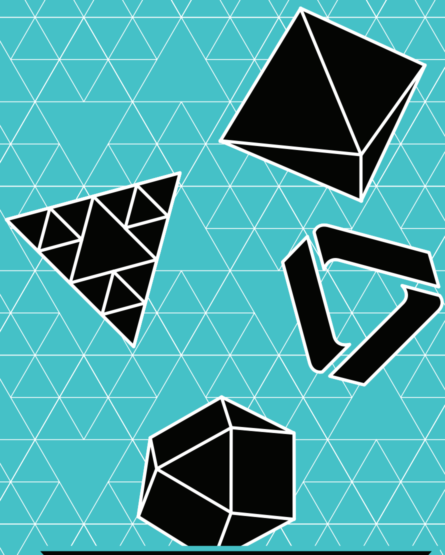


# Applied Topology

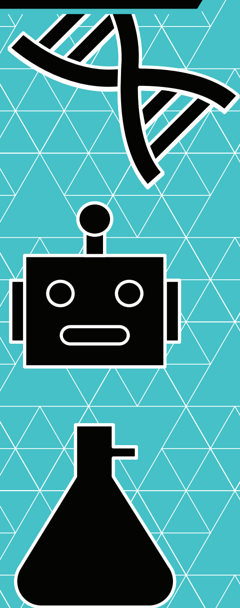
## 25.06.2017–01.07.2017

Będlewo | bcc.impan.pl/17AppTop

Scientific Committee: Alexander Dranishnikov, Michael Farber, Robert Ghrist, Yasuaki Hiraoka, Roy Meshulam, Marian Mrozek, János Pach, Günter Ziegler

Organizing Committee: Wacław Marzantowicz, Zbigniew Błaszczyk, Paweł Dłotko

"Data has shape, and shape has meaning." (G. Carlsson)



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| 16:45–18:15 | Room C<br>Roman Srzednicki, *On a problem of Whitney and the retract theorem of Ważewski* |
| 18:15 | Dinner |
| 20:00–21:30 | Room C<br>Ran Levi, *Topological analysis of neural networks* |

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| | Room A<br>Mark Grant, *Symmetrized topological complexity* |
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| | |
|---|---|
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| 10:15–11:15 | <u>Room C</u><br>Piotr Sułkowski, *Topological recursion, counting of chord diagrams, and classification of RNA complexes* |
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| 11:45–12:30 | <u>Room C</u><br>Lucile Vandembroucq, *Topological complexity of the Klein bottle*<br><br><u>Room A</u><br>Bogdan Batko, *Conley index for discrete multivalued dynamical systems and the dynamics reconstruction problem* |
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| 10:15–11:15 | Room C<br>Jarosław Buczyński, *Constructions of $k$-regular maps using finite local schemes* |
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| 11:15–11:45 | Coffee break |
| 11:45–12:30 | Room C<br>Miroslav Kramár, *Analysis of Time Scales in Complex Spatio-Temporal Systems*<br><br>Room A<br>Petar Pavešić, *Topological complexity of fibrations and covering maps* |
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| | |
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| 13:00 | Lunch |

# Introductory lectures



## Topological complexity: an algebraic topology model to the motion planning problem in robotics

Sunday
15:00–16:30

### Jesús González

Center for Research and Advanced Studies of the National Polytechnic Institute
*jechucho@gmail.com*

Early this century Michael Farber introduced the concept of Topological Complexity (TC), a model to study the continuity instabilities in the motion planning problem in robotics. Farber's model has captured much attention since then due to the rich algebraic topology properties encoded by the idea. After reviewing the basic constructions and properties of TC, I will describe some of the more recent TC developments and trends of research in this rich and fruitful area.

# Topological analysis of neural networks

Sunday 20:00–21:30

**Ran Levi**

University of Aberdeen
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A standard way to schematically represent networks in general, and neural network in particular, is as a graph. Depending on context, graphs representing networks can be directed or undirected, and in some cases carry labels or weights on their vertices and edges. With any graph one can associate a variety of combinatorial and topological objects, as well as certain algebraic invariants. The problem in using such ideas in neuroscience has been that obtaining connectivity data from a biological brain is very hard and expensive with existing techniques. Hence only very small connectivity patterns are understood, and extracting meaningful topological structures is not likely. Artificial neural networks, created by probabilistic rules can give much larger graphs, but they only provide a partial solution, as with existing knowledge their connectivity and functionality is not capable of reliably reproducing biological neuronal networks.

