigid subsets of F_N for $N \geq 3$, thus showing that spectral rigidity in the free group behaves very differently from its analogue for surfaces.

The next step in understanding spectral rigidity becomes to find sparse subsets of F_N which are spectrally rigid. Ilya Kapovich [9] proved that the trace of almost every non-backtracking random walk on the standard Cayley graph of F_N is spectrally rigid. Together with S. Francaviglia, I. Kapovich and A. Martino [6], we give a lot of explicit examples of spectrally rigid sets:

Theorem. Let $N \geq 3$, $1 \neq g \in F_N$ and a normal subgroup $H \triangleleft \mathrm{Aut}(F_N)$ which has infinite image in $\mathrm{Out}(F_N)$. Then Hg is spectrally rigid.

There are two main ingredients for the proof. Arguments using Lipschitz distortion show that the result holds for $H = \mathrm{Aut}(F_N)$ and g a basis element, i.e. the set of primitive elements is spectrally rigid. Then the general case is reduced to the set of primitive elements using the space of currents.

We record some questions that arise from this work. Given a subgroup $H < Aut(F_N)$, does the spectral rigidity of Hg depend on the non-trivial element $g \in F_N$? Is it true that the set of primitive elements is *strongly spectrally rigid*, i.e. does the translation lengths of primitive elements uniquely determine any point in the compactification of Outer Space?

Train tracks and free products

(Relative) train tracks for free groups were introduced in [4] as an analogue to the Nielsen-Thurston normal form for mapping classes. Their study enabled to establish the Scott conjecture : the rank of the fixed subgroup of an automorphism of F_N is at most N. This result was generalized for free product in [8].

The study of train tracks also led to a proof by Bestvina, Feighn and Handel [2], [3] that $\mathrm{Out}(F_N)$ satisfies the Tits' alternative: any subgroup of $\mathrm{Out}(F_N)$ is either virtually solvable or contains a non-abelian free subgroup. This had been previously shown to hold for mapping class groups in [10]. Recently, Bestvina [1] gave a short proof of the existence of train tracks for fully irreducible automorphisms using Lipschitz distortion. I am interested in finding suitable generalizations of these results to the setting of free products. In view reductions in [5], such a generalization could lead to showing that $\mathrm{Out}(G)$ satisfies the Tits alternative for any hyperbolic group G.

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Christopher Cashen

Université de Caen Basse-Normandie

I am interested in algorithmic and classification questions for finitely generated groups.

A current project with Natasa Macura is to classify mapping tori of free group automorphisms up to quasi-isometry. A related project with Gilbert Levitt is to understand isomorphisms of these mapping tori, and how different automorphisms can be if they yield isomorphic mapping tori.

I am interested in graphs of groups with cyclic edge groups, and vertex groups belonging to various different classes: abelian groups, free groups, surface groups, hyperbolic groups, etc.

Quasi-isometries of such graphs of groups are understood to varying degrees. I have completed the classification in the abelian case. The free group and surface cases are works in progress.

One attractive class that falls into this family are the Right Angled Artin Groups whose defining graph is a tree. These can be realized as a particular type of graph of groups with $\mathbb{Z}^2$ vertex groups and cyclic edge groups. From the above mentioned classification (and also known by work of Behrstock and Neumann) all such groups are quasi-isometric once the diameter of the defining tree is at least 3. The commensurability classification for these groups is not known. It is known that the diameter 3 trees are all commensurable to each other, and it is conjectured that there are infinitely many commensurability classes among the diameter 4 trees. Pallavi Dani and I are currently working on this problem.

One could also allow all of the above types of vertex groups. Such graphs of groups include many examples of limit groups, and understanding the quasi-isometry classification of this family might provide an example of a group quasi-isometric to a limit group that is not virtually a limit group. Such a group is conjectured to exist, but no example is known.

Corina Ciobotaru

Université catholique de Louvain

Locally compact groups acting on trees: algebraic structure and unitary representations

This research project is intended to achieve a doctoral thesis under the direction of Prof. Pierre-Emmanuel Caprace. The subject concerns the theory of topological locally compact groups, and more specifically the interactions between the algebraic structure of these groups and the properties of its unitary representations on Hilbert spaces.

Locally compact groups are an important family of topological groups, the most important examples are provided by the Lie groups, that could be called continuous groups of transformation, and which play an essential role in many branches of mathematics and physics. Beyond the Lie groups, the structure of locally compact connected groups has been studied extensively in the course of the first half of the twentieth century. This work culminated with the solution of Hilbert's fifth problem, implying also that any locally compact connected group is a projective limit of Lie groups.

Later in the second half of the twentieth century, the community was also interested in unrelated groups, particularly groups totally disconnected but not discrete. For one thing this family group includes very important examples consisting of the linear groups over non-Archimedean local fields, such as p-adic fields. Beyond the essential differences between topological Lie groups and p-adic groups, many properties indicate the existence of analogies that bring these two strong families among them, their linear character, first of all, but several fundamental properties of their unitary representations on Hilbert spaces (Howe-Moore property, existence of Gelfand pairs in the semi-simple case). Furthermore, locally compact totally disconnected groups also include many non-linear examples, including groups of Kac-Moody and automorphism groups of trees.

The aim of this project is to study the most basic examples of totally disconnected locally compact non-linear groups, i.e. groups of automorphisms of locally finite trees. Based on the analysis of concrete examples, we will study the following issues, where T is a locally finite semi-regular: tree

- Under what conditions a closed subgroup of Aut(T) possesses the property of Howe-Moore (decay at infinity of matrix coefficients of irreducible unitary representations)?

- Under what conditions a closed subgroup of Aut(T) has a pair of Gelfand?

Each of these two properties is satisfied by the linear simple groups, and by the full group Aut(T). We also know a few other examples of subgroups of Aut(T) having the same properties (see [2] and [1]). All these examples act doubly transitively on all ends of T, and we will seek in particular to what extent this property is related to those mentioned above. The basis of this study will be made by the previous work [1], [2], [4].

Beyond the automorphism groups of trees, this work could lead to new insights of the link between property of Howe-Moore and algebraic structure of the group, complementing the results of [3].

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Paweł Ciosmak

University of Warsaw

I am a student at the fourth year of mathematics at the University of Warsaw. I am especially interested in topology of manifolds and analysis. According with this in the past I have taken such courses as Differential Geometry, Algebraic Topology, Complex Analysis, Functional Analysis and Partial Differential Equations. In this semester, among others, I am attending Algebraic Geometry, Theory of Probability II and Lie Groups courses and two seminars: Topology and Geometry of Manifolds, Mathematical Analysis and Differential Equations. Among my interests I can also mention physics (especially quantum physics), which I was studying in the past. I made simulations on emission of quantum dots, which were presented in paper "Emission of Self Assembled CdTe/ZnTe Quantum Dot Samples with Different Cap Thickness", by S. Nowak, T. Jakubczyk, M. Goryca, A. Golnik, P. Kossacki, P. Wojnar, J.A. Gaj and me.

In the third year of my studies I have taken part in a seminar "Large scale geometry", where I had a talk about Amenability. Material, on which talks were based came from two books: "Lectures on coarse geometry" by John Roe and "Large scale geometry" by Piotr Nowak and Guoliang Yu. I was also trying to understand connections between property A and coarse embeddability into Hilbert space. Then I changed topic and I wrote my thesis about Gromov hyperbolic spaces, titled "Hyperbolic spaces and Gromov boundary". I based it on two books: "Lectures on coarse geometry" by John Roe and "Sur les groupes hyperboliques d'apres Mikhael Gromov" by E. Ghys and P. de la Harpe, finding new proofs on some results. Now I am going to start working on spectral flow of the signature operator, subject presented in the paper "The spectral flow of the odd signature operator and higher Massey products" by Paul Kirk and Eric Klassen. However, before that, I will spend some time working on Morse-Smale functions on manifolds with boundary.

I am also a member of Scientific Circle of Mathematical Physics, where last semester's topic was operator theory. In this semester theory of Dirac operators on manifolds will be presented.

Jan Czajkowski

University of Wrocław

My main research interest is percolation in hyperbolic spaces $\mathbb{H}^n$. In percolation theory one is interested in random geometric structures (e. g. random subgraphs of a given graph) and how "big" are their components, called *clusters*. I am investigating some geometric properties of clusters in Bernoulli percolation on graphs of tilings of $\mathbb{H}^3$ and in its continuous version.

I also wish to prove the existence of non-uniqueness phase in that kind of percolation on $\mathbb{H}^3$ ($\mathbb{H}^n$), i. e. that for some values of parameter p of the Bernoulli percolation there are a. s. infinitely many infinite clusters. That was shown e. g. in [10] and [11] for similar kinds of percolation on nonamenable Cayley graphs and on $\mathbb{H}^n$, respectively.

Another problem I am considering is in the setting of Bernoulli percolation on an infinite complete binary tree (or, more generally, on a hyperbolic graph) with parameter p. I am trying to verify the connection between the probability that there is an open path from the root to a point of some fixed subset A of limit (Gromov boundary in general) of the tree—as a function of p—and the Haar (or maybe Poisson in general) measure of A.

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Dieter Degrijse

Katholieke Universiteit Leuven Campus Kortrijk

I am a third year Ph.D. student at K.U Leuven. My supervisors are Prof. Paul Igodt and Dr. Nansen Petrosyan. My research area is group cohomology and the study of classifying spaces with stabilizers in a family of subgroups. In particular, I study finiteness properties of groups and their classifying spaces. Here, we mainly focus on amenable groups and word-hyperbolic groups.

A classifying space of a discrete group G for a family of subgroups $\mathcal{F}$ is a terminal object in the homotopy category of G-CW complexes with stabilizers in $\mathcal{F}$ (see [11]). Such a space is also called a model for $E_{\mathcal{F}}G$. Even though a model for $E_{\mathcal{F}}G$ always exists, in general, it need not be of finite type or finite dimensional. Questions concerning finiteness properties of $E_{\mathcal{F}}G$, such as whether for a given type of group G and family $\mathcal{F}$ there exists a finite (dimensional) model for $E_{\mathcal{F}}G$, have been particularly motivated by isomorphism conjectures in K- and L-theory (see [2] and [5]). For example, if $\mathcal{VC}$ is the family of virtually cyclic subgroups a group G and R is a commutative ring with unit, then the K-theoretical Farrell-Jones conjecture for the group G says that the assembly map $\mathrm{H}^G_n(E_{\mathcal{VC}}G, K_R) \to \mathrm{K}_n(RG)$ is an isomorphism for any $n \in \mathbb{N}$. The Baum-Connes conjecture has a similar formulation involving $E_{\mathcal{FIN}}G$, where $\mathcal{FIN}$ is the family of finite subgroups.

Finite dimensional models for $E_{\mathcal{VC}}G$ have been constructed for several interesting classes of groups: for example word-hyperbolic groups (Juan-Pineda, Leary, [7]), relative hyperbolic groups (Lafont, Ortiz, [6]), virtually polycyclic groups (Lück, Weiermann, [10]) and CAT(0)-groups (Lück, [9]). In [3], Nansen Petrosyan and I showed that every elementary amenable group G of finite Hirsch length and cardinality $\aleph_n$ admits a finite dimensional model for $E_{\mathcal{VC}}G$.

The smallest possible dimension of a model for $E_{\mathcal{F}}G$ is called the geometric dimension of G for the family $\mathcal{F}$ and denoted by $\mathrm{gd}_{\mathcal{F}}(G)$. We study the behavior of the invariant $\mathrm{gd}_{\mathcal{F}}(G)$ under group extensions and explore the connection between the invariants $\mathrm{gd}_{\mathcal{FIN}}(G)$ and $\mathrm{gd}_{\mathcal{VC}}(G)$. In particular, we consider the question whether or not one has $\mathrm{gd}_{\mathcal{VC}}(G) \leq \mathrm{gd}_{\mathcal{FIN}}(G) + 1$. It has become apparent (e.g. [1] and [8]) that the class of (right-angled) Artin groups provides a rich source of examples and counterexamples, when it comes to finiteness properties of groups and their classifying spaces. Therefore, these groups will play an important role in my future research.

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Jonas Deré

Katholieke Universiteit Leuven Campus Kortrijk

My research is focused on the study of Anosov diffeomorphisms on infra-nilmanifolds. In 1971, H.L. Porteus gave a complete algebraic characterization of the compact flat Riemannian manifolds admitting an Anosov diffeomorphism; this was done by looking at the irreducible components of the rational holonomy representation associated with the manifold. The result of Porteus was generalised recently to the class of infra-nilmanifolds modeled on a free nilpotent group with an abelian holonomy group. The aim of my research is to generalise this result to all holonomy groups, thus to find a criterion to decide whether or not a given infra-nilmanifold modeled on a free nilpotent Lie group admits an Anosov diffeomorphism. Since I only started my PhD a few weeks before this research statement, there are no results yet.

Aleksander Doan

University of Warsaw

I am a third-year undergraduate student of mathematics and theoretical physics. My scientific interests focus on the geometry and topology of manifolds, particularly on the intersections of differential geometry and algebraic topology. As regards geometric group theory, I am especially interested in hyperbolic groups and the topology of hyperbolic manifolds.

Apart from the first two years obligatory courses I attended graduate courses in differential geometry, topology and functional analysis. I have also acquired some basic knowledge of Banach algebras and C^*-algebras. Recently I have been attending courses in algebraic topology, Lie groups and hyperbolic groups as well as a graduate seminar on the topology and geometry of manifolds.

Currently I am working under the supervision of Dr Maciej Bobieński on my bachelor thesis on the Milnor fibration and the Picard-Lefschetz formula. On my own, I am trying to broaden my knowledge of differential topology and singularity theory.

Maciej Dołęga

University of Wrocław

Representation theory of symmetric group $\mathfrak{S}_n$

Representation theory of symmetric group $\mathfrak{S}_n$ is very well known for each $n \in \mathbb{N}$. However, symmetric groups can be seen as an inductive chain of groups $\mathfrak{S}_1 \subset \mathfrak{S}_2 \subset \dots$ and we can ask a lot of questions about sequences of representations corresponding to this chain; for example we can ask what can we say about asymptotics of such representations when $n \to \infty$.

One of the most efficient tool in representation theory are characters. Irreducible representations of $\mathfrak{S}_n$ are indexed by Young diagrams of size n. For a given Young diagram λ we would like to look at character $\chi_\lambda(\pi)$ where $\pi \in \mathfrak{S}_n$. There are several ways of counting this number and probably one of the most well-known is Murnaghan-Nakayama rule. Unfortunately, all these rules are very complicated when $n \to \infty$ and seem useless in this situation. The way to overcome this difficulty is to fix an element $\pi \in \mathfrak{S}_k$ and look at characters as functions defined on set $\mathbb{Y}$ of Young diagrams. More precisely, we define a normalized character Σ by a formula

$$\Sigma_\pi^\lambda = \begin{cases} n(n-1)\cdots(n-k+1)\frac{\chi_\lambda(\pi)}{\chi_\lambda(e)} & \text{if } k \leq n, \\ 0 & \text{in other cases} \end{cases},$$

where $\pi \in \mathfrak{S}_k \subset \mathfrak{S}_n$ for $k \leq n$.

There is a beautiful formula for normalized character due to Féray and Śniady [2]. We can reformulate this formula and connect it with some geometrical objects called bipartite oriented maps.

Bipartite maps

Bipartite map is a bipartite graph which is embedded into some surface, which means that a surface with removed image of this graph is homeomorphic to the collection of open discs. Formula for normalized characters found by Féray and Śniady can be expressed by some combinatorial coloring of oriented bipartite maps by Young diagram (see also [1]). We would like to find a similar formula in a much more general situation.

Jack characters

Expanding Schur symmetric functions s_λ in power symmetric functions basis we have that:

$$s_\lambda = \sum_{|\mu|=|\lambda|} \chi^\lambda(\mu)\frac{p_\mu}{z_\mu},$$

where z_μ is some combinatorial factor depending only on μ. Jack symmetric function J_λ^α is a symmetric function with an additional parameter $\alpha \in \mathbb{R}_+$ which can be treated as a continuous deformation of Schur symmetric function, i.e. $J_\lambda^1 = s_\lambda$. Expanding it in terms of power symmetric functions basis we can obtain a continuous deformation of irreducible characters:

$$J_\lambda^\alpha = \sum_{|\mu|=|\lambda|} \chi^{(\alpha,\lambda)}(\mu)\frac{p_\mu}{z_\mu}.$$

We can normalize them in the same way as irreducible characters to obtain so-called Jack characters $\Sigma_\mu^{(\alpha,\lambda)}$.

It seems that we can find a very similar formula for $\Sigma_\mu^{(\alpha,\lambda)}$ to a formula found by Féray and Śniady. The difference is that now we are considering not only orientable maps and we are considering some polynomial in α given by each map which is called by us a weight of a map. In this general situation connection with geometry seems to be much deeper than before. Our research plans are focused on understanding a mysterious weight of the map we found, which seems to code an Euler characteristic of a map and changes of orientability of a map under some transformation.

The connection between symmetric functions and geometry we found is very surprising and interesting in our opinion. There is also an interesting question to see if Jack characters are characters of some groups or algebras which deform an ordinary symmetric group.

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Spencer Dowdall

University of Illinois

My research focuses on mapping class groups and negative curvature phenomena in Teichmüller space; I am also interested in several aspects of geometric group theory. My thesis was about pseudo-Anosov dilatations in a particular subgroup of the mapping class group known as the *point-pushing subgroup*, and my recent joint work with Moon Duchin and Howard Masur has shown that Teichmüller space is hyperbolic in a certain 'statistical' sense. Currently, I am studying subgroups of the mapping class group which act convex-cocompactly on Teichmüller space, and I am also investigating almost convexity in the context of certain nilpotent and solvable groups. Below, I will briefly expand on each of these projects.

Point-pushing pseudo-Anosovs

Let $\Sigma = \Sigma_{g,n}$ denote a genus g surface with $n \geq 0$ punctures. Fix a basepoint $* \in \Sigma$. The *point-pushing homomorphism* $\mathcal{P}\colon \pi_1(\Sigma, *) \to \mathrm{Mod}(\Sigma, *)$ is a map which associates to each closed curve $\gamma \subset \Sigma$ based at $*$ an element of the *based mapping class group* of $(\Sigma, *)$. In the case that this mapping class $\mathcal{P}(\gamma)$ is pseudo-Anosov, I have shown that the pseudo-Anosov dilatation λ_γ of $\mathcal{P}(\gamma)$ is bounded, above and below, in terms of the self-intersection number of the curve γ. My research has also established bounds, in terms of g and n, on the least pseudo-Anosov dilatation attained in the subgroup $\mathcal{P}(\pi_1(\Sigma, *)) \leq \mathrm{Mod}(\Sigma, *)$.

Negative curvature in Teichmüller space

The *Teichmüller space* of Σ is the space $\mathrm{Teich}(\Sigma)$ of (isotopy classes) of marked hyperbolic structures on Σ. While not actually hyperbolic, there is a strong analogy between $\mathrm{Teich}(\Sigma)$ and a negatively curved space, and I am interested in further understanding the subtleties of this analogy. For instance, Moon Duchin, Howard Masur and I have recently shown that $\mathrm{Teich}(\Sigma)$ is *statistically hyperbolic* in the following sense: For any ball $\mathcal{B}_r(x)$ of radius r in $\mathrm{Teich}(\Sigma)$, we have shown that the average Teichmüller distance between a pair of points in $\mathcal{B}_r(x)$ is asymptotic to $2r$ as $r \to \infty$. More geometrically, our method was to show that for most pairs $y, z \in \mathcal{B}_r(x)$, the Teichmüller geodesic joining y and z dips back near the center of the ball, which is a property that is reminiscent of δ-hyperbolicity.

Christopher Leininger and I are also studying *convex cocompact subgroups* of the mapping class group. These are subgroups which, by analogy with Kleinian groups, have a well-defined limit set in the boundary of $\mathrm{Teich}(\Sigma)$ and act cocompactly on the weak convex hull of this limit set. These groups exhibit interesting dynamical properties in $\mathrm{Teich}(\Sigma)$ and are also closely related to the hyperbolicity of surface group extensions. However, our understanding of convex cocompact subgroups is severely hindered by a lack of rich families of examples. Leininger and I hope to address some of these issues by focusing on subgroups of $\pi_1(M_f) \leq \mathrm{Mod}(\Sigma, *)$, where M_f is the mapping torus of a pseudo-Anosov $f \in \mathrm{Mod}(\Sigma)$, and using the rich geometric structure of these fibered hyperbolic 3-manifolds.

Almost convexity

I am also exploring the algebraic implications of *almost convexity*. This geometric property, introduced by Cannon, depends on the Cayley graph of a group and essentially measures how difficult it is to connect two points in a metric ball without leaving that ball. When satisfied, there are efficient algorithms for constructing the Cayley graph recursively and for calculating in the group. Several authors have investigated almost convexity in specific solvable groups, and their findings have led Farb to conjecture that a finitely generated solvable group can only be almost convex if it is virtually nilpotent. I am working to prove this conjecture in the context of nilpotent-by-abelian groups. These groups have rich algebraic and geometric structures that I hope to exploit in order to show that they are generally not almost convex.

Dennis Dreesen

Université catholique de Louvain

The behaviour of (equivariant) Hilbert space compression under group constructions

Let H be a finitely generated group equipped with the word length metric relative to a finite symmetric generating subset. Uniform embeddability of H into a Hilbert space is well studied due to its relations with the Novikov and Baum-Connes conjectures [5] [7]. The Hilbert space compression of a group indicates *how well* a certain group embeds uniformly into a Hilbert space. Here, there are connections with Yu's property (A) [4].

More precisely, the Hilbert space compression of a finitely generated group G is a number between 0 and 1 that describes how close a uniform embedding $f : G \to l^2(\mathbb{Z})$ can be to being quasi-isometric. If this number is strictly greater than $1/2$, then the group satisfies Yu's property (A) [4]. The *equivariant* Hilbert space compression only takes into account those uniform embeddings which are G-equivariant relative to some affine isometric action of G on $l^2(\mathbb{Z})$ and the left multiplication action of G on itself. If this number is strictly greater than $1/2$, then the group is amenable [4].

I have studied the behaviour of equivariant and non-equivariant compression under group constructions. Exact results were obtained for the behaviour of equivariant compression under free products and HNN-extensions over finite groups. We have obtained partial results in a variety of other cases (the behaviour of compression under group extensions, under directed limits, etc) [1] [2].

I am currently working on generalizing Yu's result which states that hyperbolic groups admit a metrically proper affine isometric action on an l^p-space for p sufficiently large [6]. I am also trying to generalize our approach for calculating the equivariant compression of Baumslag-Solitar groups [3] to a more extensive class of groups.

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Matthew Durham

University of Illinois

My research, under the supervision of Daniel Groves, is focused on the actions of finite subgroups of the mapping class group of a surface, $Mod(S_{g,n})$, on its Teichmüller space, $T_{g,n}$. In particular, I am interested in understanding how much can be said in general about fixed sets and almost-fixed point sets of such subgroups in the Teichmüller and Weil-Petersson metrics. $T_{g,n}$ is dramatically different under these two metrics, so the tools used to study them and ultimately the amount which can be said about the nature of fixed and almost-fixed point sets in each will be similarly different.

The solution to the Nielsen Realization Problem by Kerckoff (and later Wolpert) showed that every finite subgroup of $Mod(S_{g,n})$ fixes a point of $T_{g,n}$, that is, arises as a group of isometries for some hyperbolic metric on $S_{g,n}$. Once realizing a finite subgroup $H \leq Mod(S_{g,n})$ as a group of isometries of $X \in T_{g,n}$, one can consider the quotient space, $X/H = O_H$, a hyperbolic orbifold. The space of hyperbolic orbifold structures on O_H, its orbifold Teichmüller space can be isometrically embedded into $T_{g,n}$ in a natural way. What's more, the embedded copy of $T(O_H)$ will be precisely the fixed point set of the action of H on $T_{g,n}$. A natural question is then how $T(O_H)$ sits inside of $T_{g,n}$ and how that is answered depends on the metric perspective taken.

The two most studied metrics on $T_{g,n}$ are the aforementioned Teichmüller metric and the Weil-Petersson metric; $Mod(S_{g,n})$ acts by isometries on $T_{g,n}$ in both metrics. In each of the metrics, $T_{g,n}$ is a uniquely geodesic metric space, but they differ as one approaches the edges of $T_{g,n}$. In particular, $T_{g,n}$ with the Teichmüller metric is complete, while with the Weil-Petersson metric it is a CAT(0) space, but not vice-versa. In both cases, I am interested in studying the level and sublevel sets of a diameter map, $diam_H : T_{g,n} \to [0, \infty)$, given by $diam_H(X) = diam_m(H \cdot X)$, where m is either the Teichmüller or Weil-Petersson metric.

