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Recursive methods of forecasting intermediate demand

Knowledge of intermediate demand is necessary in order to apply the methods of actualizations of input-output matrices performed by mean of proportional or biproportional methods. The most popular of biproportional methods is so called RAS method.

The main goal of the paper is a comparison of the traditional (direct) and, as developed by the author, recursive methods for the prediction of intermediate demand.

The RAS procedure addresses the following problem: Given $n \times n$ matrix $\mathbf{A}(0)$ and given n-element vectors for the more recent year (i.e. year 1), $\mathbf{z}(1)$, $\mathbf{u}(1)$, and $\mathbf{v}(1)$, estimate $\mathbf{A}(1)$. We denote the estimate which follows from this procedure $\mathbf{A}(1)$. If we are dealing with a 20-sector economy, we are trying to estimate 400 coefficients. These elements should be estimated under the assumption that 60 pieces of information are known. The latter are:

a) the row sums of the unknown transactions matrix $\mathbf{X}(1)$, namely $\mathbf{u}(1)$;
b) the columns sums of the same matrix, namely $\mathbf{v}(1)$;
c) the $\mathbf{z}(1)$, which are essential primarily to convert an estimate of an $x_{ij}(1)$ into an estimate of a technical coefficient $a_{ij}(1)$, since $a_{ij}(1) = x_{ij}(1)/Z_j(1)$.

According to the RAS procedure we have

$$a_{ij}(1) = r_i s_j a_{ij}(0) \quad (i = 1, \ldots, n;\ j = 1, \ldots, n)$$

where nonnegative numbers $a_{ij}(0)$ and $a_{ij}(1)$ are the values of the (i, j)-th input coefficient at times 0 and 1. We call the matrix $\mathbf{A}(1) = \{a_{ij}(1)$ biproportional to $\mathbf{A}(0) = \{a_{ij}(0)$. Knowing $\mathbf{A}(0)$ one has to determine r_i's and s_j's such that

$$\sum_{j=1}^{n} r_i s_j a_{ij}(0) Z_j(1) = U_i(1) \quad (i = 1, \ldots, n)$$

$$\sum_{i=1}^{n} r_i s_j a_{ij}(0) Z_j(1) = V_j(1) \quad (j = 1, \ldots, n)$$

The model given by the above equation is called the "biproportional input-output model". The code-name RAS of this model comes from Stone and results from the notation $r_i a_{ij} s_j$, and when input-output applications are meant, researchers often term it "RAS".

Let us employ the following symbols for the year t:

$\mathbf{z}_t$: the output vector,

$\mathbf{x}_t$: the intermediate demand vector,
$\mathbf{f}_t$: the final demand vector,
$\mathbf{A}_t$: the input-output coefficient matrix for the year t.

The fundamental identity in the input-output theory can be expressed by the equation

$$\mathbf{z}_t = \mathbf{A}_t \mathbf{z}_t + f_t. \tag{1}$$

The observation (measurement) equation is given by

$$\mathbf{y}_t = \mathbf{x}_t + \mathbf{b}_t. \tag{2}$$

In this equation $\mathbf{y}_t$ is the observed value of intermediate demand $\mathbf{x}_t$ which is disturbed by a random term (white noise) $\mathbf{b}_t$. Therefore, we can assume that $E(\mathbf{y}_t) = \mathbf{x}_t$.

Taking into account the equation (1) we can get the optimum estimate of the observed (in the future) value of $\mathbf{y}_{t+1}$ by

$$\hat{\mathbf{y}}_{t+1} = \mathbf{y}_t + \left[(\mathbf{I} - \mathbf{A}_{t_0})^{-1} - \mathbf{I} \right] (\mathbf{f}_{t+1} - \mathbf{f}_t). \tag{3}$$

Defining

$$\mathbf{y}_t^a := \left[(\mathbf{I} - \mathbf{A}_{t_0})^{-1} - \mathbf{I} \right] (\mathbf{f}_{t+1} - \mathbf{f}_t) \tag{4}$$

we can rewrite formula (3) as

$$\hat{\mathbf{y}}_{t+1} = \mathbf{y}_{t+1}^a + \mathbf{y}_t - y_t^a. \tag{5}$$

It is easy to see from (5) that the intermediate demand in the year $t+1$ can be estimated as a sum of its approximated value computed by a direct method given by (4) and the difference between the observed value and the approximated value for the year t.

In order to improve (5) let us assume $\mathbf{w}$ be a column vector of value added coefficients. For a general input-output framework of the following equality holds

$$\mathbf{e}^T = \mathbf{e}^T \mathbf{A}_t + \mathbf{w}_t^T \tag{6}$$

where $\mathbf{e}$ is a summation vector containing ones.

Therefore, assuming that $\mathbf{w}$ is known for the period t, we can now carry out the updating of the elements of $\mathbf{A}$ coming from the base year as

$$\widetilde{\mathbf{A}}_t = \mathbf{A}_{t-1} \hat{\mathbf{s}}_t \tag{7}$$

where $s_{jj}^t = \dfrac{1 - w_j^t}{\sum_i a_{ij}^{t-1}}$.

Incorporating the updating procedure directly into (4) we finally obtain

$$\mathbf{y}_t^a := \left[\left(\mathbf{I} - \mathbf{A}_{t_0} \prod_{k=t_0+1}^{t+1} \hat{\mathbf{s}}_k \right)^{-1} - \mathbf{I} \right] \mathbf{f}_t \tag{8}$$

and

$$\mathbf{y}^{a}_{t+1} := \left[\left(\mathbf{I} - \mathbf{A}_{t_0} \prod_{k=t_0+1}^{t+1} \hat{\mathbf{s}}_k \right)^{-1} - \mathbf{I} \right] \mathbf{f}_{t+1} \tag{9}$$

In the following the procedure described above based on (8) and (9) will be called the modified recursive procedure. It is easy to prove that modified recursive method is unbiased, i.e. forecasted intermediate demand and total actual demand are equal. The proof details are available from the author upon request.

To illustrate the performance of the alternative methods aimed at predicting sectoral intermediate demand, the sequence of final demand vectors recorded by the Central Statistical Office (CSO) in Poland annually over a six-year-period starting in 1995 is used. Since 2000 the CSO has ceased the separate publication of final demand according to NACE branches, though the total value of final demand is still published. The input-output balance that followed in 1995 formed the basis for calculating the direct technical coefficient matrix and the corresponding Leontief inverse. In addition to this set of data, in order to estimate intermediate demand by means of the modified recursive procedure (with updating the matrix $\mathbf{A}$), we have to know the vector of value added coefficients for the respective years. Under this study, taking the mainly methodological concern of this paper into account, these vectors are obtained directly from the available data. In a more general case, they might be estimated as well.

The calculations were carried out separately for two different aggregations (14×14 and 6×6).

At very beginning of our conclusions it should be said that Polish input-output tables over the considered period were influenced by transformation processes. This results particularly in changing the direct input coefficients, so that the assumption of the stability of the Leontief inverse which is used under the forecasting procedures does not appear still valid.

Our main goal was to compare the suggested recursive methods with traditional forecasting procedures, such as the direct method, which are based exclusively on an input-output matrix from the past. From this point of view the superiority of the modified recursive method seems to be well-supported by our results.