In this talk I will survey an on-going collaborative project where we apply topological techniques and ideas to the study of the brain. The project was motivated by the creation of a biologically accurate digital reconstruction of a small part of the cortex of a young rat by the Blue Brain Project. This digital model provided a way of extracting very accurate structural and functional information. Hence it allows us to extract large directed structural connectivity graphs that are a good approximation to the connectivity in a biological tissue. We have also developed ways of extracting time series of graphs corresponding to the reaction of the system to a variety of stimuli. These directed graphs can be studied using a combination of novel ideas and basic algebraic topology. I will describe some of our methods and the results obtained by applying them to the Blue Brain reconstruction.

# On a problem of Whitney and the retract theorem of Ważewski

Sunday
16:45–18:15

**Roman Srzednicki**

Jagiellonian University
*roman.srzednicki@im.uj.edu.pl*

In their classic book “What is Mathematics?” Richard Courant and Herbert Robbins presented a solution of Whitney’s problem of an inverted pendulum on a railway carriage. Since the appearance of the book in 1941, the solution was contested by several mathematicians, including distinguished and renowned ones. The controversy could be avoided if the opponents were more familiar with the retract theorem of Ważewski. As an applicaton of that theorem, the first rigorous argument based on the idea of Courant and Robbins was published by Ivan Polekhin in 2014.

# Plenary lectures

# Shadows of Cohen's Vanishing theorem

Thursday
15:00–16:00

**Pavle V. M. Blagojević**
Free University of Berlin
*blagojevic@math.fu-berlin.de*

The overwhelming material of the seminal Springer Lecture Notes 533 is signed by Cohen, Lada and May. Page 268 hides the Vanishing theorem of Frederick Cohen. Both the result and the proof spreading over seven pages look technical. The Vanishing theorem states that the Serre spectral sequence, for $p$ a prime, associated to the fibration

$$\mathrm{Conf}(\mathbb{R}^d, p) \longrightarrow \mathrm{E}\mathbb{Z}/p \times_{\mathbb{Z}/p} \mathrm{Conf}(\mathbb{R}^d, p) \longrightarrow \mathrm{B}\mathbb{Z}/p,$$

has only one non-zero differential:

$$\partial_{(d-1)(p-1)+1} \colon E^{i,(d-1)(p-1)}_{(d-1)(p-1)+1} \longrightarrow E^{(d-1)(p-1)+1+i,0}_{(d-1)(p-1)+1}.$$

Based on the theorem Cohen computes, in particular, the cohomology algebra of the unordered configuration space $\mathrm{Conf}(\mathbb{R}^d, p)/\mathfrak{S}_p$ with the coefficients in the field $\mathbb{F}_p$. More importantly Cohen defines Araki–Kudo–Dyer–Lashof homology operations for finite iterated loop spaces.

Over multiple decades the original content of the Vanishing theorem was forgotten, but the consequences were remembered and used over and over again. The problem of Nandakumar and Ramana-Rao about cutting of a convex polygon into convex pieces of equal area and equal perimeter brought the Vanishing theorem into the focus once again. The theorem provided multiple applications in areas like convex and combinatorial geometry, and stimulated further progress in the study of configuration spaces.

In this talk we present applications of the Cohen Vanishing theorem that range from measure partition problems, existence of $K_{s,s}$-free graphs, existence of complex skew embeddings to counting periodic trajectories of Finsler billiards.

(This lecture is based on the past and future joint work with Frederick Cohen, Michael Crabb, Michael Harrison, Roman Karasev, Wolfgang Lück, Pablo Soberón, Sergei Tabachnikov and Günter Ziegler.)