$T_{g,n}$ with the Weil-Petersson metric is a Kähler manifold and a $CAT(0)$ space, so following the work of Wolpert and others, some of the following analytic questions are of interest: For small $k > 0$, are the level sets $diam_H^{-1}(k)$ connected submanifolds and are the sublevel sets $diam_H^{-1}([0, k])$ regular neighborhoods of $Fix(H) = T(O_H)$? If so, what is the bound on k so that this is true and on what does it depend? Is $diam_H$ a Morse function and, if so, what are its critical values? In the Teichmüller metric, such finer analytic questions are less tractable. I intend to use the hierarchies of Masur-Minky and Rafi's combinatorial model to investigate the situation's coarse geometry.

Kamil Duszenko

University of Wrocław

My current research is focused on Coxeter groups and their actions on non-positively curved and negatively curved spaces. Recall that a Coxeter group is a group given by the presentation $\langle S | \{(st)^{m_{st}} = 1\}_{s,t\in S}\rangle$, where S is finite and $m_{ss} = 1$, $m_{st} = m_{ts} \in \{2, 3, 4, \ldots, \infty\}$ for $s \neq t$. To be more specific, I would like to determine the smallest possible dimension of a CAT(0) space on which a given Coxeter group can act without a global fixed point. Also, I am addressing the question if every non-affine Coxeter group acts non-trivially on a negatively curved space, or even admits a non-elementary Gromov-hyperbolic quotient. Every Coxeter group acts on the non-positively curved *Davis complex*, however, its dimension is usually far from being optimal. On the other hand, the existence of an action on a negatively curved space is known only in some specific cases. The dimension of such a negatively curved space can be bounded from below by a number related to the rank of a minimal non-affine special subgroup of W.

I have shown, among other things, that all *minimal non-affine* Coxeter groups can be mapped surjectively onto non-elementary groups acting geometrically on negatively curved spaces [1] (in particular, these quotients are Gromov-hyperbolic). As an example how such results can be applied, let us mention that the above result led to a proof that *all* non-affine Coxeter groups have unbounded reflection length, where the reflection length of an element w of a Coxeter group (W, S) is the smallest n such that w is equal to the product of n reflections, i.e. elements of the form vsv^{-1}, where $v \in V$ and $s \in S$.

[1] K. Duszenko, *Reflection length in non-affine Coxeter groups*, arXiv:1106.0619.

Ahmed Elsawy

Universität Düsseldorf

I am a PhD student under supervision of Prof. Oleg Bogopolski in Düsseldorf university. The following is a part of my Ph.D Thesis “Subgroup conjugacy separability for groups”.

On subgroup conjugacy separability for free product of groups

The subgroup conjugacy separability is a residual property of groups, which logically continues the following series of well known properties of groups: the residual finiteness (RF), the conjugacy separability (CS), and the subgroup separability (LERF).

Definition 1. A group G is called *subgroup conjugacy separable* (abbreviated as SCS), if for any two finitely generated non-conjugate subgroups $H_1, H_2 \leq G$, there exists a homomorphism ϕ from G to a finite group $\overline{G}$ such that $\phi(H_1)$ is not conjugate to $\phi(H_2)$ in $\overline{G}$.

There are a lot of papers on residually finite, conjugacy separable and LERF groups. However we know only one paper on SCS: in [2], F. Grunewald and D. Segal proved that all virtually polycyclic groups are SCS. In preprint [1], O. Bogopolski and F. Grunewald proved that free groups and some virtually free groups are SCS. Recently O. Bogopolski and K.-U. Bux proved that surface groups are SCS.

Our result is about a property, which is closely related to the SCS property.

Definition 2. A group G is called *subgroup into conjugacy separable* (abbreviated as SICS), if for any two finitely generated subgroups $H_1, H_2 \leq G$ the following holds:

if H_2 is not conjugate into H_1 in G, then there exists a homomorphism ϕ from G to a finite group $\overline{G}$ such that $\phi(H_1)$ is not conjugate into $\phi(H_2)$ in $\overline{G}$.

(Here we say that H_2 is conjugate into H_1 if there exists $g \in G$ such that $H_2^g \leq H_1$.)

Our main result is the following.

Theorem (O. Bogopolski, A.N. Elsawy). *The free product of two groups, which are SICS and LERF, is again SICS (and of course LERF).*

Future plans: I am thinking on a variant of this theorem for SCS and for amalgamated products and HNN-extensions over a malnormal cyclic subgroup. Also I am thinking on relationships between the properties RF, CS, LERF, SCS and SISC, and on the problem, which interesting classes of groups possess the last two properties.

[1] O. Bogopolski and F.J. Grunewald, *On subgroup conjugacy separability in the class of virtually free groups*, 2010. Available in arXiv:1012.5122.

[2] F.J. Grunewald and D. Segal, *Conjugacy in polycyclic groups*, Comm. Algebra **6** (1978), 775-798.

Gregory Fein

Rutgers University

I am a sixth year graduate student at Rutgers University-Newark being advised by Mark Feighn. My research interests have ranged a bit, but they all definitely fit under the umbrella of geometric group theory.

As of late, my main object of study has been a problem on the outer automorphism group of the free group, $Out(F_n)$. In a paper published earlier this year and entitled "The Recognition Theorem for $Out(F_n)$," Feighn and Handel proved a result which is able to recognize whether two outer automorphisms of the free group are the same. They did this by providing a list of qualitative and quantitative data which, taken together, uniquely determine an outer automorphism. However, their theorem only applies to what they call *forward rotationless* outer automorphisms. These are, roughly, outer automorphisms which have no periodic behavior under forward iteration. I have been working to generalize this result to all of $Out(F_n)$.

Now, it's a fact (also proved in the above paper of Feighn and Handel) that for every n, there is a constant $K = K(n)$ such that for all $\phi \in Out(F_n)$, ϕ^K is forward rotationless. This may make it seem like to generalize the recognition theorem will be a simple extension of the terms. However, once we remove the forward rotationless restriction, many serious issues arise. For example, at the heart of the recognition theorem is the ability to find a select group of preferred representative automorphisms (called *principal automorphisms*) of any outer automorphism by looking for the representatives with the largest fixed sets. However, when dealing with finite order behavior, if we take the most natural extension of the word "principal" by instead looking for large periodic sets, then we find that there are outer automorphisms which have no principal representatives under this new definition. This is because there exist finite order automorphisms with no finite order automorphism representatives. As a result, we cannot use this definition of principal, and we need to look elsewhere for our identifying data. However, I've found ways to work around this and other obstructions, and I hope to reach a final result in the near future.

I've also spent some time studying limit groups (otherwise known as fully residually free groups) from a geometric point of view. In particular, I studied some of Sela's work on Diophantine geometry and related papers on finitely generated groups acting freely on $\mathbb{R}^n$-trees.

Elisabeth Fink

University of Oxford

I am a third year PhD student at the University of Oxford and working on branch groups. Such arise as automorphism groups of rooted trees. In particular I am interested in finitely generated groups and their profinite completions. A special case of such are the so called 'spinal' groups. These are groups where the finitely many generators split up into two classes:

1. *rooted automorphisms*, which only have non trivial action at the root vertex, and
2. *spinal automorphisms*, which act on exactly two vertices on each level. On one of them as a rooted automorphism and on the other recursively as a spinal automorphism.

Such spinal groups have already been studied on regular trees. In my work, motivated by suggestion for a construction in my supervisor's paper [1], I study such groups on irregular trees. Whereas there are already good results for regular trees in the literature, irregular trees have not been studied very well yet. In my work I am currently focusing on two aspects of them:

1. Their profinite completion in comparison to their congruence and branch completion and
2. their Hausdorff dimension within the full automorphism group of the rooted tree.

Completions

Given any abstract infinite group G we can look at its profinite completion $\hat{G}$. If this group is defined as a finitely generated group on a rooted tree then considering level stabiliser subgroups we get a natural congruence filtration, leading to the congruence completion $\bar{G}$. Using the terminology from [2] we get another completion from the restricted level stabilisers, the branch completion $\tilde{G}$. With these we have natural maps

$$\hat{G} \longrightarrow \tilde{G} \longleftrightarrow \bar{G}$$

and are in particular interested in their kernels.

Hausdorff Dimension

As defined in fractal geometry we can look at the Hausdorff dimension of branch groups within the full automorphism group as described in [3]. The set of dimensions these groups can take within $[0, 1]$ is called the *Hausdorff spectrum*. B. Klopsch proved in his PhD thesis that branch groups, such as the full automorphism group on rooted trees, always have full Hausdorff spectrum. It is however not clear, which dimensions come from finitely generated groups and in particular which dimensions spinal groups can have. Some results for regular trees show that spinal groups over the binary tree can have arbitrary Hausdorff dimension within $[0, 1]$, whereas over regular trees of degree $p > 2$ they can only be rational and within a certain interval.

In my work I am giving examples of Hausdorff dimensions of spinal groups defined over irregular trees. For example trees with valency $p_i + 1$ on each level for a sequence of distinct primes p_i. I aim to do further computations and would ideally like to find the Hausdorff spectrum of spinal groups over such irregular trees. This depends very much on the choice of the valency sequence as my work shows.

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[2] MR1765119 (2002f:20044) Grigorchuk, R. I. Just infinite branch groups. New horizons in pro-pp groups, 121179, Progr. Math., 184, Birkhuser Boston, Boston, MA, 2000, 20F14 (20E08 20E18)

[3] MR1422889 (98b:20041) Barnea, Yiftach; Shalev, Aner Hausdorff dimension, pro-pp groups, and Kac-Moody algebras. Trans. Amer. Math. Soc. 349 (1997), no. 12, 50735091. (Reviewer: Marcus du Sautoy), 20E18 (17B67)

Martin Fluch

Universität Bielefeld

Classifying spaces with non-trivial stabilisers and finiteness conditions in Bredon cohomology

Classifying spaces with stabilisers in a prescribed family of groups and their finiteness conditions form an important part of various areas in pure mathematics such as group theory, algebraic topology and geometric topology. Two major examples are recent achievements in studying such celebrated conjectures as the Baum–Connes conjecture and the Farrell–Jones conjecture [1]. Progress in studying these conjectures relies heavily on understanding finiteness conditions of some specific classifying spaces.

Similar to how the cohomology of groups can be used to study the finiteness properties of Eilenberg–Mac Lane spaces $K(G,1)$ there exists a homology theory to study classifying spaces with non-trivial stabilisers: Bredon cohomology. In a nutshell it generalises classical cohomology of groups by replacing the group G by the orbit category $\mathcal{O}_{\mathfrak{F}}G$. Instead of the category of G-modules one studies the category of $\mathcal{O}_{\mathfrak{F}}G$-modules. The properties of this category depend not only on the group G but also on the family $\mathfrak{F}$ of subgroups of G. If one considers the trivial family $\mathfrak{F} = \{1\}$, then one obtains the classical case. The family $\mathfrak{F}_{\mathrm{fin}}(G)$ of finite subgroups of G is important for the Baum–Connes conjecture, and the family $\mathfrak{F}_{\mathrm{vc}}(G)$ of virtually cyclic[1] subgroups of G appears in the Farrell–Jones conjecture.

In the classical case there exist numerous invariants and finiteness conditions a group can have or satisfy, see for example [Chapter VIII][2]. Each of these conditions generalises to the Bredon setting. My research interest lies in these algebraic and geometric finiteness conditions and how they relate to each other and how they depend on the family $\mathfrak{F}$. Some problems I am working on are the following.

In the classical case and for $\mathfrak{F} = \mathfrak{F}_{\mathrm{fin}}(G)$ there are many classes of groups for which there are constructions known for classifying spaces $E_{\mathfrak{F}}G$ with nice properties [3]. However, the situation gets much more complicated for the family $\mathfrak{F}_{\mathrm{vc}}(G)$ and far less is known in this case. One interest of mine is to construct classifying spaces for groups G with virtually cyclic stabilisers and nice geometric properties, see for example [4]. At the moment I am interested in models for $E_{\mathfrak{F}_{\mathrm{vc}}}G$ where G is the infinite cyclic extension $B \rtimes \mathbb{Z}$ of a group B and B is either a free group of rank ≥ 2 or a fundamental group of a finite graph of finite groups. Another class of groups for which I want to construct models for $E_{\mathfrak{F}_{\mathrm{vc}}}G$ are generalised Baumslag–Solitar groups, that is fundamental groups of finite graphs of infinite cyclic groups.

Another project of mine is to find a counter example for the Eilenberg–Ganea conjecture for Bredon cohomology in the case that $\mathfrak{F} = \mathfrak{F}_{\mathrm{vc}}(G)$, as has been done in [5] for $\mathfrak{F} = \mathfrak{F}_{\mathrm{fin}}(G)$. The Eilenberg–Ganea conjecture, originaly formulated for the classical case, states that the geometric

[1] A group is virtually cyclic if it has a cyclic subgroup of finite index.

and cohomological dimension of a group always agree. We want to show that there exists groups G such that the geometric dimension $\mathrm{gd}_{\mathfrak{F}}G = 3$ but cohomological dimension $\mathrm{cd}_{\mathfrak{F}}G = 2$ when $\mathfrak{F} = \mathfrak{F}_{\mathrm{VC}}(G)$.

A group is of type $\mathfrak{F}$-FP_n if the trivial $\mathcal{O}_{\mathfrak{F}}G$-module $\underline{\mathbb{Z}}$ admits a projective resolution $P_* \twoheadrightarrow \mathbb{Z}$ with P_k finitely generated for all $k \leq n$. Unlike in the classical case there is hardly any result known regarding this finiteness property for general families $\mathfrak{F}$. In the classical case Brown's Criterion for FP_n provided a useful tool for studying this finiteness condition. At the moment I am working on generalising this criterion to the Bredon setting and I am interested in finding applications for it.

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Łukasz Garncarek

University of Wrocław

I am a first year PhD student at the Mathematical Institute of the University of Wrocław. My scientific interests revolve around group theory and geometry. So far, my research concerned analytic aspects of groups. Currently, I am trying to find a suitable topic for my PhD thesis.

In my MSc thesis I studied the irreducibility of some representations of subgroups of the group $\mathrm{Diff}_c(M)$ of compactly supported diffeomorphisms of a smooth manifold M. If we take a measure μ on M, having smooth density with respect to the Lebesgue measure in maps, we may define a unitary representation π of $\mathrm{Diff}_c(M)$ on $L^2(M, \mu)$ by

$$(*) \qquad \pi_s(\phi)f = \left(\frac{d\phi_*\mu}{d\mu}\right)^{1/2+is} f \circ \phi^{-1},$$

where $s \in \mathbb{R}$.

The case of the group of compactly supported diffeomorphisms preserving a measure was described by Vershik, Gelfand and Graev. In my thesis I considered the irreducibility of representations $(*)$ for the groups $\mathrm{Sympl}_c(M)$ and $\mathrm{Cont}_c(M)$ of compactly supported symplectomorphisms and contactomorphisms.

After the "large" groups of diffeomorphisms, I studied representations of Thompson's groups F and T, which are finitely presented, hence "small" in the combinatorial sense. Their natural actions on the unit interval and unit circle however still resemble the action of a "large" group, and the representations (∗) of F and T turn out to be irreducible. Moreover, representations π_s and π_t are inequivalent, provided that $s - t$ is not a multiple of $2\pi/\log 2$.

In the representation theory of $SL_2(\mathbb{R})$, the representations of the form (∗), associated to the natural action of $SL_2(\mathbb{R})$ on $\mathbb{P}(\mathbb{R}^2)$, form a part of the principal series. They are induced from one-dimensional representations of subgroups of $SL_2(\mathbb{R})$. This is not the case for the Thompson's groups. In fact, π_s are nonequivalent to representations induced from finite-dimensional representations of proper subgroups of F or T. Hence, the two possible generalizations of the principal series to F and T are disjoint.

My work on representations of the group of contactomorphisms has inspired the following question:

Problem. *Let G be a topological group. Suppose that G contains no nontrivial compact subgroups. Does it imply that the convolution algebra $\mathcal{M}_c(G)$ of compactly supported complex Borel measures on G has no zero divisors?*

The positive answer in the case of $\mathbb{R}^n$ is a variant of the Titchmarsh convolution theorem. The answer for locally compact abelian groups follows from the work of Benjamin Weiss. In a recent preprint I managed to give a positive answer for supersolvable Lie groups.

Ilya Gekhtman

University of Chicago

I am broadly interested in applying techniques from hyperbolic dynamics to Teichmüller theory. Recently, I have focused on developing analogues of Patterson-Sullivan theory for the action of subgroups of the mapping class group $Mod(S)$ on Teichmüller space $Teich(S)$ and its compactification by Thurston's sphere PMF of projective measured foliations. So far, I have been able to apply these techniques to convex-cocompact subgroups of mapping class groups, first defined in [4]. These are subgroups $G < Mod(S)$ which act cocompactly on their weak hull $WH(G)$ in $Teich(S) \cup PMF$, the union of Teichmüller geodesics between distinct points in the limit set $\Lambda(G) \subset PMF$. As a result $WH(G)$ is contained in the ϵ-thick part of $Teich(S)$ for some $\epsilon > 0$. A convex cocompact subgroup of $G < Mod(S)$ has the property that every limit point of G is uniquely ergodic, so every Teichmüller geodesic ray with vertical projective foliation $\lambda \in \Lambda(G)$ converges to λ in Thurston's compactification. As a result $WH(G)$ is contained in the ϵ thick part of $Teich(S)$ for some $\epsilon > 0$. It is known that for S a closed surface of genus at least 2, $Teich(S)$ endowed with the Teichmüller metric is not Gromov hyperbolic [7]. However, geodesic triangles with vertices in $Teich(S) \cup PMF$ and sides contained in the ϵ-thick part of $Teich(S)$ are δ-thin, where $\delta > 0$ depends only on ϵ. Using this fact, it is possible to construct the analogue of a conformal density: a G-equivariant equicontinuous family of measures ν_x $x \in Teich(S)$ on PMF with support $\Lambda(G)$ with

$$d\nu_x/d\nu_y([\alpha]) = e^{\delta(G)\beta_\alpha(x,y)}$$

where $\delta(G)$ is the exponent of convergence for G and

$$\beta_{[\alpha]}(x,y) = Ext_{[\alpha]}(x)/Ext_\alpha(y)$$

plays the role of Busemann functions for the Teichmüller metric, which are well defined for $\alpha \in MF$ uniquely ergodic. By appropriately scaling the product measure $\nu_x \otimes \nu_x$ we obtain a G invariant measure $\widetilde{\nu}$ on the space of Teichmüller geodesics and thus a geodesic flow invariant measure on the unit cotangent bundle of $Teich(S)$ which can be identified with the space $Q^1(S)$ of unit area quadratic differentials on S. This measure projects to a finite measure μ on $Q^1(S)/G$, which is the analogue of the Bowen-Margulis measure of maximal entropy in the negatively curved setting. An argument similar to [2] can be used to show μ is mixing as long as G contains pseudo-Anosovs f_1, f_2 with dilatations $\lambda(f_i)$ satisfying $\log \lambda(f_1)/\log \lambda(f_2) \notin \mathbb{Q}$ (this condition is referred to as nonarithmeticity of the length spectrum of $Teich(S)/G$). The Teichmüller geodesic flow is nonuniformly hyperbolic, and the nonuniformity can be uniformly controlled by requiring that geodesic segments spend a fixed fraction of the time in a compact subset of a stratum [1]. Using these facts I was able to prove the following.

Theorem. Let $G < Mod(S)$ be nonelementary convex cocompact, containing at least one pseudo-Anosov with axis in the principal stratum, and containing pseudo-Anosovs f_1, f_2 with dilatations $\lambda(f_i)$ satisfying

$$\log \lambda(f_1)/\log \lambda(f_2) \notin \mathbb{Q}.$$

Let $\lambda(G)$ be the Poincaré exponent of G. Let $x, y \in Teich(S)$. Then

$$\lim_{R\to\infty} e^{-\delta(G)R}|B_R(x) \cap \Gamma y| = \Lambda(x)\Lambda(y)$$

where Λ is a G invariant continuous function on $Teich(S)$. Let $N_G(R)$ be the number of pseudo-Anosov elements of G of Teichmüller translation length less than R. Then

$$\lim_{R\to\infty} N_G(R)Re^{-hR} = 1.$$

These estimates for orbit growth and growth of closed geodesics on $Teich(S)/G$ are analogous to those obtained by Roblin for quotients of $CAT(-1)$ spaces by geometrically finite groups and by Athreya, Bufetov, Eskin and Mirzakhani for the full mapping class group $Mod(S)$. I am interested in extending these results to other subgroups of $Mod(S)$. I am also exploring analogous counting problems for the Weil-Petersson metric and studying harmonic measures on PMF for the action of subgroups of $Mod(S)$.

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Dominik Gruber

Universität Wien

I have recently started working on my PhD thesis in geometric group theory under the supervision of Prof. Goulnara Arzhantseva. I have been studying lacunary hyperbolic groups and small cancellation conditions. One goal we have discussed is finding a small cancellation condition that produces a class of lacunary hyperbolic groups with useful additional properties and narrows the scope of the paper "Lacunary hyperbolic groups" by Ol'shanskii, Osin and Sapir.

Jiyoung Han

Seoul National University

The symmetric Riemannian spaces of $\mathbb{R}$-rank one are those which have no isometric embedding from $\mathbb{R}^n$, $n \geq 2$, with Euclidean metric, but only $\mathbb{R}^1$. It is known that there are only four kinds of symmetric Riemannian spaces of $\mathbb{R}$-rank one and of negative curvature; the real hyperbolic space $\mathrm{H}^n_{\mathbb{R}}$, the complex hyberbolic space $\mathrm{H}^n_{\mathbb{C}}$, the quaternion hyperbolic space $\mathrm{H}^n_{\mathbb{H}}$, and the Cayley hyperbolic plane $\mathrm{H}^2_{\mathbb{O}}$.

In short, the goal of my study is generalizing many results of the real hyperbolic spaces to other rank one spaces, which is about the asymptotic limits and the equidistribution problems of orbits of geometrically finite discrete subgroups of their isometry groups. For now, $\mathrm{H}^n_{\mathbb{C}}$ is mainly concerned with, and I intend to extend my interest to other rank one spaces after getting some results for the complex one.

One of the significant properties of $\mathrm{H}^n_{\mathbb{R}}$ to prove above problems is that there are codimension one geodesically closed subspaces in $\mathrm{H}^n_{\mathbb{R}}$ so that one can bring a special finite measure-called the Bowen-Margulis-Sullivan measure- defined on the boundary $\partial\mathrm{H}^n_{\mathbb{R}}$ into the unit tangent bundle of these subspaces. But unfortunately, there seems to be no codimension one geometrically closed subspaces in other rank one spaces, and at least it is true for the complex hyperbolic space.

Actually although they are similar in the way they were defined, the complex hyperbolic space is very different from the real hyperbolic space. For instance, the complex hyperbolic space does not have a constant curvature; it is pinched from -1 to $-\frac{1}{4}$. It means that the complex hyperbolic space is not fully homogeneous like the real one whose every geodesically closed subspace can be transformed to arbitrary geodesically closed subspace of the same dimension under the element of its isometry group.

Another significant difference is that the boundary of the complex hyperbolic space, which is of real codimension one, has a Heisenberg group structure considering the induced transformation group from the isometry group of the complex hyperbolic space. Mostly solving hyperbolic problems including equidistributions or rigidity, it is very important to understand the boundary structure and the action of the isometry group on it. In the case of real hyperbolic space $\mathrm{H}^n_{\mathbb{R}}$, the boundary of $\mathrm{H}^n_{\mathbb{R}}$ is just a Euclidean vector space $\mathbb{R}^{n-1}$, and the element of induced transformation group is euclidean similarity.

Because the one dimensional complex hyperbolic space $\mathrm{H}^1_{\mathbb{C}}$ is exactly $\mathrm{H}^2_{\mathbb{R}}$, by taking $\mathrm{H}^2_{\mathbb{C}}$ for a standard or easiest example, I am trying to understand the structure of complex hyperbolic structure such as geodesics, geodesically closed subspaces, and characters of its boundary.

