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Applications of Modified Wilson Systems

The classical Wilson bases were successfully applied to the range of problems from QAM-OFDM* coding [3], [9] to image processing. Along the recent progress to modify the classical formulae we explore potential applications of the modified Wilson systems in the image processing.

The Wilson orthonormal basis was constructed in 1991 by I. Daubechies, S. Jaffard and J.-L. Journé from elements of Gabor tight frame with redundancy 2 [6]. In particular, for the real-valued f the system composed of $(M_m f)_{m\in\mathbb{Z}}$ and

$$\left[2^{1/2}(M_{m/2}T_n f + (-1)^{m+n} M_{m/2}T_{-n} f)\right]_{n\geq 1, m\in\mathbb{Z}}$$

is an orthonormal basis in $L_2(R)$. The characterization of such atoms f is known by now [1]. Later on the case of higher even redundancies was discussed [4], [5], while the questions of other prime redundancies remained open ([8], pp. 169–172) for a longer time. The modifications by means of inner automorphisms or symplectic automorphisms proved effective [10], [11] and yet they result in the atoms no longer realvalued. Thence, the question of their effective application in image processing arises.

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* Quadrature Amplitude Modulation — Orthogonal Frequency Division Multiplexing