# The Geometry of Synchronization Problems and Learning Group Actions

Friday
10:15–11:15

**Jacek Brodzki**

University of Southampton
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We develop a geometric framework that characterizes the synchronization problem — the problem of consistently registering or aligning a collection of objects. We formulate a theory that characterizes the cohomological nature of synchronization based on the classical theory of fibre bundles. We first establish the correspondence between synchronization problems in a topological group $G$ over a connected graph $\Gamma$ and the moduli space of flat principal $G$-bundles over $\Gamma$, and develop a discrete analogue of a well-known theorem on classification of flat principal bundles with a fixed base and structural group. In particular, we show that prescribing an edge potential on a graph is equivalent to specifying an equivalence class of flat principal bundles, and we show that the existence of a solution to the synchronization problem is determined by the triviality of holonomy. We then develop a twisted cohomology theory for associated vector bundles arising from an edge potential, which is a discrete version of the twisted cohomology in differential geometry. This theory realizes the obstruction to synchronizability as a cohomology group of the twisted de Rham cochain complex. We then build a discrete twisted Hodge theory — a fibre bundle analog of the discrete Hodge theory on graphs — which geometrically realizes the graph connection Laplacian as a Hodge Laplacian of degree zero. Motivated by our geometric framework, we study the problem of learning group actions — partitioning a collection of objects based on the local synchronizability of pairwise correspondence relations. A dual interpretation is to learn finitely generated subgroups of an ambient transformation group from noisy observed group elements. A synchronization-based algorithm is also provided, and we demonstrate its efficacy using simulations and real data.

This talk is based on joint work with Sayan Mukherjee and Tingran Gao.

# Constructions of $k$-regular maps using finite local schemes

Thursday
10:15–11:15

**Jarosław Buczyński**

Mathematical Institute of Polish Academy of Sciences; University of Warsaw
*jabu@mimuw.edu.pl*

A continuous map $\mathbb{R}^m \to \mathbb{R}^N$ or $\mathbb{C}^m \to \mathbb{C}^N$ is called $k$-regular if the images of any $k$ distinct points are linearly independent. Given integers $m$ and $k$ a problem going back to Chebyshev and Borsuk is to determine the minimal value of $N$ for which such maps exist. The methods of algebraic topology provide lower bounds for $N$, however there are very few results on the existence of such maps for particular values $m$. During the talk, using methods of algebraic geometry, we will construct $k$-regular maps. We will relate the upper bounds on the minimal value of $N$ with the dimension of a Hilbert scheme. The computation of the dimension of this space is explicit for $k \leq 9$, and we provide explicit examples for $k$ at most 5. We will also provide upper bounds for arbitrary $m$ and $k$. The problem has its interpretation in terms of interpolation theory: for a topological space $X$ and a vector space $V$, a map $X \to V$ is $k$-regular if and only if the dual space $V^*$ embedded in space of continuous maps from $X$ to the base field $\mathbb{R}$ or $\mathbb{C}$ is $k$-interpolating, i.e. for any $k$ distinct points $x_1, \ldots, x_k$ of $X$ and any values $f_i$, there is a function in $V^*$, which takes values $f_i$ at $x_i$. Similarly, we can interpolate vector valued continuous functions, and analogous methods provide interesting results.

The talk is based mainly on:

## References

[1] J. Buczyński, T. Januszkiewicz, J. Jelisiejew, M. Michałek, *Constructions of k-regular maps using finite local schemes*, to appear in J. Eur. Math. Soc. (JEMS), arXiv:1511.05707.

[2] M. Michałek, C. Miller, *Examples of k-regular maps and interpolation spaces*, arXiv:1512.00609.

[3] Follow-up work with T. Januszkiewicz.

# Local to global principles for persistent homology

Monday
9:15–10:15

**Gunnar Carlsson**
Stanford University
*gunnar@ayasdi.com*

One could say that the goal of topology is to formulate “local to global principles”. Such principles have not been discussed a great deal within the applied topology/persistent homology world. I will discuss some results and questions in this direction, including excision and Künneth formulas.

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# What is persistence?

Friday
9:00–10:00

**Wojciech Chacholski**
KTH Royal Institute of Technology
*wojtek@math.kth.se*

In this talk I will describe what persistence means for us (TDA group at KTH in Stockholm), how to measure it, and how to attempt to make statistical conclusions using it. Our proposed solution is equally suitable for one and multi parameter situations. It can be used to illustrate why understanding correlations is a hard problem as we show that for correlations our persistence signatures are in general NP hard to calculate. This is in contrast with single measurement situations, where calculating the signatures requires basically a linear time. The TDA group at KTH in Stockholm consists of O. Gafvert, R. Ramanujam, H. Riihimaki, and W. Chacholski.