Tobias Hartnick

Université de Genève

I am interested in problems of the interface between geometric group theory, ergodic geometry and algebraic topology. Within this context, I am mainly interested in actions of (locally compact) groups on non-positively curved spaces. The non-positive curvature assumption allows one to define a boundary action of the group; one may then ask the question how dynamical properties of the original action are reflected by the boundary action. Specifically, I am interested in comparing cohomological invariants. Often the dynamics of the original action is related to invariants coming from (continuous) cohommology of the group in question, whereas the boundary action is reflected in the (continuous) bounded group cohomology. There is always a natural comparison map between bounded and usual cohomology; if this map is an isomorphism one speaks of *cohomological boundary rigidity*. Using the boundary realization of bounded cohomology, this sort of rigidity can be related to various classical rigidity phenomena. For example, the classical Mostow rigidity theorem for hyperbolic 3-manifolds can be deduced from cohomological boundary rigidity of $PSL_2(\mathbb{C})$ in degree 3. Unfortunately, the extent to which cohomological boundary rigidity holds or fails to hold is hardly understood, even in simple examples. It is conjectured, that semisimple real algebraic groups are cohomologically boundary rigid in arbitrary degrees. This conjecture, which would imply a similar result for lattices in semisimple Lie groups (within a stable range), is still open. In joint work with Andreas Ott, we have established various partial results towards the conjecture. In particular, we proved that the comparison map is surjective for all semisimple Lie groups of Hodge type. I am also interested in how to deduce/extend classical rigidity results using cohomological boundary rigidity (joint with M. Bucher-Karlsson). Finally, their failure of cohomological boundary rigidity in degree 2 leads to the study of quasimorphisms. In various joint papers with Gabi Ben Simon, we have realized such quasimorphisms as relative growth functions associated with group actions on posets. This provides a strong connection between the theory of partially orderable groups and bounded cohomology. Originally, this connection arose from problems in (finite and infinite-dimensional) Lie groups, but it also applies to finitely generated groups, and I am currently interested in applications of our machinery to various classes of finitely generated groups.

Another, a priori unrelated field of interested of mine, is the geometry of Kac-Moody group over local fields. In joint work with Ralf Gramlich, Ralf Köhl and Andreas Mars, we have classified connected topological twin buildings (in the sense of L. Kramer) of so-called two-spherical tree type. Their automorphism groups are fixed points of involutions of complex split Kac-Moody groups, which can therefore be regarded as non-split real Kac-Moody groups. In the non-spherical case they are non-locally compact k_ω-groups, in particular, they form an important class of compactly presented groups, which are not locally compact. They may thus serve as a testing ground for studying various properties classically studied in the context of finitely presented groups, in a broader context. For example, I would like to know which non-split real Kac-Moody groups have Property (T).

Sebastian Hensel

Max-Planck Institut, Bonn

My research interest lies in a combination of low-dimensional topology and geometric group theory. Currently I am most interested in the geometry of mapping class groups of 3–manifolds.

The *handlebody group* $\mathrm{Map}(V_g)$ is the mapping class group of a handlebody V_g of genus g. It can be identified with a subgroup of the mapping class group of the boundary surface ∂V_g. In a joint project with Ursula Hamenstädt we study the extrinsic geometry of this subgroup.

Theorem. The handlebody group $\mathrm{Map}(V_g)$ is exponentially distorted in the mapping class group of ∂V_g for any $g \geq 2$.

As a consequence, the intrinsic geometry of the handlebody group cannot simply be inferred from the geometry of surface mapping class groups. In fact, the large-scale geometry of handlebody groups shows features that distinguish it from $\mathrm{Map}(\partial V_g)$.

Theorem. For any $g \geq 3$ there is a simple closed curve $\alpha \subset \partial V_g$ such that the stabilizer of α in $\mathrm{Map}(V_g)$ is exponentially distorted.

In contrast, the stabilizer of every simple closed curve is undistorted in $\mathrm{Map}(\partial V_g)$.

Question. Are stabilizers of disks undistorted in the handlebody group?

To further compare the geometry of $\mathrm{Map}(V_g)$ to $\mathrm{Map}(\partial V_g)$, one can consider Dehn functions. Surface mapping class groups are automatic, and thus have quadratic Dehn functions. In joint work with Ursula Hamenstädt we show the following upper bound on the Dehn function of the handlebody group.

Theorem. The Dehn function of $\mathrm{Map}(V_g)$ has at most exponential growth rate.

The proof of this theorem suggests that the exponential bound is not optimal. Thus we propose

Problem. Determine a sharp bound for the growth rate of the Dehn function of the handlebody group.

Let M_g be the three-manifold obtained by doubling a handlebody V_g across its boundary surface. By a theorem of Laudenbach, the natural map from the mapping class group of M_g to $\mathrm{Out}(F_g)$ has finite kernel. This description allows to study $\mathrm{Out}(F_g)$ using topological methods. As an application of this point of view we show in a joint project with Ursula Hamenstädt

Theorem. The natural embedding of the mapping class group of a once-punctured genus g surface $S_{g,1}$ into $\mathrm{Out}(F_{2g})$ is undistorted.

David Hume

University of Oxford

My primary research interest concerns embeddability of finitely generated groups and related metric structures into Banach spaces. More specifically, one could ask for a given group G and Banach space X for properties on functions $f : \mathbb{N} \to \mathbb{N}$ which guarantee the existence of an embedding $\phi : G \to X$ such that

$$f(d_G(x,y)) \preceq \|\phi(x) - \phi(y)\|_X \preceq d_G(x,y).$$

As an example in the case of the infinite binary tree, Tessera proves that such a function f bounds an embedding into a Hilbert space if and only if

$$\sum_{n\in\mathbb{N}} \frac{1}{n}\left(\frac{f(n)}{n}\right)^2 < \infty.$$

The question is also closed for hyperbolic spaces with the same collection of functions. Some very partial results exist for free products and amalgamated products over finite subgroups (due to this definition being a quasi-isometry invariant). In particular, Dreesen proves that given embeddings

ψ_i of groups A_i into a Hilbert space $\mathcal{H}$ and a function f with the above property for both of these embeddings, then there is an embedding $\phi : A_1 * A_2 \to \mathcal{H}$ such that

$$\min\{d_G(x,y)^{\frac{1}{2}}, f(d_G(x,y))\} \preceq \|\phi(x) - \phi(y)\|_X \preceq d_G(x,y).$$

Additionally, Brodskiy and Higes proved that $A * B$ quasi-isometrically embeds in a finite product of trees whenever A and B both do. In light of Tessera's result this gives the optimal outcome in this situation, unless $A = B = \mathbb{Z}_2$.

On the other hand for each $\alpha \in [0,1]$ there exists a finitely generated group G_α such that given any f for which there exists an embedding into Hilbert space with

$$f(d_G(x,y)) \preceq \|\phi(x) - \phi(y)\|_X \preceq d_G(x,y),$$

$f(n) \preceq n^{\alpha\epsilon}$ for any $\epsilon > 0$.

Alaa Jamal Eddine

Université d'Orléans

My name is Alaa Jamal Eddine. I am in my third year of PhD thesis. My PhD Thesis deals with the Evolution Equations on discrete hyperbolic spaces. During the last two years, I intended to understand the nonlinear Schrödinger equation on discrete hyperbolic stuctures. I have obtained a complete resolution of the equation

$$\begin{cases} i\partial_t u \ \mathcal{L}u \ = \ \lambda \, |u|^{\gamma-1}u, \\ u(0) = f \end{cases}$$

on homogeneous trees $\mathbb{T}$ of degree $Q1$, $Q \geq 2$, where $\lambda \in \mathbb{R}$, and for $x \in \mathbb{T}$, $\mathcal{L}f(x) = f(x) - \frac{1}{Q1}\sum_{y;d(x,y)=1} f(y)$ is the positive discrete Laplacian on $\mathbb{T}$, and $f \in L^2(\mathbb{T})$. I have obtained a local and global well-posedness for solutions in $L^2(\mathbb{T})$ and for all $\gamma > 1$. I also obtained scattering for small L^2 data. Right now I am studying the same problem but on more abstract geometric structures, like the free product of two groups. It requires the knowledge of group theoretic aspects and abstract harmonic analysis on these groups.

Katarzyna Jankiewicz

University of Warsaw

I am a first year MSc student at the University of Warsaw. My mathematical interests are centered around analysis and geometry. During my first degree I have taken courses in differential equations, functional analysis, differential geometry, topology and algebra. Last year I attended bachelor's seminar on Lie groups and I wrote a thesis under the supervision of Prof. Stefan Jackowski about Fuchsian groups, especially (2,3,7)-triangle group and its significance. This year I am attending courses in algebraic geometry, Lie groups, hyperbolic groups, and two seminars: on topology and geometry of manifolds and on mathematical analysis and differential equations.

Pierre-Nicolas Jolissaint

Université de Neuchâtel

My main interest is the study of embeddings of metric spaces into Banach spaces. More precisely, let (X, d), (Y, δ) be two finite metric spaces. If $F : X \to Y$ is bi-Lipschitz, we define the distortion of F as

$$Dist(F) := \max_{x \neq y} \frac{\delta(F(x), F(y))}{d(x, y)} \max_{x \neq y} \frac{d(x, y)}{\delta(F(x), F(y))}.$$

Then, the Y-distortion of X is defined as the infimum of the $Dist(F)$, where the infimum is taken over all bi-Lipschitz maps from X to Y. This quantity gives an idea of how different two metric spaces can be. I am currently working on methods to find bounds for the distortion and apply them to Cayley graphs. I am also interested in making a precise link between the compression exponent of an infinite metric space and the distortion of its finite subsets with respect to the induced metric.

Paweł Józiak

University of Wrocław

1. Geometric, analytic, representation-theoretic and dynamical properties of groups, like: Kazhdan property (T), amenability, a-T-menability, weak amenability, Cowling-Haagerup constant, ergodic actions and their equivalence relations, weak containment of groups, finite dimensional approximations, notions of sofic and hyperlinear groups, their applications.

2. Geometric Methods in functional analysis: K-theory and K-homology of operator algebras, Baum-Connes conjecture, L^2-Betti numbers.

3. Graph theory: Bass-Serre theory, Expander graphs in sense of Lubotzky, approximations of certain spaces by trees.

Dawid Kielak

University of Oxford

I am interested in the nature of the apparent similarities between lattices in semi-simple Lie groups (these include many arithmetic groups, e.g. $\mathrm{GL}_n(\mathbb{Z})$), Mapping Class Groups (i.e. groups of homeomorphisms of a given surface modulo isotopy), automorphism groups of free groups (denoted $\mathrm{Aut}(F_n)$) and their quotients, outer automorphism groups of free groups (denoted $\mathrm{Out}(F_n)$).

One important property enjoyed by general linear groups over the integers is that every embedding $\mathbb{Z}^n \hookrightarrow \mathbb{Z}^m$ (with $m > n$) induces an embedding $\mathrm{GL}_n(\mathbb{Z}) \hookrightarrow GL_m(\mathbb{Z})$. An analogous statement is true for free groups and their automorphisms: writing $F_m = F_n * F_{m-n}$ allows us to construct $\mathrm{Aut}(F_n) \hookrightarrow \mathrm{Aut}(F_m)$. The situation is far less clear, however, when we focus on groups of type $\mathrm{Out}(F_n)$. Finding values of n and m for which there exist embeddings $\mathrm{Out}(F_n) \hookrightarrow \mathrm{Out}(F_m)$ is not an easy task. We can also form a more general question, and ask about all possible homomorphisms $\mathrm{Out}(F_n) \to \mathrm{Out}(F_m)$. It is worth noting that this sort of question falls under the umbrella-term of *rigidity*. It is also worth noting that the understanding of such maps is crucial if we are to understand stability phenomena of various sorts (representation, (co)homological, etc.).

More recently I have embarked on a new project, namely trying to establish if $\mathrm{Out}(F_n)$ (for $n > 3$) has property FAb, that is if all its finite index subgroups have finite abelianisations. A group G enjoys property FAb for example if it has Kazhdan's property (T). One class of groups with property (T) are groups of type $\mathrm{GL}_n(\mathbb{Z})$ for $n > 2$. It is an open and very interesting problem (with many important ramifications) if $\mathrm{Out}(F_n)$ has property (T). Proving that $\mathrm{Out}(F_n)$ does not have property FAb would immediately tell us that it does not have (T); on the other hand proving that $\mathrm{Out}(F_n)$ has FAb would already imply many consequences of (T).

The problem of having FAb or (T) is also open for the Mapping Class Groups. It is very likely that a proof showing that $\mathrm{Out}(F_n)$ does/does not have FAb/(T) can inspire (if not be generalised to) an analogous proof for the Mapping Class Groups.

Sang-hyun Kim

Korea Advanced Institute of Science and Technology

My research area is *Combinatorial and Geometric Group Theory*. The following question, due to Gromov, has intrigued me into a couple of past and ongoing projects: Does every one-ended word-hyperbolic group contain a closed hyperbolic surface subgroup?

More specifically, I study hyperbolic surface subgroups of right-angled Artin groups and other CAT(0) groups. I proved for the first time that a right-angled Artin group can contain a hyperbolic surface subgroup even when its defining graph does not have a long cycle as an induced subgraph, answering a question due to Gordon–Long–Reid. This appeared as *Co-Contractions of Graphs and Right-Angled Artin Groups*, Alg. Geom. Top. 8 (2008) pp. 849–868. Also, there is a decomposition theorem for the graphs which define right-angled Artin groups without hyperbolic surface subgroups; see *On Right-Angled Artin Groups Without Surface Subgroups*, Groups, Geom. and Dyn. 4(2) (2010) pp. 275–307.

I also consider a CAT(0) graph of free groups with cyclic edge groups. With Sang-il Oum (KAIST), I proved that a double of a rank-two free group contains a hyperbolic surface subgroup if and only if the group is one-ended; see *Hyperbolic Surface Subgroups of One-Ended Doubles of Free Groups*, arXiv:1009.3820. This depends on the tool, called *polygonality*, of words in free groups defined by Henry Wilton (University College London) and me; see *Polygonal Words in Free Groups*, Quarterly Journal of Mathematics (Advance Access published December 3, 2010) and *Geometricity and Polygonality in Free Groups*, International Journal of Algebra and Computation 21(1–2) (2011) pp. 235–256.

Another theme of my study is embedability between groups. Using mapping class groups, Thomas Koberda (Harvard University) and I also study embeddings between right-angled Artin groups. We combinatorially characterized right-angled Artin subgroups of a given right-angled Artin group when the latter was defined by either a triangle-free graph or a forest; see *Embedability between right-angled Artin groups*, arXiv:1105.5056. In particular, we completely determined when there is an embedding between right-angled Artin groups on cycles. Key ingredients of the proof are realization of right-angled Artin groups as subgroups of mapping class groups and studying the group elements corresponding to pseudo-Anosov elements.

Thomas Koberda

Harvard University

The study of mapping class groups of surfaces and their subgroups lies at the intersection of the fields of low–dimensional topology, dynamics and geometric group theory. My research is concerned with understanding the structure of mapping class groups by studying the dynamics of their actions on various objects. I have been using representation theory of finite and profinite groups to prove new facts about mapping class groups, and I have been using dynamics and mapping class groups to answer questions about the structure of right-angled Artin groups. To study right-angled Artin groups, I have been exploiting the fact that they sit inside of mapping class groups in a rich but predictable variety of ways.

Mapping class groups and covers of surfaces

While the usual homology representation of the mapping class group is not faithful, the finite covers of $\Sigma_{g,n}$ exhaust the entirety of $\pi_1(\Sigma_{g,n})$. The rational homology of each regular cover is equipped with a faithful action of the deck group, so that the data of the action of a mapping class on the rational homology of a finite cover encodes the data of the action of the mapping class on the deck group. In particular, the collection of the homology groups of all finite covers encode all the data of $\mathrm{Mod}_{g,n+1}$ acting on $\pi_1(\Sigma_{g,n})$. My research in this area is concerned with making this last statement precise, which is to say with the following general questions:

1. How are the various attributes of mapping classes which are encoded by their homological representations to be detected?

2. What new invariants of mapping classes do homology representations reveal, and how do homological representations relate to other representations of mapping class groups?

In [4], I proved that each mapping class acts nontrivially on the homology of a finite cover of $\Sigma_{g,n}$ and that one can detect the Nielsen–Thurston classification of a mapping class from the data of its actions on the homology of all finite covers of $\Sigma_{g,n}$.

One of the basic invariants of a pseudo-Anosov homeomorphism ψ is its dilatation K. Even though the dilatation of a mapping class often cannot be detected as the supremum of the spectral radii of its actions on finite covers of a base surface by [9], there are many questions which remain. One such question is whether or not for each infinite order mapping class one can find a finite cover of the base surface where the mapping class acts with infinite order on the homology. More strongly, one might wonder if given a pseudo-Anosov homeomorphism, whether there is a finite cover where the homeomorphism acts on the real homology with spectral radius off the unit circle. Questions of this ilk are the subject of my current research, and I have partial results toward resolving them and connecting them more explicitly with 3–manifold theory (see [6], for instance). On of the more important questions concerning eigenvalues is the following:

Question. Given a pseudo-Anosov homeomorphism ψ on a surface Σ, is there a finite cover Σ' where ψ acts with spectral radius greater than 1 on $H_1(\Sigma', \mathbb{R})$?

Right-angled Artin groups

Let Γ be a finite graph. The right-angled Artin group $A(\Gamma)$ is obtained by taking one generator for each vertex of Γ and declaring two generators to commute if they are connected by an edge and no further relations. From a graph Γ one can form a larger graph Γ^e whose vertices are $A(\Gamma)$–conjugates of vertices of Γ (viewed as elements of $A(\Gamma)$) and whose edges are $A(\Gamma)$–conjugates of edges of Γ (viewed as pairs of elements of $A(\Gamma)$). We will see that Γ^e contains a lot of information about $A(\Gamma)$ and has interesting intrinsic structure. I proved in [5] that sufficiently high powers of certain mapping classes generate a right-angled Artin group of the "expected" isomorphism type.

Together with Sang-hyun Kim, we used this result to study right-angled Artin groups intrinsically. We were thus able to give a mostly complete classification of right-angled Artin subgroups of right-angled Artin groups in [3]. We proved the following result, for instance:

Theorem. Let Γ be a triangle–free graph. There exists an embedding of groups $A(\Lambda) \to A(\Gamma)$ if and only if there exists an embedding of graphs $\Lambda \to \Gamma^e$.

This Theorem allows us to easily recover most known results concerning injective maps between right-angled Artin groups, and to prove many new results. For example, we can completely determine the existence of injective maps between $A(C_n)$ and $A(C_m)$, where C_n is a cycle of length n: it is exactly when $n = m + k(n-4)$.

We were also able to answer a question posed independently by C. McMullen and M. Sapir, namely that there is no 2–dimensional right-angled Artin group which contains all other 2–dimensional right-angled Artin groups. We would like to know exactly when $A(C_n)$ embeds into $A(\Gamma)$, where Γ is a general graph.

The intrinsic structure of Γ^e is also quite interesting. It behaves much like the complex of curves for a surface. One could think of Γ itself as a finite collection of simple closed curves on a surface, the Dehn twists about which generate the entire "mapping class group" on that surface. The vertices of Γ^e can be thought of as the union of the mapping class group orbits of the starting curves (i.e. "all simple closed curves on the surface"), with edges denoting disjointness. Note that this mapping class group is not a true mapping class group since one often has to pass to powers of mapping classes to have them behave properly. One can show that Γ^e is hyperbolic, which analogizes Masur and Minsky's main result in [7]. The structure of Γ^e and the subgroups of right-angled Artin groups still hold many interesting questions:

Question. How can mapping class groups and ideas related to mapping class groups be used to more precisely understand the internal structure of right-angled Artin groups? For instance, we already know that the graph Γ^e behaves much like a complex of curves in that it is Gromov hyperbolic. How far does the analogy go? Can one get acylindrical actions of right-angled Artin groups on Γ^e? If so, what would the translation lengths for hyperbolic elements mean?

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Juhani Koivisto

University of Helsinki

I am currently doing postgraduate studies and research leading to a doctoral degree at the Department of Mathematics and Statistics at University of Helsinki. My research focuses on **curvature in metric spaces**, **p-harmonic maps**, and **coarse geometry**. The research on curvature in metric spaces focuses on Gromov hyperbolic spaces and CAT(κ)-spaces. CAT(κ)-spaces are geodesic spaces whose geodesic triangles are "thinner" than their comparison triangles on a surface of constant curvature κ. These are related to recent generalisations of Ricci curvature that arise in the study of p-harmonic maps and the asymptotic Dirichlet problem on Gromov hyperbolic and related spaces. In the asymptotic Dirichlet problem a continuous boundary data is given "at infinity" and the problem is to find a p-harmonic map agreeing with the given boundary data. My aim is to generalise the work of I. Holopainen, A. Vähäkangas and U. Lang on the asymptotic Dirichlet problem on Gromov hyperbolic metric measure spaces by weakening the Gromov hyperbolicity assumption. Related to the latter and to coarse geometry is the coarse invariance of the boundary at infinity and a characterisation of isoperimetric inequalities by P. Nowak and J. Spakula using controlled coarse homology. Another research interest related to coarse geometry focuses on extending Ballman and Światkowski's work on L^2-cohomology of groups acting on simplicial complexes and the spectral condition for property (T) to Banach spaces. Here we say that a group G has property (T) if and only if every affine isometric action of G on a Hilbert space, and more generally on a Banach space, has a fixed point. In particular, I am interested in fixed point properties on L^p spaces.

Benno Kuckuck

University of Oxford

My current research is chiefly concerned with understanding the subgroup structure of direct products of groups and the interactions with higher finiteness properties of groups.

There are two main goals to this research: The first is the investigation of the subgroup structure of direct products of free groups, surface groups and limit groups. There is ample incentive to achieve a better understanding of this class of groups as it contains examples of great current interest. For one, the finitely presented residually free groups can be characterised as the subgroups of a direct product of finitely many limit groups. Also, direct products of free groups can be considered the simplest interesting examples of right-angled Artin groups, which are known to have a rich and subtle subgroup structure [2]. Previous work by Baumslag and Roseblade [1] and Bridson, Howie, Miller and Short [3] suggests that higher finiteness properties have an important part to play in the understanding of these groups: They show that while finitely generated subgroups of direct products of limit (or even free) groups can be hopelessly complicated, the subgroups of type FP_∞ are structurally very simple. Thus the properties FP_n for $n \geq 1$ seem to supply a good measure of the complexity of such a subgroup and one aim of my research is to find good ways of characterising the subgroups of type FP_n for any $n \geq 1$.

As our second goal we hope, conversely, to gain a better understanding of higher finiteness properties of groups. While the properties of finite generation and finite presentability are ubiqitous in group theory, their natural generalisations "type F_n" and "type FP_n" are still not very well understood in many respects. Powerful tools, like the Σ-invariants by Bieri, Neumann, Strebel and Renz, can be brought to bear in certain special situations and have been tied to interesting structural properties of groups. However, where these tools do not apply, we hope it will be illuminating to find new families of examples and investigate the structural consequences of the higher finiteness properties. From the above mentioned earlier work, and our results so far, it seems that direct products of groups will prove a fertile playground for analysing higher finiteness properties of groups.

The immediate objective of my current research is to make progress on two conjectures. The first characterises subgroups of type FP_k in terms of the way the subgroup is embedded inside the ambient product:

Conjecture 1. Let $\Gamma_1, \ldots, \Gamma_n$ be groups of type FP_k (with $k \geq 2$) and $G \leq \Gamma_1 \times \cdots \times \Gamma_n$ a subgroup of the direct product, which "virtually surjects to k-tuples", i.e. for $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ the image of G under the canonical projection

$$G \to \Gamma_{i_1} \times \cdots \times \Gamma_{i_k}$$

has finite index in $\Gamma_{i_1} \times \cdots \times \Gamma_{i_k}$. Then G is of type FP_k.

The second conjecture aims to describe the finiteness properties of a fibre product associated to two short exact sequences in terms of the finiteness properties of the constituent groups:

Conjecture 2. Let

$$1 \to N_1 \to \Gamma_1 \overset{\pi_1}{\to} Q \to 1$$
$$1 \to N_2 \to \Gamma_2 \overset{\pi_2}{\to} Q \to 1$$

be short exact sequences of groups, such that N_1 is of type F_n, Γ_1 and Γ_2 are of type F_{n1} and Q is of type F_{n2}. Then the fibre product

$$P := \{(\gamma_1, \gamma_2) \in \Gamma_1 \times \Gamma_2 \mid \pi_1(\gamma_1) = \pi_2(\gamma_2)\}$$

is of type F_{n1}.

The $k = 2$ and $n = 1$ cases, respectively, of these two conjectures have been proven by Bridson, Howie, Miller and Short [4] and in [5] I prove a general weakened form of these conjectures and certain special cases of the full conjectures.

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Sang-hoon Kwon

Seoul National University

Geometric group theory attracts me as I am interested in two main questions of the Erlangen program:

1. Given a geometry, for example, the Euclidean geometry, the hyperbolic geometry or the projective geometry, find the transformation group which preserves properties of the figures in this geometry.

2. Conversely, given a group, find a geometry such that the group acts on it and preserves its geometric properties.

I would like to learn and get ideas from the young geometric group theory meeting, especially, I would like to discuss about discrete subgroups, right-angled Artin groups and Kazhdan's property (T).

I study mainly dynamics of homogeneous space, for example, Riemannian symmetric space. More precisely, I am working on ergodic theory of the action of Lie group over various fields. For instance, every (relative) rank-1 Lie group over a non-Archimedean local field acts on Bruhat-Tits tree which is a negatively curved metric space. Tree is a special case of Tits buildings which were first introduced by Tits in a series of papers from 1950s. It is a $CAT(0)$-simplicial complex Δ which can be expressed as the union of subcomplexes Σ (called apartments) satisfying the following axioms:

(B0) Each apartment Σ is a Coxeter complex.

(B1) For any two simplices $A, B \in \Delta$, there is an apartment Σ containing both of them.

(B2) If Σ and Σ' are two apartments containing A and B, then there is an isomorphism $\Sigma \to \Sigma'$ fixing A and B pointwise.

It is in some sense a generalized symmetric space. Also primary properties of buildings are of combinatorial nature, for example, the incidence relations between simplexes. It also has many applications to group theory such as group cohomology and geometry, topology and representation theory.

There are many well-known results about ergodicity and equidistributed property of the orbit on hyperbolic manifolds or hyperbolic groups. As a classical result, the geodesic and horocycle flows are mixing and spheres are equidistributed on the hyperbolic surface. Eskin-McMullen(1992) showed similar results to those on a general affine symmetric space and Oh-Shah showed for geometrically finite hyperbolic 3-manifold case and also for some hyperbolic groups. My purpose is to find similar (or different) properties for Bruhat-Tits trees and buildings. Therefore, I am interested in studying Lie-theory, Coxeter groups and more generally Right-angled Artin groups.

I am also interested in Kazhdan's property (T) for the application to expander graphs. Property (T) was first used by Margulis to construct a family of expander graphs. Expander graphs often come from uniform spectral gap. Recently, Bekka-Lubotzky proved that an automorphism group of tree can always have the tree lattice which does not have a spectral gap.

I am sure that the young geometric group theory meeting will be great chance for me to learn and interact with other young mathematicians working in this field. Also, I hope to contribute to the meeting by sharing my ideas with them.

Robin Lassonde

University of Michigan

My research is on geometric group theory. In this area, one uses group actions on spaces to gain information about groups. I am particularly interested in group actions on trees and, more generally, group actions on $CAT(0)$ cubical complexes. Over the past two decades, $CAT(0)$ cubical complexes have become an important tool in mathematics, with applications in geometric group theory, analysis, and robotics. In my dissertation [10], I rework the theory of intersection numbers of splittings of a group, removing finite generation assumptions (for both the ambient group and the subgroups associated to the splittings). The key construction takes finitely many splittings of a group, and produces a canonical $CAT(0)$ cubical complex on which the group acts. I would like to use the key construction to define canonical group theoretic JSJ decompositions.

A splitting of a group G is a decomposition of G as an amalgamated free product [14] or an HNN extension [7]. Both amalgamated free products and HNN extensions were initially described in terms of normal forms for words. In 1977, Serre discovered that splittings can be described as group actions on trees. This topic is known as "Bass-Serre theory" [19], [20]. I think of a splitting of a group G as a simplicial tree on which G acts without inverting any edges, with no fixed points, and with exactly one edge orbit. This is the same as having a non-trivial graph of groups structure for G with exactly one edge, and the aforementioned tree is the Bass-Serre tree for the splitting. However, from the tree point of view, it is much easier to decide what an "isomorphism of splittings" should mean: two splittings of G isomorphic if their trees are G-equivariantly isomorphic.

One motivating example of splittings is that of simple closed curves on an (orientable) surface. By Van Kampen's theorem, any π_1-injective simple closed curve γ on a surface induces a splitting of the fundamental group of the surface as follows. Take the preimage of γ in the universal cover of the surface, and let T denote its dual tree. Let G denote the fundamental group of the surface. After choosing basepoints, we have an action of G on the universal cover, which induces an action on T. This gives a splitting of G over a subgroup isomorphic to $\mathbb{Z}$.

Now we consider intersection number. Given two simple closed curves on a surface, to find the intersection number of the two induced splittings, simply homotope the curves to geodesics and count the number of points of intersection. In [15], Scott defined the intersection number of two splittings (or, more generally, two almost invariant subsets) of a finitely generated group. This definition agrees with the computation for splittings induced by simple closed curves on a surface. I show that that Scott's definition for intersection number of splittings also works when the ambient group is not finitely generated [10]. Whereas he uses the geometry of the Cayley graph to show that the definition is symmetric, I use the geometry of trees.

What happens when two splittings have an intersection number of zero? First we look at the case of splittings σ_1 and σ_2 induced by simple closed curves γ_1 and γ_2 on a surface. Homotope γ_1 and γ_2 to geodesics. These geodesics must be disjoint (since otherwise the splittings would have positive intersection number). Take the preimage of γ_1 and γ_2 in the universal cover of the surface. Let T denote the dual tree to this preimage. Then T is a common refinement of the Bass-Serre trees for the two splittings. A natural question to ask is, given arbitrary splittings (not just those induced by simple closed curves on surfaces), whether there is an algebraic tool to choose suitable representatives for splittings. The answer is "yes;" given any collection $\mathcal{C}$ of splittings of any group G, if $\mathcal{C}$ has pairwise intersection number zero, and C satisfies a sandwiching condition, then one can choose nice representatives for the splittings and produce a common refinement for their trees [10]. This result was previously proved by Scott and Swarup for splittings of a finitely generated group over finitely generated subgroups [16]. Whereas sandwiching is automatic in Scott and Swarup's setting, in the non-finitely generated case, it is necessary to explicitly assume a sandwiching condition; otherwise the theorem is false. "Most" (in some sense) collections of splittings satisfy sandwiching.

What happens when splittings (or, more generally, almost invariant sets) have positive intersection number? When splittings have positive intersection number, it is not possible to construct a G-tree whose edge orbits bijectively correspond to the splittings; however, one can construct a relevant $CAT(0)$ cubical complex. A **cubical complex** C is a CW-complex whose cells are standard Euclidean cubes of varying dimensions, such that the intersection of any two cells is either empty or a common face of both. C is called a $CAT(0)$ **cubical complex** if, in addition, C is simply connected, and the link of any vertex (i.e. 0-cube) is a flag complex. If the splittings are induced by simple closed curves on a surface, arrange the curves in general position, and look at the preimage of the curves in the universal cover of the surface. This preimage consists of a collection of lines, some of which may intersect. The dual graph to these lines in the universal cover gives part of a $CAT(0)$ cubical complex. For example, any intersection of lines corresponds to a cycle of length four, which we fill in with a square. For arbitrary splittings, one would like an algebraic tool for constructing a $CAT(0)$ cubical complex, such that intersections of hyperplanes correspond to positive intersection number of splittings. In [11], Niblo-Sageev-Scott-Swarup did this for splittings of a finitely generated group over finitely generated subgroups, by generalizing a construction of Sageev [13] (note that the constructions in [11], [13] also worked for almost invariant sets). In the case when the splittings have pairwise intersection number zero, one gets a tree, i.e. a 1-dimensional $CAT(0)$ cubical complex, that is a common refinement of the Bass-Serre trees for the splittings. In the case of positive intersection number, one gets a higher dimensional $CAT(0)$ cubical complex. I adapt the construction from [11], [13] to work for splittings of an arbitrary group over arbitrary subgroups [10]. The difficult part of the argument is proving that the resulting $CAT(0)$ cubical complex is nonempty.

Now I briefly describe a potential application of this $CAT(0)$ cubical complex. Problems in geometric group theory are often motivated by results on 3-manifolds. Classical JSJ theory began with work by Waldhausen [21] in 1969 and was developed in 1979 by Jaco and Shalen [8], and Johannson [9]. A simplified version of the main theorem is as follows.

Theorem. Let M be a compact irreducible orientable 3-manifold. Then there exists a minimal collection of disjoint embedded annuli and tori, such that after cutting M along the annuli and tori, each connected component is either a Seifert fibered space, an I-bundle, or is atoroidal and acylindrical. Moreover, this collection is unique up to isotopy.

Theorem describes splittings of $\pi_1(M)$ over $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$. If M happens to be closed, then only tori are needed.

Over the next couple of decades, geometric group theorists developed group theoretic analogues of classical JSJ theory [18], [1], [12], [2], [3], [4], [17], [5], [6]. The idea is to describe splittings of a group G over a specified class of subgroups. Ideally one would like group theoretic JSJ decompositions that are canonical, and can simultaneously handle splittings over multiple types of edge groups (for example, virtually polycyclic groups of arbitrary lengths). However, previous attempts have had limited success developing decompositions addressing these issues.

One method for constructing group theoretic JSJ decompositions is using algebraic regular neighborhoods. Given a collection $\mathcal{C}$ of almost invariant subsets of a group G, an **algebraic regular neighborhood** of the collection is a bipartite G-tree (denote the vertex colors by V_0 and V_1) where each element of $\mathcal{C}$ is "enclosed" (in some sense) in a V_0-vertex, each V_0-vertex encloses at least one element of $\mathcal{C}$, and any splitting of G having intersection number zero with every element of $\mathcal{C}$ is enclosed in a V_1-vertex [17]. Scott and Swarup proved the existence of algebraic regular neighborhoods for a finite collection of almost invariant sets, and proved uniqueness of algebraic regular neighborhoods for an arbitrary collection of almost invariant sets. I have proved that one can make a more direct construction of algebraic regular neighborhoods by using the aforementioned $CAT(0)$ cubical complex [10]. I would like to utilize this construction to develop new canonical group theoretic JSJ decompositions.

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Adrien Le Boudec

Université Paris-Sud 11

The general framework of my thesis is geometric group theory, which closely interacts with many other areas of mathematics, like hyperbolic geometry, algebraic topology or computational group theory for instance. More precisely, part of my current focus is on Lie groups and their asymptotic cones.

The concept of asymptotic cone was introduced by Gromov in 1981. Roughly speaking, an asymptotic cone of a metric space is what one sees when one looks at the space from infinetely far away. One of the reasons why it makes it a relevant notion is that there exist characterizations of important families of groups, like abelian groups, nilpotent groups or hyperbolic groups, in terms of asymptotic cones.

Recently, the importance of having cut-points in asymptotic cones has been pointed out. Drutu, Mozes and Sapir established a link between the existence of cut-points in the asymptotic cones of a group and the divergence of the group, which estimates the length of a path connecting two points avoiding a large ball.

The purpose of my work, that I am just starting, will be to characterize connected Lie groups whose asymptotic cones have cut-points. I am also interested in the same question as well as variants for other classes of groups, like p-adic Lie groups or arithmetic lattices.

Jean Lécureux

Technion, Haifa

My research is about buildings and their boundaries. I'm especially interested in non-affine buildings. They are CAT(0) spaces with some interesting groups acting on them, which behave quite similarly to p-adic algebraic groups. One typical example of such exotic buildings are Kac-Moody buildings.

In a joint work with P.-E. Caprace, we defined a new, "combinatorial", boundary of buildings. This boundary should be seen as an analogue of the maximal Satake-Furstenberg compactification of symmetric spaces. Our main result is that this boundary parametrizes (up to finite index) the maximal amenable subgroups of the boundary: stabilisers of points are amenable, and any amenable group acting on the building fixes a point, up to finite index. One of the main interests of this theorem is that it applies to any kind of building, without any hypotheses. Another interest is that we understand the structure of amenable subgroups acting on buildings. Namely, they are an extension of a "locally finite" radical by a virtually abelian subgroup of the Weyl group.

I also extended this theorem to prove that the action of the isometry group on the boundary of the building is amenable. This implies some non-trivial properties on groups acting on buildings (for example, they satisfy the Novikov conjecture).

The structure of this boundary is also interesting. It is a stratified space, each stratum consisting of a union of subspaces corresponding to a blow-up of a point of the CAT(0) boundary. In particular, there is an interesting subspace of this boundary which is the closed strata. In the case of p-adic algebraic groups, this is a copy of G/P; however, in the general case, it is not a homogeneous space.

As in the classical, algebraic case, an interesting question would be to see whether the Poisson boundary of a group Γ acting on such a building can be mapped into these strata. In the classical case, this is an important step in the proof of Margulis' superrigidity theorems. There are several ways to get this kind of results. The first one is to study random walks on buildings and their asymptotic properties. Another possibility is to use the strong ergodic properties of the Poisson boundary, together with the amenability of the action, in order to get such a map.

Paul-Henry Leemann

Université de Genève

My research centers around free groups and graphs. I'm very interested in the topological approach of Stallings which represents subgroups of free groups as covering of graphs, more precisely in the combinatorial variant used by Kapovich and Myasnikov. These helps make connections between combinatorial and algebraic properties of subgroups of a free group and geometric properties of their Schreier graphs (like vertex-transitivity).

I'm also interested in $\mathrm{Out}(F_n)$ and Outer Space. With A. Talambutsa we are trying to translate the result of Ladra, Silva and Ventura about the relation between the length of an (outer) automorphism and the length of its inverse in a combinatorial statement. Indeed, the proof uses results of Algom-Kfir and Bestvina about the (non-symmetric) metric of Outer Space and it seems to be a heavier machinery that is strictly necessary.

For the future, I plan to learn more about Auter Space in the attempt to generalize the precedent result to $\mathrm{Aut}(F_n)$. I also want to study Artin groups to see in which sense statement about free groups could be generalized.

Furthermore, more as a hobby, I want to see what happens to some of the mentioned results if instead of looking at F_n we take free group of infinite rank.

Arie Levit

Hebrew University

I'm a graduate student in my final year. My research interests are limit groups and CAT(0) cubical complexes. As part of my thesis work, I am currently interested in developing techniques for explicitly obtaining codimension-1 subgroups in the fundamental group of a graph of groups "recursively" - given such subgroups in the vertex groups. I am also interested in finding geometric conditions for the resulting action to be "nice" - for example, proper and cocompact.

Since limit groups can be defined in terms of ω-residually free towers which are essentially graphs of groups, I am hoping that this will lead to an explicit cubulation of limit groups. Alternatively I'm interested in generalizing these methods to more general families of graphs of groups with certain given restrictions, such as hyperbolicity.

For an overview of limit groups, I would recommend C. Champetier and V. Guirardel's article *Limit groups as limits of free groups.* CAT (0) cubical complexes and codimension-1 subgroups are discussed e.g. in M. Sageev's thesis *Ends of group pairs and non-positively curved cube complexes.*

I am about to finish the MA and continue to PhD studies, and I would like to widen my interests within the scope of geometric group theory and I'm excited about this opportunity to familiarize myself with new ideas.

Joel Louwsma

University of Oklahoma

My research is in the areas of geometric group theory and low-dimensional topology. I am broadly interested in ideas related to stable commutator length, quasimorphisms, and bounded cohomology. Groups in which I have a particular interest are braid groups, mapping class groups, and outer automorphism groups of free groups.

Stable commutator length, a kind of relative Gromov–Thurston norm, has been the subject of much recent interest. One way to study stable commutator length is through a duality, discovered by Bavard, with the theory of quasimorphisms. This duality implies that every homogeneous quasimorphism gives a lower bound on the stable commutator length of a given element. One can ask when these bounds are sharp, in which case the quasimorphism is said to be extremal for the element in question. Although extremal quasimorphisms are known to exist, few examples of them have been found, due largely to the fact that the space of all homogeneous quasimorphisms is poorly understood for most groups. Finding extremal quasimorphisms would give an indirect way to compute stable commutator length, something that can presently be done for few groups, such as free groups and virtually free groups.

My thesis studies when the rotation quasimorphism on the modular group $\mathrm{PSL}(2,\mathbb{Z})$ is extremal. Much of this work is based on a program I have written to compute stable commutator length in the modular group. This was used to generate a significant amount of experimental data about when the rotation quasimorphism is and is not extremal, and a number of interesting patterns were observed in this data.

Recently, Danny Calegari and I have used a geometric approach to prove one of these experimental observations. It follows from work of Calegari that the rotation quasimorphism is extremal for a hyperbolic element of the modular group if and only if the corresponding geodesic on the modular surface $\mathbb{H}^2/\mathrm{PSL}(2,\mathbb{Z})$ virtually bounds an immersed surface. Calegari and I show how to explicitly construct such immersed surfaces under suitable hypotheses. Specifically, we show that, for every hyperbolic element of the modular group, the product of this element with a sufficiently large power of a parabolic element is represented by a geodesic on the modular surface $\mathbb{H}^2/\mathrm{PSL}(2,\mathbb{Z})$ that virtually bounds an immersed surface. This implies that the rotation quasimorphism is extremal for such elements.

Although this shows that the rotation quasimorphism is extremal for certain families of elements, the exact condition controlling its extremality is still unclear. I am currently studying this further and hope to characterize the elements of the modular group for which the rotation quasimorphism is extremal, for example in terms of the arithmetic codings of geodesics on the modular surface described by Svetlana Katok and her collaborators.

Extremal quasimorphisms always exist, and therefore when the rotation quasimorphism is not extremal there must be some other quasimorphism that is extremal. A number of quasimorphisms on $\mathrm{PSL}(2,\mathbb{Z})$ can be constructed from counting quasimorphisms on F_2 defined by Brooks. I plan to use a modification of my program for computing stable commutator length on the modular group to determine values of some of these quasimorphisms, and I hope to analyze the differences between the extremality of these quasimorphisms and the extremality of the rotation quasimorphism.

The three-strand braid group B_3 is a central extension of the modular group, and therefore our procedure for computing stable commutator length in the modular group extends to B_3. The rotation quasimorphism lifts from $\mathrm{PSL}(2,\mathbb{Z})$ to B_3, and our result gives information about when this lifted quasimorphism is extremal. The procedure for computing stable commutator length does not generalize to higher-strand braid groups, however, and therefore I would like to understand *why* certain quasimorphisms are extremal for three-strand braids, with the goal of carrying these reasons over to higher-strand braid groups. This would give an indirect method for computing the stable commutator length of higher-strand braids.

Michał Marcinkowski

University of Wrocław

I am a first year PhD student at the University of Wrocław. I am interested in geometric group theory, recently especially in sofic, Kahzdan and a-T-menable groups.

In my master thesis I was interested in the notion of induced affine representation with connection to Gromov's a-T-menability.

We say that a group G is a-T-menable (or it has the Haagerup property) if it admits a metrically proper affine representation.

Given an affine representation α, we can decompose it to a linear representation π of G and a π-cocycle $b\colon G \to \mathcal{H}$, s.t. $\alpha = \pi b$. It is well known that isometric classes of affine representations with linear part π can by classified by the first cohomology group $H^1(G,\pi)$ of G with coefficients in the representation π.

In my thesis I consider the notion of induced affine representation. Given a discrete groups $H < G$, we can ask if there exists a map $\sim\colon H^1(H,\pi) \to H^1(G,\widetilde{\pi})$ where $\widetilde{\pi}$ is the induced linear representation, with some functorial properties, such as $\restriction \circ \sim = Id_{H^1(H,\pi)}$, where $\restriction$ is the canonical restriction. Unfortunately, there is no way to induce affine representation in general (there are simple examples where the canonical restriction is not surjective). Nevertheless, there is a short list of cases where one can induce a representation:

- When $[G\colon H]$ is finite.
- When $H < H \times_\Gamma G$ and Γ is finite.
- When Γ is an appropriate lattice in a $\mathbf{R}$-rank 1 Lie group [2, chap. 3.III].

The second dot gives the proof of theorem from [1, Chapter 6.2] that amalgamation of two a-T-menable groups along finite group is again a-T-menable.

The motivation to develop the notion of affine representation was to obtain more geometrical proofs of being a-T-menable (e.g. the known fact that if N is a-T-menable and G is amenable, then the extension of G by N is a-T-menable).

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Timothée Marquis

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Recall that a Kac–Moody group $\mathcal{G}(k)$ over a field k acts by automorphisms on a twin buiding $X \times X_-$. The induced action of $\mathcal{G}(k)$ on each factor is non-discrete and so we may consider the closure of the image of $\mathcal{G}(k)$ in the topological group $\mathrm{Aut}(X)$; this group, denoted $\widehat{\mathcal{G}}(k)$, is called a **complete Kac–Moody group** over k.

In this work, we study the properties of subgroups of topological Kac–Moody groups. We will first focus on complete Kac–Moody groups over finite fields, which we will study as locally compact groups by analogy with semisimple algebraic groups over locally compact fields. In particular, we have just proved the following statement.

Theorem. A complete Kac–Moody group $\widehat{\mathcal{G}}(\mathbb{F}_q)$ over a finite field $\mathbb{F}_q$ is **topologically Noetherian**: every ascending chain of open subgroups possesses a maximal element.

Note that in the case of semi-simple groups over local fields, G. Prasad proved a stronger property: every proper open subgroup is compact (see [3]). However, this property fails for Kac–Moody groups. Our strategy consisted in showing that every open non-compact subgroup of $\widehat{\mathcal{G}}(\mathbb{F}_q)$ is in fact essentially parabolic, therefore implying Theorem (see [1]).

We are currently trying to establish the following conjecture.

Conjecture. The pointwise fixer H of an apartment of X in $\widehat{\mathcal{G}}(\mathbb{F}_q)$ is virtually torsion-free.

However technical in appearance, this conjecture, once confirmed, would have remarkable consequences: it would imply amongst other things the existence of a uniform lower bound on lattices of $\widehat{\mathcal{G}}(\mathbb{F}_q)$ analoguous to the one obtained by A. Lubotzky and Th. Weigel [2] for the group $\mathrm{SL}_2(\mathbb{F}_p(\!(t)\!))$.

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Alexandre Martin

Université de Strasbourg

My current research focuses on boundaries of groups in the sense of Bestvina and their generalisations. In [2], Bestvina defined a notion of boundary, called a $\mathcal{Z}$-boundary, that has strong implications in geometric group theory. This notion of boundary was further generalised to the notion of an equivariant $\mathcal{Z}$-compactification by the work of Farrell and Lafont [7], who proved the Novikov conjecture for groups admitting what they call an $E\mathcal{Z}$-structure. A stronger notion of boundary for groups with torsion, which implies a stronger version of the Novikov conjecture, has been developed by Carlsson and Pedersen [4] and Rosenthal [11], [12]. Such compactifications have also turned out to be useful in proving the Farrell-Jones conjecture for various classes of groups.

The existence of $E\mathcal{Z}$-structures is known for groups admitting a classifying space for proper actions with a sufficiently nice geometry, such as hyperbolic [3], [9], systolic [10] and CAT(0) groups.

In my work, I adress the following combination problem: Given a group G acting cocompactly by simplicial isometries on a simplicial complex X, are there natural conditions under which it is possible to build an $E\mathcal{Z}$-structure for G, assuming that the stabilisers of simplices of X all admit an $E\mathcal{Z}$-structure?

There are already some special cases for which such a combination theorem is know to hold. For instance, Tirel [13] explained how to build a $\mathcal{Z}$-boundary for free and direct products of groups admitting $\mathcal{Z}$-boundaries. Furthermore, Dahmani [5] built an $E\mathcal{Z}$-structure for a torsionfree group that is hyperbolic relative to a group admitting an $E\mathcal{Z}$-structure.

My research deals with actions on CAT(0) spaces such that every simplex stabiliser admits an $E\mathcal{Z}$-structure. I am able to construct a Bestvina boundary for G, along with an $E\mathcal{Z}$-structure, in the case of an acylindrical action (that is, there is a uniform upper bound on the diameter of subspaces of X fixed by an infinite subgroup of G) with some algebraic restrictions on the way stabilisers of simplices embed in one another. The general theorem, although too technical to be stated here, has the following corollary.