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# Estimating the Reach of a Manifold

Friday
15:00–16:00

**Frédéric Chazal**
Inria Saclay
*frederic.chazal@inria.fr*

Various problems in manifold estimation, and topological and geometric inference make use of the so-called reach (also known as the conditioning number or feature size) which is a measure of the regularity of the manifold.

In this talk, we will investigate into the problem of how to estimate the reach of a manifold $M$ from point clouds randomly sampled on $M$. We propose an estimator of the reach (in the framework where the tangent spaces of $M$ are known) and we obtain upper and lower bounds on the minimax rates for estimating the reach.

This is a joint work with E. Aamari, J. Kim, B. Michel, A. Rinaldo and L. Wasserman.

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## On topological complexity and LS-category

Tuesday
9:00–10:00

**Alexander Dranishnikov**

University of Florida
*dranish@math.ufl.edu*

The topological complexity $TC(X)$ of a configuration space $X$ is a numerical invariant that measures stability of navigation algorithms. By the definition of $TC(X)$ is the minimal number $k$ such that $X \times X$ can be covered by $k+1$ open sets each of which can be deformed to the diagonal $\Delta X$. The LS-category $cat(Y)$ of a space $Y$ is the minimal number $k$ such that $Y$ can be covered by $k+1$ open sets each of which can be deformed in $Y$ to a point. We give an answer to the question whether $TC(X) = cat(X \times X/\Delta X)$.

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## Topology of complexes arising in models for Distributed Computing

Monday
10:30–11:30

**Dmitry Feichtner-Kozlov**

University of Bremen
*dfk@math.uni-bremen.de*

We shall talk about various simplicial models for distributed computing. The main model consists of iterated chromatic subdivisions and corresponds to the well-used model in distributed computing. Using the Weak Symmetry Breaking as an example we shall translate questions of distributed computing into purely combinatorial-simplicial questions for the corresponding complex. We shall see how this approach leads to many interesting new and open questions in various fields of mathematics.

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## Topology meets neuroscience

Tuesday
15:00–16:00

**Kathryn Hess**
EPFL
*kathryn.hess@epfl.ch*

I will present an overview of applications of topology to neuroscience on a wide range of scales, from the level of neurons to the level of brain regions. In particular I will describe collaborations in progress with the Blue Brain Project on topological analysis of the structure and function of digitally reconstructed microconnectomes and on topological classification of neuron morphological types. I will then briefly sketch applications of topology to the analysis of brain imaging data.

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## Limit theorem for persistence diagrams and related topics

Monday
15:00–16:00

**Yasuaki Hiraoka**
Tohoku University
*hiraoka@tohoku.ac.jp*

In this talk, I will present a recent result about convergence of persistence diagrams on stationary point processes in $\mathbb{R}^N$. Several limit theorems such as strong laws of large numbers and central limit theorems for random cubical homology are also shown. If I have time, recent progress on higher dimensional generalization of Frieze zeta function theorem is also presented.

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## Concurrency Theory and Subspace Arrangements

Tuesday
10:15–11:15

**Roy Meshulam**
Technion – Israel Institute of Technology
*meshulam@math.technion.ac.il*

Concurrency theory in computer systems deals with properties of systems in which several computations are executing simultaneously and potentially interacting with each other. We will be concerned with Dijkstra's classical

PV-model of concurrent computation. In this model, an execution corresponds to a directed path (d-path) in a (time-flow directed) state space, and d-homotopic (preserving the directions) d-paths represent equivalent computations. In this paper we consider a special class of PV-models in which the access and release of every resource happen without time delay. We describe the homotopy type of such model in terms of a complement of a certain arrangement of subspaces in products of simplices. This approach facilitates a combinatorial computation of the Poincaré polynomial of the space of d-paths, and yields a number of applications including a simple proof of a result of Raussen and Ziemiański and a determination of the connectivity of space of d-paths in terms of combinatorial data of the PV-program.

Joint work with Martin Raussen.

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# A dual of Tarski's plank problem: Using a fixed point theorem

Wednesday
9:00–10:00

**János Pach**

Courant Institute; EPFL; Renyi Institute
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A slab (or plank) of width $w$ is a part of the $d$-dimensional space that lies between two parallel hyperplanes at distance $w$ from each other. According to the translative covering conjecture, any slabs $S_1, S_2, \dots$ whose total width is divergent have suitable translates that altogether cover $\mathbb{R}^d$. We show that this statement is true if the widths of the slabs, $w_1, w_2, \dots$, satisfy the slightly stronger condition $\limsup_{n\to\infty} \frac{w_1+w_2+\ldots+w_n}{\log(1/w_n)} > 0$. This can be regarded as a converse of Bang's theorem, better known as Tarski's plank problem.

We apply our results to a problem on simultaneous approximation of classes $F$ of real functions. We say that a sequence of positive numbers $x_1 \leq x_2 \leq \dots$ controls all elements of $F$ if there exist $y_1, y_2, \dots \in \mathbb{R}$ such that for every $f \in F$, there exists an index $i$ with $|p(x_i) - y_i| \leqslant 1$. We find necessary and sufficient conditions for a sequence to have this property for various function classes. Our proofs are besed on combinatorial and topological ideas.

Joint work with A. Kupavskii and G. Tardos.

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# Topological recursion, counting of chord diagrams, and classification of RNA complexes

Wednesday
10:15–11:15

**Piotr Sułkowski**

University of Warsaw; California Institute of Technology
*psulkows@fuw.edu.pl*

I will introduce the topological recursion, which is a universal formalism — originating in the realm of matrix models — which assigns an infinite family of symplectic invariants to a given algebraic curve. I will illustrate the power of this formalism by showing how it can be used to solve the problem of counting of chord diagrams and classification of RNA complexes.

# “Half-plenary” lectures

# Conley index for discrete multivalued dynamical systems and the dynamics reconstruction problem

Wednesday
11:45–12:30

**Bogdan Batko**

Jagiellonian University
*bogdan.batko@uj.edu.pl*

Motivated by the problem of reconstructing dynamics from samples we revisit the Conley index theory for discrete multivalued dynamical systems [T. Kaczynski, M. Mrozek, Topology Appl. 65 (1995), 83–96].

We introduce a new, less restrictive definition of the isolating neighborhood. It turns out that then the main tool for the construction of the index, i.e. the index pair, is no longer useful. In order to overcome this obstacle we use the concept of weak index pairs. We present the new construction of the index based on weak index pairs, and discuss properties of the index, namely Ważewski property, the additivity property, the homotopy (continuation) property and the commutativity property.

We also show that our approach can be useful in the reconstruction of the qualitative features of an unknown dynamical system on the basis of the available experimental data only.

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# Solving Equations and Optimization Problems with Uncertainty

Thursday
11:45–12:30

**Peter Franek**

Institute of Science and Technology Austria
*peter.franek@gmail.com*

We study the problem of detecting zeros of continuous functions that are known only up to an error bound, extending the earlier theoretical work with explicit algorithms and experiments with an implementation. Further, we show how to use the algorithm for approximating worst-case optima in optimization problems in which the feasible domain is defined by the zero set of a function $f\colon X \to \mathbb{R}^n$ which is only known approximately.

The algorithm first identifies a subdomain $A$ where the function $f$ is provably non-zero, a simplicial approximation $f'\colon A \to S^{n-1}$ of $f/|f|$, and then verifies non-extendability of $f'$ to $X$ to certify a zero. Deciding extendability is based on computing the cohomological obstructions and their persistence. We describe an explicit algorithm for the primary and secondary obstruction, two stages of a sequence of algorithms with increasing complexity. Using elements and techniques of persistent homology, we quantify the persistence of these obstructions and hence of the robustness of zero.

We provide experimental evidence that for random Gaussian fields, the *primary obstruction* — a much less computationally demanding test than the secondary obstruction — is typically sufficient for approximating robustness of zero.

Joint work with Marek Krčál and Huber Wagner.

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## Symmetrized topological complexity

Monday
12:00–12:45

**Mark Grant**

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There are (at least) two variants of Farber's topological complexity in the literature which impose an additional symmetry constraint on motion planners, namely that the motion from $B$ to $A$ should be the reverse of the motion from $A$ to $B$. These are the *symmetric topological complexity* $\mathrm{TC}^S(X)$ of Farber–Grant, and the *symmetrized topological complexity* $\mathrm{TC}^\Sigma(X)$ of Basabe–González–Rudyak–Tamaki. The latter has the advantage of being homotopy invariant. In this talk I will present upper and lower bounds for $\mathrm{TC}^\Sigma(X)$. The upper bound comes from equivariant obstruction theory, and the lower bounds from the cohomology of the symmetric square $SP^2(X)$. I'll also show that symmetrized topological complexity coincides with its "monoidal" version, where the motion from $A$ to $A$ is required to be constant. These results allow the calculation of the symmetrized topological complexity of spheres.