Theorem 1. Let $G(\mathcal{Y})$ be a strictly developable simple complex of groups over a finite simplicial M_κ-complex. Assume that:

- The universal covering of $G(\mathcal{Y})$ is CAT(0) for the simplicial metric associated to the induced M_κ-complex structure,
- The local groups are hyperbolic, and all the inclusions are quasiconvex embeddings,
- The action of $\pi_1(G(\mathcal{Y}))$ on the universal cover of $G(\mathcal{Y})$ is acylindrical.

Then $\pi_1(G(\mathcal{Y}))$ admits an $E\mathcal{Z}$-structure.

The construction follows a strategy used by Dahmani [6] to amalgamate Bowditch boundaries of relatively hyperbolic groups in the case of acylindrical graphs of groups.

As an application of the previous construction, we prove a combination theorem for hyperbolic groups, in the case of acylindrical simple complexes of groups of arbitrary dimension.

Theorem 2 [Combination Theorem for Hyperbolic Groups]. Let $G(\mathcal{Y})$ be a strictly developable simple complex of groups over a finite simplicial M_κ-complex. Assume that:

- The universal covering of $G(\mathcal{Y})$ is hyperbolic and CAT(0) for the simplicial metric associated to the induced M_κ-complex structure,
- The local groups are hyperbolic, and all the inclusions are quasiconvex embeddings,
- The action of $\pi_1(G(\mathcal{Y}))$ on the universal cover of $G(\mathcal{Y})$ is acylindrical.

Then $\pi_1(G(\mathcal{Y}))$ is hyperbolic. Furthermore, the local groups embed in $\pi_1(G(\mathcal{Y}))$ as quasiconvex subgroups.

This theorem is proved by showing that $\pi_1(G(\mathcal{Y}))$ is a uniform convergence group on the boundary constructed in Theorem 1. An advantage of such a dynamical approach is that it yields an explicit model for the Gromov boundary of $\pi_1(G(\mathcal{Y}))$.

Such a result is already known for acylindrical graphs of groups: the hyperbolicity is a direct consequence of the much more general combination theorem of Bestvina and Feighn [1], while the quasiconvexity of vertex stabilisers follows from a result of Kapovich [8]. To our knowledge, Theorem 2 is the first result that partly generalises these results to complexes of groups of arbitrary dimension.

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Eduardo Martínez-Pedroza

Memorial University of Newfoundland

The principal theme of my research program is the study of countable infinite groups. My contributions are part of the program of classifying and understanding the structure of certain classes of discrete groups. I am particularly interested in fundamental groups of 3-manifolds and groups admitting actions on negatively curved spaces.

Residual finiteness and Subgroup separability. A group is residually finite if the intersection of all finite index subgroups equals the trivial subgroup. A group is subgroup separable if each finitely generated subgroup equals the intersection of all finite index subgroups containing it. An outstanding question in geometric group theory is whether all hyperbolic groups are residually finite. On the other hand, subgroup separability of Kleinian groups, discrete subgroups of $PSL(2,\mathbb{C})$, is of interest among 3-manifold topologists due to its relation with the virtual Haken conjecture. The following two results were obtained in joint work with J.Manning.

Theorem 1 ([11]). If all hyperbolic groups are residually finite, then all finitely generated Kleinian groups are subgroup separable.

This result answers a question by Agol, Groves and Manning [1]. It relies on a generalization of the Klein-Maskit combination theorems developed by the author in [12]. This use of combination theorems in connection with subgroup separability was further developed by Chesebro, Deblois, and Wilton proving separability for a class of hyperbolic 3–manifolds groups [2]. Another result from [11] is the following.

Theorem 2 ([11]). If fundamental groups of compact hyperbolic 3–manifolds are subgroup separable, then fundamental groups of finite volume hyperbolic 3–orbifolds are subgroup separable..

Coherence and Local Quasiconvexity. A group G is *coherent* if finitely generated subgroups are finitely presented. The coherence of certain classes of groups has been of interest. One of the most relevant results in the area is that 3-manifold groups are coherent [7]. The search for conditions implying coherence is part of my research program. The following two results were obtained in joint work with D.Wise by extending results in [6].

A one relator product of a pair of groups A, B is the quotient $(A * B)/\langle\langle r^m \rangle\rangle$ where r is an element in the free product $A * B$ and $\langle\langle r \rangle\rangle$ denotes its normal closure.

Theorem 3 ([9]). Let A and B be coherent groups, let $r \in A * B$ be a cyclically reduced word of length at least 2, and $m > 0$ such that $3|r| < m$. Then the group $(A * B)/\langle\langle r^m \rangle\rangle$ is coherent.

A question in [3] asks whether groups of the form $(A * B)/\langle\langle r^m \rangle\rangle$ with A and B finite and large m are virtually torsion free. Theorem 3 can be considered evidence of a positive answer to this question; it was known that for large values of m, if $(A * B)/\langle\langle r^m \rangle\rangle$ is virtually torsion free then it is coherent [6]. An application of Theorem 3 was found by Matsuda, Oguni and Yamagata. They exhibit finitely generated groups with universal relatively hyperbolic structures where every finitely generated subgroup is relatively quasiconvex [5].

A (p, r)–*Gromov polyhedron* X is a simply-connected 2-complex where each 2-cell is a p-gon, and each 0-cell x has $\mathsf{link}(x)$ isomorphic to the complete graph K_r [4]. The $\mathsf{link}(x)$ is the graph whose vertices represent 1-cells of X adjacent to x, and two vertices are connected by an edge if the corresponding 1-cells are adjacent to a common 2-cell.

Theorem 4 ([9]). Let G act properly and cocompactly on a (p, r)–Gromov polyhedron. If $6r \leq p$, then G is locally quasiconvex and, in particular, coherent.

Relatively Hyperbolic Groups. The above results on coherence and subgroup separability rely on my research contributions in the study of relatively hyperbolic groups. My most important contributions in the area are the combination theorems for relatively quasiconvex subgroups [12], a new approach to relative quasiconvexity with D.Wise [10], and the study of different relatively hyperbolic structures on a group in connection with quasiconvexity [8].

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Sebastian Meinert

Freie Universität Berlin

My research is centered around the study of deformation spaces of metric simplicial G-trees for certain finitely generated groups G.

For a finitely generated group G, let $\mathcal{T}_G$ be the space of all metric simplicial G-trees, up to equivariant isometry, that are nontrivial (i.e. G does not fix a point in T) and minimal (i.e. there is no proper G-invariant subtree). We equip $\mathcal{T}_G$ with the equivariant Gromov-Hausdorff topology, which is described in [2]. There is an obvious action of the outer automorphism group $Out(G)$ on $\mathcal{T}_G$ by precomposing the G-actions on the trees with automorphisms of G.

A subgroup of G is called an *elliptic subgroup* of $T \in \mathcal{T}_G$ if its action on T has a fixed point. To each tree $T \in \mathcal{T}_G$ we associate a *deformation space* $\mathcal{D} \subseteq \mathcal{T}_G$, consisting of all trees in $\mathcal{T}_G$ that have the same elliptic subgroups as T. There is also an associated *projectivized deformation space* $\mathcal{PD} := \mathcal{D}/\mathbb{R}_+$, which is the quotient of $\mathcal{D}$ by the action of the group of positive real numbers by rescaling the metrics on the trees. If $\mathcal{D}$ contains a tree with finitely generated vertex stabilizers then $\mathcal{D}$ and $\mathcal{PD}$ are contractible (cf. [2]).

Under certain assumptions on the elliptic subgroups of T the action of $Out(G)$ on $\mathcal{T}_G$ restricts to an action on $\mathcal{D}$ and descends to an action on $\mathcal{PD}$. For example, denote by F_n the free group on n generators, the action of $Out(F_n)$ on $\mathcal{T}_{F_n}$ descends to an action on the projectivized deformation space of trees on which F_n acts freely. This space is known as Culler-Vogtmann Outer space and denoted by X_n. Since all vertex stabilizers of trees in X_n are trivial and hence finitely generated, Outer space is contractible. In fact, it can be shown that Outer space is a *classifying space for the family of finite subgroups* of $Out(F_n)$. That is, it is a contractible $Out(F_n)$-CW complex whose isotropy groups are all finite and for all finite subgroups $H \leq Out(F_n)$ the fixed point set ${X_n}^H$ is contractible.

Classifying spaces for families of subgroups, especially of $Out(G)$ for a finitely generated group G, are of great interest in algebraic K-theory of group rings. For example, classifying spaces for the family of virtually cyclic subgroups (i.e. subgroups which contain a cyclic subgroup of finite index) appear in the statement of the Farrell-Jones conjecture. We can prove existence of these spaces, but there is a lack of explicit models. In [1] Clay has, by putting restrictions on G^2, described a certain class of projectivized deformation spaces $\mathcal{PD}$ that are acted upon by $Out(G)$. He has also given a criterion when a subgroup $H \leq Out(G)$ fixes a point in $\mathcal{PD}$, and from a result in [2] we may conclude that the fixed point set of H is contractible if it is nonempty. By following these ideas and the ideas brought up in the study of Outer space, I am looking for explicit models of classifying spaces for families of subgroups within the theory of deformation spaces.

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Bartosz Naskręcki

Adam Mickiewicz University

My current research is mainly focused on arithmetical aspects of theory of modular forms - holomorphic functions on upper half plane with nice symmetry properties under congruence subgroups of special linear group over $\mathbb{R}$ of dimension two. Modular forms can be expanded as a Fourier series with the coefficients carrying important arithmetical data. Modular forms can be gathered into families of functions having fixed weight and level (according to their transformation behaviour under symmetry group) and the form finite dimensional vector spaces over $\mathbb{C}$. They are acted upon by a family of operators called Hecke operators which transform in a specific fashion the Fourier coefficients. Simultaneous eigenforms with respect to the whole family of Hecke operators exist and they have extremely nicely behaved Fourier coefficients (e.g. lying in a common number field).

I'm currently interested in relating those coefficients via congruence relation modulo powers of prime numbers for different special classes of those functions. The importance of the topic is emphasized in the existence of representation of absolute Galois group of rational numbers going tho special linear group over finite fields. The work of Shimura, Serre, Deligne et al. made it possible to study Galois representations via arithmetical properties of Fourier coefficients of certain modular forms.

I study computationally the representations attached to different modular forms and try to predict certain regular behaviour in terms of congruences modulo high powers of primes.

Tomasz Odrzygóźdź

University of Warsaw

I'm a student of 4th year at College of Inter-Faculty Individual Studies in Mathematics and Natural Sciences. My main field of study is mathematics, second field is physics. In mathematics I'm highly interested in Differential Geometry, Algebraic Topology and Functional Analysis, in physics my main interests are Theory of Relativity and Quantum Mechanics. Therefore, apart from obligatory subjects, I have taken, inter alia, following courses:

[2] We require G to act on a locally finite simplicial tree without an invariant point or line, such that the edge stabilizers are virtually polycyclic subgroups of Hirsch length n, and such that the action is minimal and irreducible.

- Algebraic Topology: category theory, homotopy groups, fibrations, cofibrations and its applications in Differential Geometry,
- Differential Geometry: differential forms, connections, foliations, characteristic classes, theory of Lie groups,
- Functional Analysis: theory of linear operators on Banach and Hilbert Spaces,
- Complex Analysis: multi-valued functions, Riemann Surfaces - highly connected to theory of covering spaces (topology),

Since academic year 2010/2011 I'm a vice-chairman of Scientific Circle of Mathematical Physics. In last year we were studying C*-algebras and Representaion Theory, now we are trying to learn advanced Differential Geometry, which is necessary to work in General Relativity. Last year I was attending a seminar on Coarse Geometry conducted by Sławomir Nowak and Tadeusz Koźniewski. The topic of my bachelor thesis concerned Property A and Uniform Roe Algebras. I used methods from Functional Analysis to prove facts in Geometric Group Theory. This year I attend the seminar Topology and Geometry of Manifolds, which is highly related to Differential Geometry. In case of Physics, I'm a member of a team working on problems in Quantum Theory of Information. It's a new branch, only about 20-30 people all over world know about this theory. We are dealing with problems on borderline between Relativity Theory and Quantum Mechanics, such as: what happens with entangled particles when one of them falls into a black hole. In my research career I'm planning to work betwen mathematics and physics, probably in Differential Geometry, maybe I'll be trying to construct Quantum Gravity Theory. I also like Algebra, Geometry, Topology and their connections. This year I'm learning Algebraic Geometry and high level Probablity Theory (it's connected with Functional Analysis, so it's very interesting to me).

Hester Pieters

Université de Genève

After finishing my master thesis on hyperbolic Coxeter groups and the Leech lattice at the Radboud University Nijmegen, I began in September 2011 a PhD under the supervision of Michelle Bucher at the University of Geneva.

I am interested in the connection between the topology and the geometry of manifolds. On Riemannian manifolds, typically of nonpositive curvature, invariants from topology such as for example characteristic numbers are often proportional to geometric invariants such as the volume. An example are the so-called Milnor-Wood inequalities which relate the Euler class of flat bundles to the Euler characteristic of the base manifold.

I study in particular cohomology classes of complex hyperbolic manifolds. For this I use techniques from bounded cohomology.

Let M be a manifold and let $\beta \in H^q(M;\mathbb{R})$ be a cohomology class. The Gromov norm $\|\beta\|_\infty$ is by definition the infimum of the sup-norms of all cocycles representing β:

$$\|\beta\|_\infty = \inf\{\|b\|_\infty \mid [b] = \beta\} \in \mathbb{R}_{\geq 0} \cup \{+\infty\}$$

This is a way of assigning a numerical invariant to a cohomological invariant. It has for instance been computed for the Kähler class. However, the value of this norm is only known for a few cohomology classes. I would like to determine its value for certain cohomology classes of complex hyperbolic manifolds, especially in top dimension. This could lead to new Milnor-Wood type inequalities and computations of the simplicial volume of complex hyperbolic manifolds.

Thibault Pillon

Université de Neuchâtel

After my master thesis on a notion of irreducibility of affine isometric actions of groups on a Hilbert space, I began my graduate studies under the direction of Alain Valette in August.

The main concern of my work is the l^p compression exponent of groups. In [3] Gromov introduced the notion of coarse embeddability of a metric space into another one because he believed a group coarsely embeddable into a Hilbert space would satisfy the Novikov conjecture [4]. This fact has been proved by Yu [8]. Moreover, together with Skandalis and Tu, Yu proved that groups which embed coarsely into a Hilbert space also satisfy the coarse Baum-Connes conjecture [7]. Guentner and Kaminker introduced the notion of the compression exponent of a group as a quantitative measure of 'how well' does a group embed into a Banach space. Let X be a Banach space. The compression exponent $\alpha_X^*(G)$ of a finitely generated group G equipped with the word metric is defined as the supremum over all $\alpha \geq 0$ such that there exists a map $f : G \to X$ satisfying

$$\frac{1}{C}d(x,y)^\alpha \leq ||f(x) - f(y)|| \leq Cd(x,y).$$

One can define the equivariant compression exponent $\alpha_X^\#(G)$ by restraining the supremum over all equivariant mappings, namely those mappings which are orbits of an affine isometric action of G on X. The study of the compression exponent with Hilbertian targets, or l^p targets, has led to strong results involving e.g. amenability or the speed of random walk on a group [5], [1], [6]. It is then natural to study the behavior of the compression exponent under group constructions. The group constructions I am interested in are amalgamated free products over finite subgroups, HNN extensions over finite subgroups and more generally fundamental groups of graphs of groups with finite edge groups. In his PhD thesis, Denis Dreesen gave a partial answer in the case of the non-equivariant compression and a full answer in the case of the equivariant compression in a Hilbertian target [2]. The first objective of my thesis is to complete and generalize his work to l^p targets.

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Wojciech Politarczyk

Adam Mickiewicz University

My main research interests lie in the topology and geometry of manifolds, especially 4-dimensional manifolds. Recently I finished my Master Thesis under supervision of prof. Krzysztof Pawałowski. The thesis was concerned with the stable classification of 4-manifolds.

Suppose we have a group G, which has a finite presentation $\mathcal{P}$, and a smooth, closed, connected and orientable 4-manifold X with free fundamental group F_α, where α is the number of generators in $\mathcal{P}$. Using $\mathcal{P}$ and X one can construct a smooth, closed, connected and orientable manifold $Y(X, \mathcal{P})$ such that $\pi_1(Y(X, \mathcal{P})) \cong G$. Now if we take a different presentation $\mathcal{P}'$ of G or take different manifold X', we will obtain another manifold $Y(X', \mathcal{P}')$ with the fundamental group isomorphic to G. One can ask whether these two manifolds are homotopy equivalent, homeomorphic, diffeomorphic. Answering such questions is extremely difficult, thus can consider another equivalence relation, which is very important in the study of 4-manifolds, namely stable diffeomorphism.

Definition. Suppose that M and N are two smooth 4-manifolds, then M and N are stably diffeomorphic if there exist $k, l \in \mathbb{N}$ and a diffeomorphism

$$f\colon M\#k(S^2 \times S^2) \longrightarrow N\#l(S^2 \times S^2).$$

Using Kreck's modified surgery approach (see for instance [2]) I was able to prove

Theorem. Manifolds $Y(X, \mathcal{P})$ and $Y(X', \mathcal{P}')$ are stably diffeomorphic iff

1. $\sigma(X) = \sigma(X')$ and $w_2(X) = w_2(X')$ or
2. $\sigma(X) = \sigma(X')$ and $w_2(X) \neq 0$ and $w_2(X') \neq 0$,

where $\sigma(X)$ is the signature of X and $w_2(X)$ is the second Stiefel-Whitney class of X.

The other topic I am working on is concerned with homology 4-spheres, smooth and closed 4-manifolds with the same homology as the 4-sphere. More precisely there are two problems I am trying to solve:

1. Which groups can occur as fundamental groups of homology 4-spheres? In 1960's Kervaire proved (see [1]) that if $n \geq 5$ then G is a fundamental group of a homology n-sphere iff

   $$(*) \qquad H_1(G; \mathbb{Z}) = H_2(G; \mathbb{Z}) = 0.$$

   The condition $(*)$ is necessary in dimension 4, however we are far from proving theorem similar to this in [1].

2. Classification of smooth homology 4-spheres with fixed fundamental group G. The first step to do this is the stable classification, which boils down to computing proper bordism groups and deciding which elements can be represented by homology 4-spheres.

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Maria Beatrice Pozzetti

Eidgenössische Technische Hochschule Zürich

I am a first year PhD student at ETH in Zürich under the supervision of Prof. Alessandra Iozzi. My scientific interests lie mainly in the area of geometric group theory. At the moment I am learning various topics related to locally compact groups and Lie groups such as lattices, representations, amenability, property (T). I am particularly interested in the theory of continuous bounded cohomology [5] and its applications to rigidity questions and maximal representations [3], [1], [2], but at the moment I am still deciding on the topic of my PhD thesis and for this reason I think that the Young Geometric Group Theory meeting would be a great opportunity for me.

I did my Master in Pisa and there I learned many topics related to geometric group theory: apart from the relationships between bounded cohomology of discrete groups and spaces on one hand and quasimorphisms of discrete groups on the other (that I studied for my Bachelor thesis), I followed a course on Gromov hyperbolicity and I read Mineyev's papers [6], [7] about the relationships between bounded cohomology and Gromov-hyperbolicity. I also learned the theory of CAT(κ) spaces.

For my Master thesis I worked on Bucher's explicit calculations of the proportionality coefficient between the Riemannian volume and the simplicial volume of manifolds covered by $\mathbb{H}^2 \times \mathbb{H}^2$ [4]. In this context elements playing a key role are continuous cohomology and continuous bounded cohomology of locally compact topological groups and the subtle relations between these theories.

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Tomasz Prytuła

University of Warsaw

I am at the first year of master level studies in mathematics at the University of Warsaw. My mathematical interests are mainly commutative algebra and geometry. During bachelor studies I took, beyond standard ones, courses in algebra, topology, differential geometry, functional analysis, and PDEs. I also attended a seminar about Lie groups which allowed me to write a bachelor thesis about conjugacy in compact Lie groups. This year I'm attending courses in algebraic geometry, Lie groups and hyperbolic groups, two courses in algebra (mostly about group algebras and representation theory) and a seminar about the topology and geometry of manifolds.

Janusz Przewocki

Polish Academy of Sciences

I am interested in many fields of mathematics, yet now I am working in the area of topology. Being more specific my current work concerns Milnor-Thurston homology theory.

It is a version of homology theory where the chain modules consist of measures on singular simplices with a compact support. It was first used by Thurston [1] in his proof of Gromov's Theorem, where it is very convenient to use chains with infinite number of simplices. Furthermore the theory was independently developed by Hansen [2] and Zastrow [3].

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Piotr Przytycki

Polish Academy of Sciences

It does not require saying what metric curvature is to define what a *CAT(0) cube complex* is: it is a simply connected cube complex whose all links are flag. I am interested in groups acting by isometries on such complexes, usually properly, but not necessarily cocompactly. Or rather, I would like to show that some particular widely studied classes of groups admit such an action. The main method for achieving that is to use a construction due to Sageev [4], which requires codimension 1 subgroups. Geometrically, these are the stabilizers of "walls" in the arising cube complex, which have to be located also in the original space we aim to cubulate.

There is a series of algebraic consequences for groups acting geometrically on CAT(0) cube complexes. Their Dehn function is quadratic, word and conjugacy problems are decidable. This follows from the fact that these groups are biautomatic, and another consequence of that is that all solvable subgroups are virtually abelian. They satisfy the Tits alternative [5] and rank rigidity theorem [2]. They fail to have property (T) unlike many other groups in geometric group theory. Their class is rich enough to contain groups that are not residually finite [6] and even infinite simple [1]. It is not known if they all satisfy the $\mathbb{Z} \oplus \mathbb{Z}$ conjecture. The class of word-hyperbolic groups acting geometrically on a CAT(0) cube complex seems to be better behaved, but still might possibly contain non-residually finite groups.

There are particular actions on CAT(0) cube complexes, discovered by Haglund and Wise [3], called *special.* If the action is free, then these are the actions for which the quotient complex has hyperplanes with good properties. Namely, we require that hyperplanes are two-sided, have no self-intersections, no self-osculations and moreover, that there are no interosculations for pairs of hyperplanes. Complexes with these properties (called *special*) are exactly the ones which admit a local isometry into the Salvetti complex — a classifying space of a right-angled Artin group. Hence groups acting specially on CAT(0) cube complexes are subgroups of right-angled Artin groups, in particular they are linear, hence residually finite. When we study the cubulations of various families of groups, we are thus very much interested in determining whether virtually, i.e. up to passing to a finite index subgroup, the group acts specially.

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Doron Puder

Hebrew University

Let me state my relevant results as well as some plans for further research. The first result is a new criterion, and algorithm, to detect primitive words, as well as free factors: An element $w \in F_k$, the free group on k generators, is called primitive if it belongs to a basis (i.e. a free generating set) of F_k. A new analysis of Stallings core graphs representing finitely generated subgroups of F_k, yields a new algorithm to answer the following question: Assume that two f.g. subgroups $H, J \leq F_k$ are given (in terms of finite generating sets). Determine whether H is a free factor of J. In particular, this enables us to identify primitive words. (The new algorithm is different than the famous Whitehead method to address the same problem).

The second result shows that measure preserving words are primitive. A word $w \in F_k$ is called measure preserving if for every finite group G, the word map $w : G^k \to G$ induces uniform distribution on G (given uniform distribution on G^k). It is an easy observation that a primitive word is measure preserving. It is conjectured that the converse is also true. We prove it for F_2, and in a very recent joint work with O. Parzanchevski, prove the conjecture in full. As an immediate corollary we prove another conjecture involving $\hat{F}_k$, the profinite completion of F_k: If $w \in F_k$ belongs to a basis of $\hat{F}_k$, then it is primitive.

I hope to use similar techniques to establish further results in the field. One direction is to further analyse properties of the lattice of subgroups of F_k. Another possible direction is to consider words that are measure preserving on, say, compact Lie groups. Are these also primitive?

Yulan Qing

Tufts University

Let X be a CAT(0) space and let Γ be a finitely generated group that acts on X properly, cocompactly and by isometries. The geometry and topology of the ideal boundary of the space X, denoted $\partial_\infty X$, can be used to study the structure of Γ. In order to study the topology of $\partial_\infty X$, we first study how well behaved the topology is with respect to the space. Gromov's "geometric independence" states: *If a finitely generated group G acts discretely, cocompactly and by isometry on two Gromov hyperbolic Hadamard spaces X_1, X_2, then there is a G-equivariant homeomorphism $\partial_\infty X_1 \to \partial_\infty X_2$.* Croke and Kleiner showed that such is no longer the case when the hyperbolicity assumption is dropped [2]. They gave a construction for a family of CAT(0) spaces $\{X_\alpha : 0 < \alpha \leq \frac{\pi}{2}\}$ that each admit a geometric action by the same group G. They showed that $\partial_\infty X_\alpha \neq \partial_\infty X_{\frac{\pi}{2}}$ for all $0 < \alpha < \frac{\pi}{2}$. Julia Wilson showed that in fact $\partial_\infty X_\alpha \neq \partial_\infty X_\beta$ for all $\alpha \neq \beta$, so that G is a CAT(0) group with uncountably many non-homeomorphic boundaries [3].

In this study, we change the lengths of the essential loops that are isometrically glued together in this example and show that:

Theorem. On the components of ∂X other than the safe-path component, the G-equvariant action does not extend to a homeomorphism under the visual topology between ∂X and $\partial X'$, where X' has a different set of length data.

We are also in the process of building a homeomorphism between the boundaries with varied lengths data that is not induced by the algebra, i.e. to show that the two spaces are indeed homeomorphic.

The study opens up a variety of new questions. We want to study more properties of the set of points in the boundary that correspond to infinite itinerary geodesics. We want to compare them to a Cantor set. We would also like to build different classes of templates [2]. Moreover, we will generalize the result to a larger class of right angled Artin groups.

During the study, we have observed Cantor sets with a non-uniform probability measure. We would like to make study this example's boundary with probability measure as well.

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Colin Reid

Université catholique de Louvain

My research is in the area of totally disconnected topological groups. I intend to learn more about geometric methods in group theory to complement the (mostly algebraic) methods I have used so far.

An important class of residually finite groups are the just infinite groups, that is, those groups with no proper infinite images. In a sense these are the 'smallest' infinite residually finite groups and there are several well-studied examples, for instance most arithmetic groups are just infinite or nearly so. More generally, every infinite finitely generated abstract group has a just infinite image, and an analogous result holds for a large class of profinite groups, including all finitely generated pro-p groups. However, the general theory of just infinite groups is at present fairly mysterious, especially in the case of hereditarily just infinite groups (those in which every subgroup of finite index is just infinite). I have contributed to this theory, especially in the profinite case. I proved the following characterisation for profinite groups, generalising an earlier result for pro-p groups: an infinite profinite group G is just infinite if and only if, for every open normal subgroup K, there are only finitely many open normal subgroups of G not contained in K. In other words, all but finitely many finite images of G are extensions of G/K. I have also shown that a pro-p group is hereditarily just infinite if and only if the Frattini subgroup is just infinite. I used these results to prove that all just infinite profinite groups arise as inverse limits with a certain structure, and that conversely any inverse system with the specified structure has a just infinite limit. In particular, I have found a recipe for producing many examples of hereditarily just infinite groups, all of whose composition factors are non-abelian. (The existence of such groups was first discovered in a 2010 paper of J. Wilson.)

I have also contributed to the Sylow theory of profinite groups, which serves as a connection between the theory of pro-p groups (which are better understood) and the theory of profinite groups in general. In particular, I have investigated a natural generalisation to profinite groups of the generalised Fitting subgroup of a finite group. In finite group theory, this subgroup is important because it always contains its own centraliser, so the group can be understood through automorphisms of the generalised Fitting subgroup. The situation for profinite groups is more complicated, but there is a canonical decomposition: a profinite group has a unique largest image with trivial generalised pro-Fitting subgroup, and in the kernel, the generalised pro-Fitting subgroup contains its own centraliser. I have published some further results on the structure of finite and profinite groups with bounded prime divisors and whose Sylow subgroups have a bounded number of generators; in particular, such groups are either (pro-)nilpotent or have a non-nilpotent finite image of bounded size, and there is a bound on the index of the (pro-)Fitting subgroup. I am currently developing a local Sylow theory of totally disconnected, locally compact (t.d.l.c.) groups: given a t.d.l.c. group G, there is a t.d.l.c. group $G_{(p)}$ which is locally virtually pro-p and which maps continuously to G with dense image.

My current project with Prof. Pierre-Emmanuel Caprace and Prof. George Willis concerns the local structure of t.d.l.c. groups, especially those that are topologically simple. We have established a division of all compactly generated, topologically simple t.d.l.c. groups into a small number of types (which are determined by any open subgroup), and shown for some of these types that the group must be abstractly simple. We have also shown that many examples of profinite groups U cannot occur as open compact subgroups of a compactly generated topologically simple, t.d.l.c. group G, for instance this is the case if U has a non-trivial abelian normal subgroup. The key tool is the structure lattice, which is a modular lattice formed by taking all closed normal subgroups of open compact subgroups, modulo commensurability. From this one can obtain further lattices that admit a Boolean algebra structure, and which measure the extent to which the group is locally decomposable. In the compactly generated, topologically simple case, the action of the group on these Boolean algebras exhibits many useful properties, such as having dense orbits on the Stone space and having an element α such that every nonzero element of the algebra contains a G-translate of α. These properties can be translated back into intrinsic properties of the group, such as whether or not the group is abstractly simple, and also encapsulate 'branching' properties of the actions of the group on geometric objects such as trees. One important case of the theory we are developing is t.d.l.c. groups that are locally hereditarily just infinite, so that the modular lattice has just two elements. I plan to apply my knowledge of just infinite groups in this case to shed light on a currently poorly-understood class of t.d.l.c. groups.

Luis Manuel Rivera-Martinez

Universität Wien

My research topic is about metric approximation of infinite groups by finite ones. More specific, I am interested in questions related to sofic groups [2]. Sofic groups were introduced by M. Gromov in relation with the Gottschalk's Surjunctivity Conjecture (the name was coined by B. Weiss). In the last years, several results has showed the importance of knowing that a given group is sofic. For example, Gromov shows that sofic groups are surjunctive. It remains an open problem if there exists a non-sofic group.

Also I am interested in another classes of groups that have the metric approximation property [3]. For example, the class of weakly sofic groups (w-sofic groups) [1] whose definition is a natural extension of the definition of sofic groups, where instead of the normalized Hamming metric on symmetric groups its use general bi-invariant metrics on finite groups.

Every sofic group is w-sofic, but the converse implication is unknown. Also, we don't know any example of a non w-sofic group. A result in shows that the existence of a non w-sofic group is equivalent to some conjecture about the closure of products of conjugacy classes in the profinite topology on free groups.

[1] L. Glebsky and L. M. Rivera, *Sofic groups and profinite topology on free groups*, Journal of Algebra, **320** (9), (2008), 3512–3518.

[2] V. Pestov, *Hyperlinear and sofic groups: a brief guide*, The Bulletin of Symbolic Logic, **14** (4) (2008), 449–480.

[3] A. Thom, *About the metric approximation of Higman's group*, to appear in the Journal of Group Theory, arXiv:1008.1931v2

Moritz Rodenhausen

Universität Bonn

Since April 2010, I have been PhD student with Prof. Carl-Friedrich Bödigheimer at the University of Bonn. For the period April through September 2011 I was in Oxford as a visitor with Prof. Martin Bridson.

Most recently I study isometric group actions on CAT(0)-spaces. Roughly speaking, a CAT*(0)-space* X is a geodesic metric space whose geodesic triangles are no thicker than comparison triangles in the Euclidean plane having the same sidelengths. For an isometry $\gamma : X \to X$ the *translation length* is

$$|\gamma| := \inf_{x \in X} d(x, \gamma(x)).$$

γ is called *semisimple* if the infimum is attained for some $x \in X$ and *parabolic* otherwise. In the semisimple case, γ is *elliptic* if $|\gamma| = 0$, i.e. if γ has a fixed point, and *hyperbolic* for $|\gamma| > 0$.
If G is a group acting on a complete CAT(0)-space, then we ask: What $g \in G$ always have to act by zero translation length? It is well-known that this is the case for all g of finite order and those such that the infinite cyclic subgroup $\langle g \rangle \subset G$ is distorted. Therefore this problem is more interesting for elements generating *undistorted* (i.e. quasi-isometrically embedded) infinite cyclic subgroups, e.g. Nielsen generators of $\mathrm{Aut}(F_n)$.

I have shown that the translation lengths of two commuting isometries α and β satisfy the following parallelogram law:

$$|\alpha\beta|^2|\alpha\beta^{-1}|^2 = 2(|\alpha|^2|\beta|^2).$$

As an application I can show that Nielsen generators in $\mathrm{Aut}(F_n)$ for $n \geq 4$ have to act by zero translation length. Bridson has shown this for $n \geq 6$ and for $n \geq 4$ in the special case of semisimple actions in [1]. In the same paper Bridson constructs actions of $\mathrm{Aut}(F_3)$ such that Nielsen generators act as hyperbolic isometries.

It follows from elementary CAT(0) geometry that $g \in G$ has to act by zero translation length if it has finite order in the abelianization of its centralizer. For a second proof of my result I want to understand the centralizer $C(\rho) \subset \mathrm{Aut}(F_n)$ of a Nielsen generator $\rho \in \mathrm{Aut}(F_n)$. By some reduction arguments similar to [2], I know an explicit finite generating set for $C(\rho)$. I can compute that ρ is a commutator in $C(\rho)$, i.e. vanishes in the abelianization $H_1(C(\rho))$.

Now there are the following open questions: Is $C(\rho)$ finitely presented? What are the centralizers of other elements of $\mathrm{Aut}(F_n)$, e.g. $C(\phi)$ for $\phi(a_1) = a_1 w$, $\phi(a_i) = a_i$ for $i \geq 2$, where $a_1, \ldots, a_n$ is a basis of F_n and w an arbirary word in $a_2, \ldots, a_n$? How do the latter more general automorphisms act on complete CAT(0)-spaces?

[1] M. R. Bridson: *The Rhombic Dodecahedron and Semisimple Actions of Aut(F_n) on CAT(0) Spaces* 2011

[2] W. Magnus, A. Karrass, D. Solitar: *Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations*, Interscience Publishers 1966

Pascal Rolli

Eidgenössische Technische Hochschule Zürich

The topics of my research are quasimorphisms and bounded cohomology. I am currently working on generalizations of my own combinatorial construction of a class of quasimorphisms $q_\sigma : \mathbb{F} \to \mathbb{R}$, $\sigma \in \ell^\infty$, defined on free groups $\mathbb{F}$. (http://arxiv.org/abs/0911.4234). This construction is rather different from previously known quasimorphisms, in particular it is linear and leads to a rather simple proof of the infinite-dimensionality of the second bounded cohomology of non-abelian free groups.

Firstly, I recently reinterpreted the construction using actions on trees. This led to a uniform description of a class of quasimorphisms assigned to an action of a group Γ on a tree $\mathcal{T}$. It applies in particular to amalgams $\Gamma = A *_B C$ and HNN-extensions $\Gamma = A*_\varphi$ acting on their Bass-Serre trees. I am currently studying these situations and I have obtained certain non-triviality results. The relevant bounded cohomology spaces have been investigated by K. Fujiwara who proved infinite-dimensionality using a generalization of a construction by R. Brooks. Again, it is my aim is to utilize the linearity of my construction to obtain infinite-dimensional subspaces of bounded cohomology.

Secondly, I've been studying twisted 2-cocycles for linear isometric actions of free products $\Gamma = A * B$ on Hilbert spaces. These are maps that reduce to quasimorphisms when the action is trivial. The construction given in the aforementioned article has a natural generalization to this setting. I was able to prove that every action of Γ on a finite-dimensional Hilbert space $\mathcal{H}$ admits non-trivial twisted 2-cocycles. In the case of $\mathcal{H}$ being infinite dimensional, most importantly if $\mathcal{H} = \ell^2(\Gamma)$, the arguments are no longer valid. I'm currently trying to fix this by using the CAT(0)-structure of the metric in $\mathcal{H}$.

Furthermore, I constructed a quasimorphism in a purely geometric way, using the action of a free group on a certain CAT(0)-square complex quasi-isometric to the group's Cayley graph.

All my work so far relies on groups acting on geometric objects. The Young Geometric Group Theory meeting seems to be a great opportunity to learn more about the topic and to interact with other motivated students and researchers.

Jordan Sahattchieve

University of Michigan

Quasiconvex Subgroups of $F_m \times \mathbb{Z}^n$

Which subgroups of CAT(0) groups are themselves CAT(0)? If Γ is a Gromov hyperbolic group and H is a subgroup, there is a well-known sufficient condition for H to be itself hyperbolic. A subgroup H is called *quasiconvex* if its orbit is a quasiconvex set, meaning, any geodesic connecting points in the orbit is contained in some fixed bounded neighborhood of the orbit. We then have the following elementary fact: if H is quasiconvex, H is hyperbolic. This observation naturally gives rise to the following question: *Is a quasiconvex subgroup of a CAT(0) group CAT(0)?* There are no known counterexamples, however, the question remains open in the general case. In my thesis, I have proved that quasiconvex subgroups of a $F_m \times \mathbb{Z}^n$ act cocompactly on the convex hulls of their orbits. These groups are not hyperbolic and the CAT(0) space on which they act has many flats which are not isolated.

Bounded Packing in Polycyclic Groups

Given a finitely generated group G and a codimension-1 (equivalently, the number of ends of H in G is at least two) subgroup H, one constructs a non-positively curved cube complex and an essential action of G on it. This construction is due to Sageev. The term *codimension-1* comes from the following classical example: if M is a 3-manifold with fundamental group G and N is an immersed incompressible surface, then the image of $\pi_1(N)$ under the immersion map is a codimension-1 subgroup of G. In their paper, *Packing subgroups in relatively hyperbolic groups*, Wise and Hruska define *bounded packing*, a property of H which guarantees that Sageev's cubing arising from the pair (G, H) will be finite dimensional. I have proved than any subgroup of $G = \mathbb{Z}^n \rtimes_\phi \mathbb{Z}$ has bounded packing if ϕ is a hyperbolic automorphism. Simultaneously and independently, W. Yang showed that any separable subgroup has bounded packing in the ambient group which coupled with a result of Macev on separability of subgroups in polycyclic groups settles the general case.

Current Interests

I am currently interested in questions on 3-manifolds, Artin groups, and of course CAT(0) and hyperbolic groups.

Andrew Sale

University of Oxford

The Conjugacy Problem asks whether, given a finitely generated group, there exists an algorithm which determines if the input, two words u, v on a generating set, represent conjugate elements in the group. We can extend this problem further by asking if there is a relationship between the length of a pair of conjugate elements and the minimal length of a conjugator between the two. Grunewald and Segal [1] solved the conjugacy problem for arithmetic groups, after Grunewald [2] and Sarkisjan [5] independently solved it for $\mathrm{SL}_n(\mathbb{Z})$. However the question of conjugacy length in these groups remains open.

Let G be a connected semisimple real Lie group of rank at least 2, which is the identity component of the group of isometries of a Riemannian symmetric space X with no Euclidean factors. Let Γ be a non-uniform irreducible lattice in G. We denote by Δ_{mod} the quotient $\partial_\infty X/G$, where $\partial_\infty X$ denotes the boundary of X. For each hyperbolic element $a \in G$ we can define its *slope* as follows: in X a family of parallel geodesics are translated by a. Pick one of these geodesics, $c : \mathbb{R} \to X$, oriented so that a translates it in the positive direction. The slope of a is the image of $c(\infty)$ in Δ_{mod}. Because all geodesics in X translated by a are parallel, the slope is well-defined.

For the following we fix a basepoint $p \in X$. By a result of Lubotzky, Mozes and Raghunathan [3], to study the lengths of elements in Γ it is enough to look at the distance these elements move the basepoint p.

Masur and Minsky [4] showed that one can obtain a linear upper bound for the length of conjugators between pseudo-Anosov elements in mapping class groups. With this in mind we look first at the semisimple elements of a lattice Γ.

Theorem. For each $\xi \in \Delta_{\mathrm{mod}}$ there exists a constant $\ell_\xi > 0$ such that a pair of hyperbolic elements a, b in Γ of slope ξ are conjugate in Γ if and only if there exists $g \in G$ such that $gag^{-1} = b$ and

$$d_X(p, gp) \leq \ell_\xi(d_X(p, ap)d_X(p, bp)).$$

The proof of the Theorem involves looking at the distance from p to the set $\mathrm{MIN}a = \{x \in X \mid d_X(x, ax) \leq d_X(y, ay) \ \forall\ y \in X\}$.

We are looking also for a similar result for unipotent elements. Here we do hope to obtain a uniform bound. In the following, N is a maximal connected unipotent subgroup of G.

Conjecture. Let u, v be conjugate unipotent elements in $N \cap \Gamma$. We can construct $g \in G$ such that $gug^{-1} = v$. For such g we have:

$$d_G(1, g) \preceq d_G(1, u)d_G(1, v).$$

By the Iwasawa decomposition of G we can express any element of N in the form:

$$\exp\left(\sum_{\lambda \in \Lambda} Y_\lambda\right)$$

where Λ are the positive roots of a reduced root system and Y_λ belong to root-spaces of $\mathfrak{g}$. If we find such expressions for u and v, and if furthermore we see that whenever λ is simple in Λ the corresponding entry in the sum for both u and v is non-zero, then we can prove the conjecture. Extending this method to all elements in $N \cap \Gamma$ seems possible, with techniques motivated by methods which work for upper-triangular unipotent matrices in $\mathrm{SL}_n(\mathbb{Z})$.

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Marco Schwandt

Universität Bielefeld

Finiteness properties of groups are of interest in their own right. A group G is said to be *of type* F_m, if it admits a $K(G,1)$-space with finite m-skeleton. In view of this definition F_1 is equivalent to the group being finitely generated and F_2 to being finitely presented. With $\phi(G)$ we want to denote the *finiteness length* of G, i.e. , the largest m for which G is of type F_m.

One method of studying such finiteness properties is to consider actions of the group on certain "nice" spaces and to analyze these actions using geometric arguments to deduce finiteness properties.

Let us discuss an example of this. Consider the linear algebraic group SL_3 over a global function field K. Let S be a finite non-empty set of places of K and $\mathcal{O}_S$ the ring of S-integers. Moreover let $\mathcal{B} \leq \mathrm{SL}_3$ be the group of upper triangular matrices, it is a Borel subgroup and as such a minimal parabolic subgroup of SL_3. Let $\mathcal{P}$ denote the subgroup of SL_3 consisting of matrices of the form

$$\begin{pmatrix} * & * & * \\ 0 & * & * \\ 0 & * & * \end{pmatrix}.$$

It contains $\mathcal{B}$, therefore it is parabolic, and we have a chain $\mathcal{B} \leq \mathcal{P} \leq \mathrm{SL}_3$ in the poset of parabolic subgroups of SL_3.

Now let G be the S-arithmetic group $\mathrm{SL}_3(\mathcal{O}_S)$ and define P and B accordingly.

In this situation it is known by a theorem of Bux (cf. [2]) that the finiteness length of B is equal to $|S|-1$. More generally he proved that the finiteness lengths of Borel subgroups of semi-simple groups $\mathcal{G}$ do not depend on the local ranks of the given group. In a more recent work Bux, Köhl and Witzel (cf. [1]) proved that the finiteness length of $\mathcal{G}(\mathcal{O}_S)$ does depend on the local ranks of $\mathcal{G}$, given nice enough additional properties of $\mathcal{G}$. Since it is known that the local ranks of SL_3 are always 2 and SL_3 satisfies the properties of [1], we obtain $\phi(G) = 2\,|S|-1$ in our example. We have:

$$\begin{array}{r|ccccc} & \mathcal{B} & \leq & \mathcal{P} & \leq & \mathrm{SL}_3 \\ \phi(\_(\mathcal{O}_S)) & |S|-1 & & ? & & 2\,|S|-1 \end{array}$$

This leads to various obvious questions. What is the finiteness length of P? Does it depend on the local ranks of SL_3?

Moreover it is of interest what happens for parabolic subgroups $\mathcal{P}(\mathcal{O}_S)$ of reductive groups $\mathcal{G}$ in general. The above results can be applied, and yield the finiteness length of $\mathcal{G}(\mathcal{O}_S)$ and $\mathcal{B}(\mathcal{O}_S)$, but not of $\mathcal{P}(\mathcal{O}_S)$. An obvious assumption would be, that the finiteness lengths of parabolic subgroups are monotonic if we consider a chain such as above. Furthermore, can a "height" be determined, at which the finiteness length becomes dependent on the local ranks of the group?

There are certainly enough questions arising from the above example alone.

Since both of the above cited results were proven by using tools of the theory of affine buildings and groups acting on them, it can be expected, that the same kind of tools could answer at least some of the before mentioned questions.

[1] Kai-Uwe Bux, Ralf Gramlich, and Stefan Witzel, *Higher finiteness properties of reductive arithmetic groups in positive characteristic: the rank theorem*, 2011.

[2] Kai-Uwe Bux, *Endlichkeitseigenschaften auflösbarer arithmetischer Gruppen über Funktionenkörpern*, 1998.

Alessandro Sisto

University of Oxford

My main research topic so far has been the geometry of asymptotic cones. Indeed, I obtained results on asymptotic cones of relatively hyperbolic groups and more generally asymptotic cones of groups containing global cut-points [6], some of them discovered independently by Osin and Sapir, which I then used to show that all asymptotic cones of a given 3-manifold group are bilipschitz equivalent [4]. In [7] I showed that there exist exactly three real trees up to isometry that can occur as asymptotic cones of groups, that all separable asymptotic cones are proper, and related results, including the fact that a group with "few" separable asymptotic cones is virtually nilpotent (relying on results by Gromov and Point). In some sense in the opposite direction Lars Scheele and I [3] described some examples of metric spaces and groups exhibiting exotic behaviours with respect to the operation of iteration of asymptotic cones. Finally, asymptotic cones are used to prove quasi-isometry rigidity results in a joint work with Roberto Frigerio and Jean-François Lafont [1] for a certain class of manifolds admitting a geometric decomposition similar to that of Haken 3-manifolds. In the same paper we also show several other kinds of rigidity results. Our motivation in considering that class of manifolds, other than the analogy with 3-manifolds, was to provide examples of rigidity phenomena in a non-$CAT(0)$ context, and indeed many of the manifolds we considered do not admit a $CAT(0)$ metric.

I also wrote a paper together with Roberto Frigerio on a characterization of hyperbolic spaces and real trees in terms of the ratio of the hyperbolicity constant of geodesic triangles and their perimeters [2] and a paper on an application of ultrafilters in combinatorics [5].

I am currently in the final stages of the preparation of a joint paper with John MacKay about quasi-isometric embeddings of the hyperbolic plane in relatively hyperbolic groups and a joint paper with David Hume on embeddings of graph manifold groups inside products of trees. Finally, I am exploring possible analogies between relatively hyperbolic groups, mapping class groups, right angled Artin groups and graph manifold groups inspired by the alternative definition of relative hyperbolicity I gave in [6].

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[4] A. Sisto - *3-manifold groups have unique asymptotic cones*, arXiv:1109.4674 (2011).

[5] A. Sisto - *Exponential triples*, Electronic Journal of Combinatorics, 18(1), P147 (2011).

[6] A. Sisto - *Projections and relative hyperbolicity*, arXiv:1010.4552 (2011).

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Rizos Sklinos

Hebrew University

My research is focused on model theory and its interactions with geometric group theory. Much attention has been given to the subject after Sela [6] and Kharlampovich-Myasnikov [1] independently proved that non abelian free groups share the same common theory. Furthermore, Sela [7] proved, using geometric methods, that this common theory is stable. Apart from the fact that this introduces new methods to the study of free groups, it also shows the necessity of a more thorough understanding of geometric techniques in model theory.

The theory of free groups is considered as a new leaf in stability theory. It is a source of many model theoretic phenomena, occurring in a natural theory. For example, it is non equational (Sela [7]), non superstable (Poizat [5], Gibbons) as well as connected, i.e. there is a model with no definable proper subgroup of finite index (Pillay [3], Poizat).

In my thesis I try to give a model theoretic point of view on the subject. In stable groups there is a distinguished family of types called the generic types. As the theory of free groups is connected there is a unique generic type over any set of parameters. To be even more precise, every generic type in this theory is the non forking extension of the generic type over the empty set. An interesting property of this type is that any maximal independent set of realizations in a free group has the cardinality of the rank of the free group (Pillay [4]). Thus, one should expect that the generic type has weight 1. It is quite surprising that it has infinite weight. Moreover, I found an infinite independent sequence of realizations of the generic type in F_ω with the property that each member of the sequence forks with a single realization of the generic type. Intuitively speaking, weight can be thought of as an abstract exchange principle. As bases of free groups are not subject to some exchange principle, this explains the "bad" behavior of the generic type.

Using the sequence witnessing the infinite weight, I proved that uncountable free groups are not $\aleph_1$-homogeneous. On the other hand, in a joint paper with C. Perin [2] we prove that finitely generated free groups are homogeneous (as first order structures). This answers a question of A. Nies, who proved that F_2 is homogeneous and asked whether this is true for higher rank free groups. The techniques used in this paper are purely geometric.

Perin, in her thesis, shows that an elementary subgroup of a finitely generated free group is a free factor. I proved [8], using some forking calculus, that this result does not extend to F_ω. Thus, there is an elementary subgroup of F_ω which is not a free factor.

Again, in joint work with Perin we give an algebraic description of forking in standard models of the theory of free groups, i.e. free groups. So, our result is: two tuples of elements of a free group are independent over the empty set if and only if they belong to associated free factors, i.e.

free factors whose bases together extend to a basis of the whole group. We also give a description of forking over standard models, and we expect to extend the result over arbitrary subsets of free groups.

Another interesting question concerns centralizers of elements in models of the theory of free groups. The conjecture being that they are pure groups, i.e. all the definable structure comes from multiplication alone. It is quite hard to obtain (non) definability results without a thorough understanding of the quantifier elimination (up to boolean combination of AE formulas) of the theory of free groups. However, I have used some geometric stability theory to recast the above conjecture to the following: any infinite definable set in the centralizer finitely covers the centralizer, i.e. finitely many translates of the set cover the centralizer. A related result of independent interest is that any U-rank 1 expansion of $(\mathbb{Z}, +, 0)$ is a pure group. Also, in a theory without the finite cover property when a family of groups is uniformly definable, then to each formula we can assign a natural number n, so that for every group in the family either n translates of an instance of this formula covers the group or does not finitely cover the group.

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Amit Solomon

Hebrew University

I am just about to start my graduate studies (towards the M.Sc degree), whence I do not have a research statement. I am interested generally in geometry (and analysis), and have not yet found my specific area of interest. I consider doing my thesis under the guidance of Tsachik Gelander (CAT(0) spaces, geometric group theory, and much more) or Jake Solomon (Differential geometry, symplectic geometry, etc.), perhaps jointly with Emmanuel Farjoun (algebraic topology, homotopy theory). As you can see, almost any geometric subject I encountered so far with my poor background, swept me off my feet, and I cannot yet decide between these beautiful areas. Among other topics I am interested in, are Khovanov homology, Knots, and geometry of "nice" spaces (Banach, Hilbert, etc.).

I hope to learn more about geomertic group theory in the workshop and hopefully that will help me decide on a direction to follow.

Piotr Sołtan

University of Warsaw

The main focus of my research is in the theory of quantum groups on operator algebra level. This theory uses the language of C*-algebras and von Neumann algebras to study objects generalizing locally compact groups in the spirit of Alain Connes's non-commutative geometry ([2]). From another point of view the theory of quantum groups may be regarded as a blend of non-commutative geometry and abstract harmonic analysis. The main references on quantum groups are [12], [13], [14], [16] and [15].

The subject of quantum groups may be divided into two main branches: compact quantum groups and non-compact quantum groups. While the first is very well studied, the latter is still rather mysterious. Accordingly my own research is divided between the two branches, although I have done work which tries to bridge the gap ([5], [7] and [4], [3] where the notion of a quantum Bohr compactification has been introduced). On the "compact" side much attention is being paid to actions of quantum groups, while the "non-compact" part of the theory still needs a better look at the basic axioms. The respective areas are addressed in my papers [6], [8] and [9], [10].

The fundamental examples in the theory of quantum groups come from group theory. More precisely given any locally compact group G the C*-algebras objects $C_0(G)$, $\mathrm{C}^*(G)$ and $\mathrm{C}^*_r(G)$ are canonically endowed with quantum group structure. In particular this is the case for a *discrete* group G. In the last case there is a great abundance of tools coming from geometric group theory. I would like to study the possible generalizations of these tools to discrete quantum groups possibly along the lines of [11]. Moreover, there is some recent work on quantum isometry groups of spectral triples (non-commutative analogs of differential manifolds) which is very strongly based on geometric aspects of discrete group theory.

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Aneta Stal

University of Warsaw

I am a student of the 5th year at the College of Inter-Faculty Individual Studies in Mathematics and Natural Sciences. I'm currently working on my master thesis: "Etale extensions of ring spectra". I am interested in the topology of manifolds, algebraic topology, category theory and their applications in physics. My bachelor thesis in physics was titled: "Geometric formulation of gauge theory". Recently I have taken the following courses connected to conference topics: Topology II, Algebraic Topology I, II, Differential Geometry I, II and a seminar in Topology and Geometry of Manifolds. This semester I am attending courses in Homological Algebra, Lie groups, Functional Analysis and a seminar in Classical Algebraic Structures.

Emily Stark

Tufts University

I am interested in questions in geometric group theory and low-dimensional topology, with a primary focus on cube complexes. A cube complex is a CW-complex in which each cell is topologically a cube, and the attaching maps restrict to linear homeomorphisms on the faces. Regarding each cube as a unit cube embedded in Euclidean space, a cube complex admits a piecewise Euclidean metric. Alternatively, a cube complex may be given a piecewise hyperbolic metric by viewing each cube as a polyhedron in hyperbolic space, with the gluing maps given by isometries. In either case, the distance between two points in a cube complex may be defined as the infimum of the lengths of paths between the points. Bridson showed that these pseudometrics turn a cube complex into a complete geodesic space [1]. A cube complex is CAT(κ) if, with the induced path metric, the cube complex is a CAT(κ) metric space.

Gromov pioneered the study of CAT(κ) cube complexes; he characterized the global geometry of a cube complex in terms of a local combinatorial condition [2]. Indeed, this condition proves useful in studying the large-scale geometry of a CAT(κ) cube complex. Gromov's link condition, for example, may be used to prove that hyperplanes in a CAT(κ) cube complex are convex and separate the complex into two convex complementary halfspaces [3]. In turn, an understanding of large-scale geometrical properties translates to knowledge of group theoretic properties in the case of groups that act geometrically on some CAT(κ) cube complex. For example, the properties of hyperplanes are exploited in the proof of Sageev and Wise that groups that act properly on a finite dimensional cube complex satisfy the Tits alternative [4].

My current research concerns the curvature properties of cube complexes. A long-standing problem in geometric group theory asks to reconcile the notions of hyperbolicity in the non-manifold setting. In particular, if a δ-hyperbolic group acts geometrically on a cube complex, then the Švarc-Milnor Lemma shows that this large-scale hyperbolicity of the group translates to large-scale hyperbolicity of the cube complex. However, it is unknown whether there exists any cube complex that a δ-hyperbolic group acts on which can be given a CAT(-1) metric. I am interested in the extent to which the distinction between large-scale and local hyperbolicity can be detected in the case of groups acting on cube complexes.

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Markus Steenbock

Universität Wien

The research topic we focus on concerns infinite groups. The methods combine algebraic, geometric, and probabilistic tools.

We plan to study the famous Kaplansky conjecture in the context of random groups. This outstanding conjecture, also known as zero divisor conjecture, states that the group ring of every torsion-free group over an integral domain has no zero divisors.

Our main objective is to establish the Kaplansky conjecture for random finitely presented groups.

Recall that every group can be given by a group presentation $\langle G, R\rangle$, and it is called finitely presented if both, the set of generator G and the set of relators R, are finite sets. Intuitively, a random group is a group defined by a random choice of relators R. The idea is to determine what properties occur typically, that is most probably for such choices.

The following are our starting points.

Kaplansky's conjecture has been shown for various classes of groups, for example, unique product groups. On the other hand, it is known that there are infinitely many torsion free groups without unique product property. In the construction of such examples of groups, Rips and Segev use graphical presentations and small cancellation conditions [Rips & Segev'1987].

Random finitely presented groups are torsion-free and hyperbolic.

Delzant showed that Kaplansky's conjecture hold for any hyperbolic group with a certain large translation length property [Delzant'1997].

Thus, we plan to show that random finitely presented groups are unique product groups.

Step 1: Study the original construction of Rips-Segev.

- generalize Rips-Segev's torsion-free groups without unique product property
- estimate the size of the class of (generalized) Rips-Segev's groups
- investigate whether or not they satisfy Kaplansky's conjecture

Step 2: Study conditions for large translation length.

- formulate combinatorial conditions on the relators which imply large translation length
- determine whether or not these conditions are very probable to occur, that is if a random group typically has the large translation property

As a result of our study, we will obtain first results on the Kaplansky conjecture in the context of random groups. Thus, we plan to show, that Kaplansky's conjecture holds for a typical finitely presented random group. Otherwise, we will determine a class of finitely presented groups which are potential counterexamples. Our investigation also contributes to the study of purely algebraic properties of hyperbolic groups. Moreover, our work will contribute in providing potential counterexamples to other outstanding conjectures.

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Harold Sultan

Columbia University

My principle areas of research are geometric group theory and the asymptotic geometry of Teichmüller space. I have been studying the large scale geometry of Teichmülller space via an analysis of the geometry of the pants complex, $\mathcal{P}(S)$. In my paper [9], I study the tree graded structure of $\mathcal{T}(S)$. In particular, I prove the following characterization of the *canonical finest pieces* in the tree-graded structure of $\mathcal{T}_\omega(S)$, which is motivated by a similar characterization for mapping class groups in [4], and for right angled Artin groups in [1]. The characterization of finest pieces is given in terms of the complex of separating multicurves, $\mathbb{S}(S)$ which is defined and explored in [9].

Theorem 1 ([9]). For any surface S, any asymptotic cone $\mathcal{P}_\omega(S)$, and any points $a_\omega, b_\omega \in \mathcal{P}_\omega(S)$, the following are equivalent:

1. No point separates a_ω and b_ω,
2. In any neighborhood of a_ω, b_ω, respectively, there exists a'_ω, b'_ω, with representative sequences (a'_n),(b'_n), such that $\lim_\omega d_{\mathbb{S}(S)}(a'_n, b'_n) < \infty$.

The *divergence* of a metric space measures the inefficiency of detour paths. In particular, a geodesic γ has superlinear divergence if and only if its ultralimit γ_ω in some asymptotic cone contains a global cutpoint. Hence, Theorem is relevant to the study of divergence. In particular, using Theorem 1, in conjunction with a careful analysis of the connected components of $\mathbb{S}(S_{2,1})$, I prove the following, resolving open problems of Behrstock-Druţu-Mosher in [3], Behrstock-Druţu in [2], and Brock-Masur in [5]:

Theorem 2 ([9]). $\mathcal{T}(S_{2,1})$ is unique amongst Teichmuller spaces of all other surfaces in the following two senses. First, it is thick of order two. Second, it has superquadratic yet subexponential divergence.

In addition to my research relating to the large scale geometry of $\mathcal{T}(S)$, I also have several papers with results related to combinatorial complexes and CAT(0) geometry which I will summarize presently.

In my paper [7], using graph theoretic methods, for all surfaces of finite type I provide asymptotically sharp bounds on the maximal distance in the pants complex from any pants decomposition to the set of pants decompositions containing a separating curve, $\mathcal{P}_{sep}(S)$. In particular, I prove that for closed surfaces this distance is a logarithmic function of the genus.

In my paper [8], using some nice properties of Farey graphs I prove that the separating curve complex $\mathbb{S}(S_{2,0})$ satisfies a quasi-distance formula and is δ-hyperbolic, answering a question of S. Schleimer.

Finally, in my paper [10], I use methods of CAT(0) geometry and asymptotic cones to prove that Morse and (b,c)-contracting quasi-geodesics are equivalent in CAT(0) spaces. As a corollary I provide a converse to a characterization of Morse geodesics in the asymptotic cone thereby resolving an open question of Druţu-Mozes-Sapir in [6] and Behrstock-Druţu in [2], for CAT(0) spaces.

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Michał Szostakiewicz

University of Warsaw

My strongest mathematical background is in dynamical systems.

I have defended MSc in mathematics with thesis named "Dimension of some fractal construction", which dealt with Hausdorff and box dimension of a particular fractal set.

Right now I'm almost finished with my PhD thesis in mathematics: "Ergodic properties of holomorphic maps of complex projective spaces", which will be defended at Institute of Mathematics of the Polish Academy of Sciences. My advisor is prof. Anna Zdunik from the University of Warsaw. I study mainly statistical properties with heavy geometrical, lgebraic and combinatorial machinery.

I'm also MSc in computer science with thesis: "Fractal compression and its modifications". I currently work as a researcher in Interdisciplinary Centre for Mathematical and Computational Modelling of Warsaw University. I deal mainly with the problem of matching meshes with a given data signal.

Krzysztof Święcicki

University of Warsaw

Currently I am on master level studies in math and physics at the University of Warsaw. My specialization program in math is topology and geometry of manifolds. I took 2 semesters long classes in topology, algebra and functional analysis and one year seminar in Lie groups. Right now I am attending courses in algebraic geometry, algebraic topology, differential geometry, Lie groups, harmonic analysis which I should be about to end in the January. I do not possess any knowledge in geometric group theory, but I would love to learn it.

Łukasz Świstek

University of Warsaw

My main area of interest in mathematics is noncommutative algebra. I'm mostly interested in noncommutative/nonassociative algebras over a field. I've written my bachelor's thesis about Clifford algebras and Spin groups. I'm also interested in topics around Bott's periodicity theorem (mainly in its applications to Clifford algebras).

I don't know a lot about geometric group theory but I really want to know much more. In this semester I'm attending a lecture about hyperbolic groups.

During my studies I've done the following subjects connected to conference topics: Topology I and II, Algebra I, II and III, Algebraic Topology I and II. I've also attended a seminar about Lie groups. I've tried a little bit of algebraic geometry too.

Samuel Taylor

University of Texas at Austin

My main research interests are geometric group theory and coarse geometry with applications to low-dimensional topology. In particular, I am interested in the geometry and the subgroups structure of the mapping class group along with other geometric and combinatorial objects associated to surfaces, e.g. Teichmuller space, the curve complex. To understand the geometry/topology of surfaces and these related spaces, one makes heavy use of methods from geometric group theory and analogies to other well-studied objects. This is reflected in trying to understand to what extent the mapping class group resembles rank 1 (or higher rank) lattices in semi-simple Lie groups, to make precise the degree to which Teichmuller space resembles a space of non-positive curvature, and using the actual hyperbolicity of the curve complex to answer these and other related questions. Here are a few specific research areas of interest:

Recently, there has been an interest in understanding the structure of right-angled Artin subgroups of the mapping class group. Koberda proves that beginning with a collection of Dehn twists and (partial) pseudo-Anosov maps supported on connected subsurfaces, there is an N so that the subgroup generated by the Nth powers of these generators is a right-angled Artin group. Similarly, Clay, Leininger, and Mangahas give conditions for when partial pseudo-Anosov mapping classes supported on disjoint or overlapping subsurfaces generate right-angled Artin groups that are quasi-isometrically embedded in the mapping class group and Teichmuller space (via the orbit map). Their methods make use of the Masur-Minsky distance estimates in the mapping class group and analogous formulas for distance in Teichmuller space by Rafi. This produces examples of surfaces subgroups of the mapping class group that act co-compactly on quasi-isometrically embedded copies of the hyperbolic place in Teichmuller space. I am interested in using these methods to better understand undistorted subgroups of the mapping class group.

Another question in which I am interested involves quasi-isometric embeddings between mapping class groups and other combinatorial complexes. Behrstock, Kleiner, Minsky, and Mosher, and independently Hamenstadt, proved that the mapping class group is quasi-isometrically rigid, that is any quasi-isometry is a bounded distance from an actual isometry of the space. Using this result, Schleimer and Rafi proved quasi-isometric rigidity of the curve complex also produces quasi-isometric embeddings between curve complexes induced by orbifold covers. Their methods, along with the Masur-Minsky distance formula for the marking complex, also show that orbifold covers produce quasi-isometric embedding between mapping class groups. In a slightly different direction Aramayona, Leininger, and Souto produce injective homomorphisms between mapping class groups induced through covering spaces, and depending on the covers used, these injections can be either quasi-isometrically embedded (as the the Schleimer-Rafi examples) or badly distorted.

Other than these specific questions, I am broadly interested in hyperbolic groups, groups actions on CAT(0) spaces, and hyperbolic geometry. Attending the meeting would give me a broader view of current research in geometric group theory and the opportunity to interact with other graduate students and researchers in the area.

Ewa Tyszkowska

University of Gdańsk

A Klein surface X is a 1-dimensional complex manifold for which the changes of coordinates are either conformal or anticonformal. If all changes of coordinates are conformal then X is orientable and it is called a Riemann surface. Let g be a topological genus of X and let k be the number of its boundary components. There is a Riemann surface X^+ of topological genus $p = \eta g + k - 1$ and an involution $\tau : X^+ \to X^+$ such that $X^+/\tau = X$, where $\eta = 2$ or 1 according to X being orientable or not. The integer p is called the algebraic genus of X.

The study of Klein surfaces is based on the theory of NEC groups which are discrete groups of isometries of the hyperbolic plane $\mathcal{H}$ with compact orbit space. A compact Klein surface X of algebraic genus $p \geq 2$ is isomorphic to orbit space $\mathcal{H}/\Gamma$, where Γ is a surface NEC group. Furthermore, a finite group G is an automorphism group of X if and only if $G \cong \Lambda/\Gamma$ for some NEC group Λ.

I do research connected with automorphisms of Klein surfaces. Most of my results concern topological classification of conformal actions on Riemann surfaces and the number of ovals of their symmetries.

We say that a finite group G acts on a topological surface X if there exists a monomorphism $\varepsilon : G \to \mathrm{Hom}^+(X)$, where $\mathrm{Hom}^+(X)$ is the group of orientation-preserving homeomorphisms of X. Two actions of finite groups G and G' on X are topologically equivalent if the images of G and G' are conjugate in $\mathrm{Hom}^+(X)$. There are two reasons for the topological classification of finite actions rather than just the groups of homeomorphisms. Firstly, there is one-to-one correspondence between the equivalence classes of group actions and conjugacy classes of finite subgroups of the mapping class group and so such a classification gives some information on the structure of this group. Secondly, the enumeration of finite group actions is a principal component of the analysis of singularities of the moduli space of conformal equivalence classes of Riemann surfaces of given genus since such a space is an orbit space of Teichmüller space by a natural action of the mapping class group.

A symmetry of Riemann surface X is an antiholomorphic involution ϱ. Under the correspondence between curves and surfaces the fact that a surface X is symmetric means that the corresponding curve can be defined over the reals. Furthermore the non-conjugate, in the group of all automorphisms of X, symmetries correspond to non-isomorphic, over the reals, real curves and finally if X has genus g then the set of fixed points $\mathrm{Fix}(\varrho)$ of ϱ consists of k disjoint Jordan curves called ovals which varies between 0 and $g + 1$ and is homeomorphic to a smooth projective real model of the corresponding curve.

Richard D. Wade

University of Oxford

My research involves the study of automorphisms of free groups, and more generally, automorphisms of right-angled Artin groups. In particuar, I've been looking at homomorphisms from $\mathrm{Out}(F_n)$ and $\mathrm{Out}(A_\Gamma)$ to lattices in higher-rank Lie groups, such as $\mathrm{SL}_k(\mathbb{Z})$ (when $k \geq 3$).

In a joint paper with Martin Bridson [1], we prove the following:

Theorem 1. Let Λ be an irreducible lattice in a real, higher-rank, semisimple Lie group with no compact factors. Then every homomorphism $\Lambda \to \mathrm{Out}(F_n)$ has finite image.

This generalises an earlier theorem of Bridson–Farb, who proved the above result for uniform lattices. One reason why some thought the above result might be true comes from Margulis' superrigidity theorem, which implies the following:

Theorem 2 ((A consequence of) Margulis' Superrigidity Theorem). Let G be a real semisimple Lie group with finite centre, no compact factors, and $\mathrm{Rank}_{\mathbb{R}} G \geq 2$. Let Λ be an irreducible lattice in G. If $\mathrm{Rank}_{\mathbb{R}} G \geq n$, then every homomorphism $f : \Lambda \to \mathrm{GL}_n(\mathbb{Z})$ has finite image.

As $\mathrm{Out}(A_\Gamma)$ lies somewhere between $\mathrm{Out}(F_n)$ and $\mathrm{GL}_n(\mathbb{Z})$, I wanted to see if the above two results could extend to $\mathrm{Out}(A_\Gamma)$. To do this you need some kind of notion of 'rank' for $\mathrm{Out}(A_\Gamma)$.

We defined the *SL-dimension* (I dislike this phrase now, as I think the word rank fits better!) of $\mathrm{Out}(A_\Gamma)$ to be the largest 'obvious' copy of $\mathrm{SL}_k(\mathbb{Z})$ in $\mathrm{Out}(A_\Gamma)$. This is the size of a maximal subgraph $\Gamma' \subset \Gamma$ such that $A_{\Gamma'} \cong \mathbb{Z}^k$ *and* every automorphism of $A_{\Gamma'}$ extends to an automorphism of A_Γ. We write $k = d_{\mathrm{SL}}(\mathrm{Out}(\Gamma))$. We show the following:

Theorem 3 ([2]). Let G be a real semisimple Lie group with finite centre, no compact factors, and $\mathrm{Rank}_{\mathbb{R}} G \geq 2$. Let Λ be an irreducible lattice in G. If $\mathrm{Rank}_{\mathbb{R}} G \geq d_{\mathrm{SL}}(\mathrm{Out}(A_\Gamma))$, then every homomorphism $f : \Lambda \to \mathrm{Out}(A_\Gamma)$ has finite image.

In particular, this implies:

Corollary. If $k \geq 3$ then $\mathrm{Out}(A_\Gamma)$ has a subgroup isomorphic to $\mathrm{SL}_k(\mathbb{Z})$ if and only if $k \leq d_{\mathrm{SL}}(\mathrm{Out}(A_\Gamma))$.

I'm not sure of any applications of the last couple of results, but I hope that the techniques used in the proof (which, admittedly, I've not talked about!) may be useful elsewhere.

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[2] Richard D. Wade, Johnson homomorphisms and actions of higher-rank lattices on right-angled Artin groups, *arXiv:1101.2797*, 2011.

Pei Wang

Rutgers University

My first time to really come into contact with geometric group theory was about two years ago by studying Peter Scott and Terry Wall's note on "Topological methods in group theory" in professor Feighn's reading course. It was really amazing to me that so much topology could be applied to solve group problems.

After then, I took professor Feighn as my advisor and started reading basic papers about R-trees and group actions on trees. Here are some of them:

"The accessibility of finitely presented groups" by Dunwoody

"Bounding the complexity of simplicial group actions" by Bestvina and Feighn

"R-trees in topology, geometry and group theory" by Bestvina

"Stable actions of groups on real trees" by Bestvina and Feighn

"Group actions on R-trees"; by Culler and Morgan

And since January 2011, I've been meeting with a small group of students focused on geometric group theory. The schedule for that group can be viewed here: http://andromeda.rutgers.edu/ gm-fein/grouptheoryseminar.html

At that period I read some papers about limit groups, outer space and $\mathrm{Out}(F_n)$. Here are some of them:

"Notes on Sela's Work: Limit groups and Makanin-Razborov Diagrams" by Bestvina and Feighn

"Train tracks and automorphisms of free groups" by Bestvina and Handel

"A Bers-like proof of the existence of train tracks for free group automophisms" by Bestvina

"Hyperbolicity of the complex of free factors" by Bestvina and Feighn

Recently I'm planning to work on some problems on my own. Slowly I'm trying to read Sela's third paper of a series "Diophantine geometry over groups III: rigid and solid solutions". See if I could get some good ideas from there.

This summer I went to a workshop and conference "the geometric group theory and logic: the elementary theory of groups" in Chicago. It was a really nice experience to meet and talk to so many graduate students and mathematicians sharing common interests in geometric group theory. So I'm really looking forward to learn more in the young geometric group theory meeting in Poland this winter.

Sarah Wauters

Katholieke Universiteit Leuven Campus Kortrijk

In his PhD thesis [2], M. Hartl constructs an exact sequence of a functor (defined in e.g [1, Chapter IV], involving the category of central group extension of torsion-free n-step nilpotent groups. As a result, for certain finitely generated nilpotent groups G, Hartl obtains a group extension with $\mathrm{Aut}(G)$ as extension group, and well-understood kernel and quotient. Furthermore, he explicitly gives the associated cocycle by means of a Bockstein morphism, thereby completely determining $\mathrm{Aut}(G)$.

As an interesting application, we can compute a presentation for the automorphism group of the free 2-step nilpotent group on k generators, extending a result in [3]. Using the same techniques, we will try to find presentations for the automorphism groups of both finitely generated free 3-step nilpotent groups and arbitrary finitely generated 2-step nilpotent groups, such as the Heisenberg groups. However, this involves elaborate computations. By refining the methods, e.g. by extending the construction to an exact sequence of a functor covering also the non-central extensions, we hope to speed up the computations and extend the class of groups for which we can apply these methods.

[1] Baues, H. J., *Algebraic homotopy*, volume 15 of *Cambridge Studies in Advanced Mathematics*, Cambridge University Press, Cambridge, 1989.

[2] Hartl, M. *Abelsche Modelle Nilpotenter Gruppen.* PhD Thesis, RFW-Univ. Bonn, 1991.

[3] Lin, W. *Presentations of the automorphism groups of free nilpotent groups.* Comm. Algebra, 1996, 24 11, 3402–3411.

Christian Jens Weigel

Universität Gießen

Currently I am working on my PhD-thesis in Gießen, Germany, under the supervision of Prof. Bernhard Mühlherr.

The first topic we worked on in the context of my PhD-thesis was the isomorphism problem for Coxeter groups, based on the research of Caprace, Mühlherr, Weidmann and others. Solving the isomorphism problem means answering the question whether two Coxeter groups given by two different Coxeter matrices are isomorphic or not. Regarding this Mühlherr made a conjecture, which gives implicitly the existence of an algorithm to decide this question based on the two Coxeter diagrams.

The conjecture was already known to be true for several classes of Coxeter groups, which was shown with different methods of approach, among which are geometric arguments as well as the application of Bass-Serre-theory. We tried to show it for a broader class of Coxeter groups, and succeeded in doing so for Coxeter groups with a diagram not containing certain rank 3 subdiagrams. To do this we employed geometric methods on the Cayley graph of the respective Coxeter groups (to appear in Innovations in Incidence Geometry).

Following this work we started working on Weyl groupoids based on the research of Cuntz and Heckenberger, who classified spherical Weyl groupoids and showed that spherical Weyl groupoids correspond to crystallographic hyperplane arrangements, which can be interpreted as simplicial decompositions of the sphere.

Crystallographic hyperplane arrangements occur naturally when considering the geometric realization of a Weyl group W with fundamental reflections S. More precisely, the set $Fix(s)$ for any reflection $s \in S^W$ is a hyperplane and the fixed point sets of all other reflections endow a simplicial structure on $Fix(s)$. It can be seen, by considering simple examples like the Weyl group A_3, that the resulting tessellation is not so symmetric as to provide a group acting transitively on its chambers, which are the connected components of $Fix(s) \setminus \bigcup_{s \neq t \in S^W} Fix(t)$. However, the resulting chambers are open simplicial cones, and the sets $Fix(t)$, $s \neq t \in S^W$ induce a hyperplane arrangement, which is in this case also crystallographic. This has already been done by Cuntz and Heckenberger in the spherical case.

The term "crystallographic" here refers to a property of the associated root system of a hyperplane arrangement, which is a subset of the dual space. In the case of an arrangement coming from a spherical Weyl group this root system coincides with the root system of the Weyl group, and the crystallographic property also coincides with the term known for Coxeter groups in general.

We are currently trying to generalize this notion to simplicial decompositions of the Euclidean space, which give rise to hyperplane arrangements in a higher dimensional embedding Euclidean space. This way it is possible to define a root system for the hyperplane arrangement and to establish the notion of crystallographic root systems for these objects.

Associated to such a crystallographic hyperplane arrangement is again a Weyl groupoid, which arises by considering the maps which exchange adjacent chambers (maximal simplices), and which allows a more algebraic approach.

This generalizes the affine Weyl groups, which arise as special cases and which also yield tessellations of the Euclidean space. It is our goal to see whether affine Weyl groupoids can always be seen as sub-arrangements of an affine Weyl group or if there exist exceptions, and, if so, to classify them. We would like to achieve this at least for the rank two case, yet a complete classification might be achievable as well.

Jacek Wieszaczewski

University of Wrocław

My current research is focused on boundaries of hyperbolic groups. I am looking for new examples of topological spaces that can be such boundaries.

Let us take an orientable surface of genus two equipped with a metric of constant negative curvature. We can choose a (totally geodesic) circle C on it so that there is one hole on either side of that circle. Cutting along C gives us two identical halves of this space. Each of them has a connected and totally geodesic boundary. We can take three such halves now and glue all of them together identifying their boundaries. What we get is a manifold almost everywhere (except the circle where the gluing took place).

Many spaces H can be created this way, using varying initial manifolds (also of higher dimensions) and gluing in different ways. For each of them the universal cover is what I call ,,branched hyperbolic space". The boundary at infinity of this cover is also the boundary of the fundamental group of H (which is hyperbolic). The aim of my research is to classify those boundaries for as wide class of spaces H as possible.

Stefan Witzel

Universität Münster

My main interest is in geometric group theory. More specifically, the groups I have been studying are S-arithmetic lattices in semisimple Lie groups and groups acting on buildings. The properties I try to understand are topological and homological finiteness properties which generalize the properties of being finitely generated and being finitely related.

Past research: Finiteness properties of S-arithmetic lattices

A classifying space for a group Γ is a CW complex X whose fundamental group is Γ and whose universal cover is contractible. A group is said to be of type F if it admits a compact classifying space. This is a very strong condition satisfied for example by free groups, free abelian groups, and surface groups. One way to weaken it is to demand that Γ admit a classifying space of finite dimension, in which case the least such dimension is called the geometric dimension of Γ. Another way is to require that Γ admit a classifying space whose n-skeleton is compact. In that case Γ is said to be of type F_n. It is not hard to see that Γ is of type F_1 iff it is finitely generated and of type F_2 iff it is finitely presented. A group is of type F_∞ if it is of type F_n for all n.

For many groups the distinction between these properties is not relevant: they are either of type F_∞ (hyperbolic groups for example) or not even finitely generated. The first separating examples were [13] (finitely generated but not finitely presented) and [14] (finitely presented but not of type F_3). A large class of naturally occurring examples of groups that are of type F_{n-1} but not of type F_n for every n consists of S-arithmetic groups in positive characteristic.

To describe them let $\mathbf{G}$ be an almost simple algebraic group defined over a rational function field k, for example $k = \mathbf{F}_q(t)$. Let T be a maximal set of nonequivalent valuations on k and for each $\nu \in T$ let k_ν be the completion of k at with respect to ν. Let $S \subseteq T$ be a nonempty finite subset and let $\mathcal{O}_S = \{x \in k \mid \nu(x) \leq 1 \text{ for } \nu \in T \setminus S\}$. Let N be the sum over the local ranks $\sum_{\nu \in S} \mathrm{rk}_{k_\nu} \mathbf{G}$. Then

Theorem. *The group $\Gamma = \mathbf{G}(\mathcal{O}_S)$ is of type F_{N-1} but not of type F_N.*

For example $\mathrm{SL}_{n+1}(\mathbf{F}_q[t])$ is of type F_{n-1} but not of type F_n.

This theorem is proved in [3] in joint work with Kai-Uwe Bux and Ralf Köhl and answers a question that goes back at least to [7, p. 197]. Many partial cases were known before: Bux and Wortman have shown that Γ is not of type F_N ([8], see also [10]) and that the full statement holds if $\mathbf{G}$ has k-rank 1 [9]. For higher rank Abramenko [1] proved the statement in the case where $\mathbf{G}$ is a classical group (SL_{n+1}, Sp_{2n}, SO_{2n}, SO_{2n+1}) and $\mathcal{O}_S = \mathbb{F}_q[t]$ with q large compared to n.

Bux, Köhl and I generalized Abramenko's result to arbitrary $\mathbb{F}$-groups $\mathbf{G}$ without a bound on q [2]. In my PhD thesis [15] I proved the analogous statement for rings of Laurent polynomials $\mathcal{O}_S = \mathbb{F}_q[t, t^{-1}]$. The Rank Theorem was then again obtained in joint work.

Research project: Geometric aspects of reduction theory

Each of the the groups $\mathbf{G}(\mathcal{O}_S)$ above is a lattice in $G = \prod_{\nu \in S} \mathbf{G}(k_\nu)$ (embedded diagonally). The same holds if k is a number field and S contains all Archimedean valuations of T.

The theory leading to these statements is called reduction theory. In the number field case it was developed by Borel and Harish-Chandra [4] and, using different methods, by Godement [11]. Harder [12] then developed his reduction theory in the function field setting.

In the article [3] Harder's reduction theory has been formulated in more geometric terms. Classical reduction theory for arithmetic groups over function fields is also very geometric. This naturally leads to the question as to which extent there is a geometric formulation of reduction theory, uniform for number fields and function fields. Once such a formulation is found, one may also ask whether there is a geometric way to prove it.

As a first step in this direction I am currently reflecting possibilities to give a uniform geometric characterization of when an S-arithmetic subgroup is a *uniform* lattice (so that G/Γ is compact).

Research project: Bredon homological finiteness conditions

If X is a classifying space for Γ with finite n-skeleton, the universal cover $\tilde{X}$ admits a free Γ-action which gives rise to a free resolution $F_* \to \mathbb{Z} \to 0$ of the trivial $\mathbb{Z}\Gamma$-module $\mathbb{Z}$ with F_i finitely generated for $i \leq n$. The existence of such a resolution means, by definition, that Γ is of type FP_n. Being of type FP_n is therefore closely related though slightly weaker than being of type F_n.

In this setting one may replace X by a classifying space for proper actions or more generally by a classifying space for actions with stabilizers in a family $\mathfrak{F}$ of subgroups of Γ (these feature for example in the Baum–Connes- and the Farrell–Jones conjecture). On the homological side one is then lead to study modules over the orbit category $\mathcal{O}_{\mathfrak{F}}\Gamma$ as introduced by Bredon [5].

The most important tool for establishing finiteness properties in the classical situation is a criterion due to Brown [6]. Together with Martin Fluch I am currently working toward an analogue of this criterion in the setting of Bredon-homology.

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Wenyuan Yang

Université Paris-Sud 11

I have defended my thesis this May in Université Lille 1 under the supervision of Leonid Potyagailo. My thesis focuses on the study of geometric and algebraic properties of relatively hyperbolic groups. In particular, I investigated peripheral structures of a relatively hyperbolic group. Usually a group may be hyperbolic relative to different collections of subgroups. Such a collection of subgroups is referred to as a peripheral structure. In my thesis, the notion of parabolically extended structures is introduced to capture the relation between different structures endowed on a relatively hyperbolic group. We give a characterization of parabolically extended structures as a generalization of Osin's results.

One corollary of our results roughly says that if the group under consideration acts geometrically finitely on its Floyd boundary, then every peripheral structure is parabolically extended with respect to the canonical one for the geometrical finite action on Floyd boundary. However, there indeed exist examples of relatively hyperbolic groups which do not act geometrically finitely on their Floyd boundary. This follows from Behrstock-Drutu-Mosher's work on Dunwoody's inaccessible group.

I am now a postdoc in Orsay. I am interested to study the boundary of relatively hyperbolic groups via analytic approach. Cubulating groups and some related topics are going to occupy my future research.

Gašper Zadnik

University of Ljubljana

My research is in the area of CAT(0) spaces. My degree (2010) is the review of geometry of CAT(0) spaces which admit some proper cocompact isometric action and algebraic properties of groups which act so. I still follow the spirit of the degree. At the begining of my PhD-study I studied Riemannian symmetric spaces $SL(n,\mathbb{R})/O(n)$ from the viewpoint of CAT(0) geometers and answered the question about the classification of nonidentity component of isometries of $SL(3,\mathbb{R})/O(3)$ posed by K. Fujiwara in his talk, see [1], Problem 4.1. Article is in preparation.

Now I am concentrated on so called *flat closing problem*, which asks whether a group G acting properly and cocompactly by isometries on a CAT(0) space X contains $\mathbb{Z}\times\mathbb{Z}$, if X contains isometrically embeded copy of $\mathbb{R}\times\mathbb{R}$ (see [2] for example). I am especially interested in example where X is two-dimensional CAT(0) cube complex. There are some indications that the answer should be negative already in this case, although there is no known counterexample in general.

This is an important problem because it is also related to the question about obstructions for CAT(0) group to be hyperbolic (in the sense of M. Gromov). It is known that a group containing $\mathbb{Z} \times \mathbb{Z}$ is not hyperbolic. At the other hand, proper cocompact CAT(0) space is hyperbolic iff it does not contain isometrically embeded $\mathbb{R} \times \mathbb{R}$. But CAT(0) space and group acting on it properly cocompactly by isometries are quasiisometric, hence the space is hyperbolic iff the group is.

There are also many greater goals, let me mention two examples:

- duality condition for CAT(0) groups (does for every geodesic line c in CAT(0) space X there exist a sequence of elements $(g_n)_{n\in\mathbb{N}}$ from a group G acting properly cocompactly by isometries on X such that $g_n x$ tends to one and $g_n^{-1}x$ to another endpoint of c in ∂X for some (any) $x \in X$);
- are π and ∞ the only posible diameters of Tits boundary of CAT(0) spaces, admitting some proper cocompact isometric action (or, more general, action with full limit set at the boundary)?

I hope I will get some ideas to attack them during solving some easier problems.

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Matthew Zaremsky

Universität Bielefeld

Overview: My research deals with a variety of topics in group theory and geometric group theory, with a particular emphasis on the theory of buildings and on homological stability of certain families of groups. My thesis work, supervised by Peter Abramenko, was concerned with analyzing transitivity properties of groups acting on buildings. More recently I have been collaborating with Kai-Uwe Bux, analyzing homological properties of the group $\Sigma Aut(F_n)$ of symmetric automorphisms of the free group, using discrete Morse theory and some constructions similar to Culler-Vogtmann Outer space. I am currently working on a Morse-theoretic proof that $\Sigma Aut(F_n)$ has trivial rational homology, which is a stark contrast to the *purely* symmetric case where the groups are not even homologically stable.

Symmetric automorphisms: An automorphism of F_n is called *symmetric* if it sends the generators and their inverses to conjugates of each other. Let $\Sigma Aut(F_n)$ be the group of symmetric automorphisms of F_n. There is a natural simplicial complex C_n on which $\Sigma Aut(F_n)$ acts, which is analogous to the spine K_n of Culler-Vogtmann Auter space. In Auter space, the spine is built out of "marked" basepointed graphs with fundamental group F_n, and the space C_n is essentially the subspace consisting only of *cactus graphs*, i.e., graphs in which each edge is contained in a unique cycle. So far, by analyzing C_n using discrete Morse theory, I have proved that the groups $\Sigma Aut(F_n)$ have stable rational homology, and am now working on a proof that the rational homology is even trivial.

Future work: A natural next step is to analyze the group $\Sigma Aut_m(F_n)$ of *partially symmetric* automorphisms of F_n, where only the first m generators must map to conjugates of each other. If m is 0 or n, we recover $Aut(F_n)$ and $\Sigma Aut(F_n)$, where we have homological stability, so it would be interesting to see if the $\Sigma Aut_m(F_n)$ are homologically stable.

Buildings: One definition of a building Δ is as a simplicial complex made up of a choice of *apartment system* $\mathcal{A}$ with certain properties, for instance a large degree of symmetry. The need to keep track of a choice of apartment system is alleviated by a more combinatorial, though equivalent definition of a building. Under the combinatorial approach, all the structure of the building is encoded just in its set $\mathcal{C}$ of chambers (maximal simplices) and its *Weyl distance function* $\delta : \mathcal{C} \times \mathcal{C} \to W$, a function assigning to each pair of chambers an element in the Weyl group W of the building.

Let G be a group acting on a building Δ. The action is called *strongly transitive* if G acts transitively on the set of pairs (C, Σ) where C is a chamber in the apartment Σ. The action is called *Weyl transitive* if for each $w \in W$, G acts transitively on the set of pairs (C, D) with $\delta(C, D) = w$. We also introduce a new notion, that of *weak transitivity.* A group G acts weakly transitively on a building Δ if there exists an apartment Σ such that $Stab_G(\Sigma)$ acts transitively on the chambers of Σ. It turns out that a Weyl transitive action is strongly transitive if and only if it is weakly transitive. My thesis work was concerned with finding examples of Weyl transitive actions that are not weakly transitive, and determining their "degree of failure" to be weakly transitive.

Future work: Weyl transitivity is a relatively recent notion, with some powerful applications toward limiting the subgroup structure of a group. One class of groups with mysterious subgroup structure is anisotropic algebraic groups $G(k)$, though it turns out over global fields k these do admit Weyl transitive actions. It would interesting to see whether we could say something new about the subgroup structure of anisotropic groups using Weyl transitivity.

Paweł Zawiślak

University of Warsaw

My current research is focused on the properties of trees of manifolds with boundaries and boundaried trees of manifolds.

Trees of manifolds are inverse limits of certain inverse systems of compact manifolds. The most familiar examples of such spaces are the Pontryagin spheres, orientable and nonorientable, which are trees of 2-tori and of projective planes, respectively.

First such a space was constructed by W. Jakobsche [5] as a potential counterexample to the Bing-Borsuk conjecture. Earlier similarly constructed spaces were considered in a different context by L.S. Pontryagin [9] and R.F. Williams [12]. Then F.D. Ancel and L.C. Siebenmann [1] noticed that the tree of some homological 3-spheres can be identified with a compactification of the Davis contractible 4-manifold which covers a closed 4-manifold [3].

Properties of trees of orientable manifolds were investigated later by W. Jakobsche. In [6] he showed m-homogeneity of these spaces (for every natural m). Moreover, he showed that a tree of orientable homology spheres is a cohomology manifold, the space which can be often identified with the fixed-point set of a topological action on a manifold or a cohomology manifold. He also showed that such homogeneous cohomology manifolds appear as compactifications of contractible 4-manifolds, or orbit spaces of actions of 0-dimensional infinite compact groups. Some results of W. Jakobsche were extended to the nonorientable case by P.R. Stallings [11].

In the original Jakobsche definition a tree of manifolds there is the inverse limit $\varprojlim (L_n, \alpha_n)$ of a certain inverse system of orientable manifolds L_n and maps $\alpha_n : L_n \to L_{n-1}$ between them [6]. This inverse system has the property that every element L_{n+1} is, up to a homeomorphism, a connected sum of its predecessor L_n and a finite number of closed orientable manifolds $L_{n,1}, \ldots, L_{n,k_n}$ which belong to a countable family $\mathcal{L}$. Jakobsche showed that if the inverse system (L_n, α_n) satisfies some natural assumptions, then the inverse limit $\varprojlim (L_n, \alpha_n)$ depends only on the family $\mathcal{L}$. We denote this inverse limit by $\mathcal{X}(\mathcal{L})$.

Later P.R. Stallings generalized this definition to the case of nonorientable PL-manifolds [11]. A natural question about generalisations of the above definitions appears.

The first natural generalisation is the notion of boundaried trees of manifolds. These spaces are inverse limits $\varprojlim (L_n, \alpha_n)$ of certain inverse systems of compact manifolds L_n with boundaries and maps $\alpha_n : L_n \to L_{n-1}$ between them. Every element L_{n+1} of such a system arises from its predecessor by performing finitely many operations of one of the following types: a connected sum with a compact manifold (with or without boundary) belonging to a countable family $\mathcal{L}^0$, or a boundaried sum with a compact manifold with boundary belonging to a countable family $\mathcal{L}^1$.

At the moment the following properties of boundaried trees of manifolds are known:

- I showed [14] that if the inverse system (L_n, α_n) satisfies some natural assumption generalizing Jakobsche's assumptions, then the inverse limit $\varprojlim (L_n, \alpha_n)$ depends only on the families $\mathcal{L}^0$ and $\mathcal{L}^1$ (we denote this inverse limit by $\mathcal{X}(\mathcal{L}^0, \mathcal{L}^1)$),
- J. Świątkowski showed (private conversation) that the topological dimension of the space $\mathcal{X}(\mathcal{L}^0, \mathcal{L}^1)$ is equal to the dimension of L_1 minus 1 (provided that the family $\mathcal{L}^1$ is not empty).

The following properties of boundaried trees of manifolds are conjectured:

- no local cut-points,
- the group of homeomorphisms of $\mathcal{X}(\mathcal{L}^0, \mathcal{L}^1)$ acts transitively on points coming from the interior of L_n and from the connected components of the boundary of L_n.

D. Osajda showed in [8] that the Gromov boundary of a 7-systolic simplicial complex X (see [7] for the definition) is homeomorphic to the inverse limit $\varprojlim (S_n, \Pi_n)$ of a system of combinatorial spheres S_n centered at a fixed vertex $v \in X$ and natural projections $\Pi_n : S_n \to S_{n-1}$ between them. In the case when the complex X is a normal pseudomanifold of dimension 3, it turns out that these spheres are surfaces and the Gromov boundary $\partial_G X$ is a tree of surfaces (orientable or not, according to the orientability or nonorientability of X) [13].

H. Fischer showed in [4] that trees of manifolds appear naturally as boundaries of right-angled Coxeter groups with manifold nerves. More precisely, if the nerve of a right-angled Coxeter group is PL-homeomorphic to the triangulation of a closed, oriented manifold M, then the $CAT(0)$ boundary of this group is homeomorphic to the tree $\mathcal{X}(\{M, \bar{M}\})$, where $\bar{M}$ is M with the opposite orientation.

In [10] P. Przytycki and J. Świątkowski showed that every closed 3-manifold M admits a "flag-no-square" triangulation. It follows that there exist hyperbolic right-angled Coxeter groups whose nerves are PL-homeomorphic to M, and thus whose Gromov boundaries are homeomorphic to $\mathcal{X}(\{M, \bar{M}\})$.

J. Świątkowski showed (private conversation) that if X is a $CAT(0)$ complete, piecewise Euclidean or piecewise hyperbolic, oriented simplicial pseudomanifold of dimension $n+1$, whose singularity locus has dimension 0 (i.e. links of points are n-spheres, except the vertices, where triangulations of closed n-manifolds from a family $\mathcal{M}$ can appear), then the $CAT(0)$-boundary of X is homeomorphic to $\mathcal{X}(\mathcal{M})$. Such pseudomanifolds exist due to the Charney-Davis hyperbolisation procedure [2].

Thus the natural question appears if boundaried trees of manifolds occur as boundaries of nonpositively curved spaces and groups.

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Joanna Zubik

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The main object of my recent research is AHA property (asymptotic hereditary asphericity). The concept of AHA was introduced by Januszkiewicz and Świątkowski (see [2]). It was inspired by the property of hereditary asphericity for topological spaces (Davermann [1]). AHA was used in [2] to show that there exist hyperbolic groups of any cohomological dimension, which contain no subgroups isomorphic to fundamental groups of closed nonpositively curved Riemannian manifolds of dimension greater than 2.

We say that a metric space X is AHA, if for every $r > 0$ there is $R > r$ such that for every $A \subset X$, any simplicial map $f : S \to Rips_r(A)$ (S is any triangulation of the sphere S^k, $k \geqslant 2$) has a simplicial extension $F : B \to Rips_R(A)$ for some triangulation B of the ball B^{k+1} such that

$\partial B = S$. AHA is a quasi-isometry invariant. Thus it makes sense to speak about AHA property for finitely generated groups.

Not much is known about AHA property. I study behaviour of AHA property under free amalgamated product and HNN extensions. Even an attempt to prove the natural conjecture that a free product of two AHA groups is again AHA presents difficulties. The reason is that we don't have asymptotic analogues of many concepts of algebraic topology.

I introduced two slightly weaker properties than AHA: $AHA(-)$ (where we require that appropriate maps defined on spheres have extensions to maps on simply connected manifolds) and $AHA(k)$ (which concerns only maps defined on spheres of dimension k). Both of them are still sufficient for the purposes for which AHA was applied for initially in [2].

I have shown that a free product of two $AHA(-)$ (respectively $AHA(k)$) groups with amalgamation along a finite subgroup is again $AHA(-)$ (respectively $AHA(k)$). I have shown an analogous fact for HNN extensions. Now I study wider classes of subgroups.

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