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## Analysis of Time Scales in Complex Spatio-Temporal Systems

Friday
11:45–12:30

**Miroslav Kramár**

Inria

*miroslav.kramar.1@gmail.com*

In this talk we will introduce the methods of topological data analysis. Namely, the persistence diagrams which are a relatively new topological tool for describing and quantifying complicated patterns in a simple but meaningful way. We will demonstrate this technique on patterns appearing indense granular media. This procedure allows us to transform experimental or numerical data, from experiment or simulation, into a point cloud in the space of persistence diagrams. There are a variety of metrics that can be imposed on the space of persistence diagrams. By choosing different metrics one can interrogate the pattern locally or globally, which provides deeper insight into the dynamics of the process of pattern formation. We will use these metrics to identify the important time scales at which behavior of the system changes. We will also discuss a physical interpretation of these time scales.

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## Topological methods for a faster materials discovery

Monday
12:00–12:45

**Vitaliy Kurlin**

University of Liverpool

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The traditional approach to a discovery of new materials is to generate a large database of hypothetical molecular structures and then run expensive simulations predicting various chemical properties. Only few structures with a low energy have a chance to be physically synthesized and to remain stable in practice. Topological Data Analysis can speed up a prediction of most promising materials by describing the overall shape of a molecular data cloud. The talk will report the state-of-the-art results for the recent database of hydrogen organic frameworks introduced in the paper from Nature 543 (2017), 657–664.

The work in progress is joint with many colleagues at the Materials Innovation Factory in the University of Liverpool, UK.

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# On the Topology of Steel

Thursday
11:45–12:30

**Frank Lutz**

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Polycrystalline materials, such as metals, are composed of crystal grains of varying size and shape. Typically, the occurring grain cells have the combinatorial types of 3-dimensional simple polytopes, and together they tile 3-dimensional space.

We will see that some of the occurring grain types are substantially more frequent than others — where the frequent types turn out to be "combinatorially round". Here, the classification of grain types gives us, as an application of combinatorial low-dimensional topology, a new starting point for a topological microstructure analysis of steel.

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# Stable Local Persistent Homology

Tuesday
11:45–12:30

**Washington Mio**

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Persistent homology has been widely used for probing and getting insights into the global organization of data across spatial scales. Localized forms of persistent homology enables us to uncover richer information about the shape of data. However, naive localization is prone to instability due, for example, to sampling and noise. I will discuss a formulation in which local homology across scales is viewed as a continuous path in the space of persistence diagrams, stable with respect to the Wasserstein distance. Time permitting, I also will illustrate how local homology can be used in shape analysis by applying the method to synthetic data and to quantitative trait loci analysis of tomato leaf and root morphology, where the key goal is to elucidate the genetic basis of morphological variation.

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## Topological complexity of fibrations and covering maps

Friday
11:45–12:30

**Petar Pavešić**

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Topological complexity of a map was initially introduced as a measure of the manipulation complexity of a mechanical device with a given forward kinematic map. The resulting concept is not homotopy invariant because kinematic maps that appear in robotics usually have singularities. On the other hand, the topological complexity of a fibration is homotopy invariant, so we may consider the approximation of the topological complexity of an arbitrary map by the topological complexity of the associated fibration. In our talk we will discuss the topological complexity of fibrations and their relation with the complexity of general maps. In particular, we will consider the important special case of covering maps and their complexity.

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## Topological complexity of the Klein bottle

Wednesdy
11:45–12:30

**Lucile Vandembroucq**

University of Minho
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Dranishnikov has recently established that Farber's topological complexity of the non-orientable surfaces of genus at least 4 is maximal. In this talk, we will determine the topological complexity of the Klein bottle and extend Dranishnikov's result to all the non-orientable surfaces of genus at least 2.

This is a work in collaboration with Daniel C. Cohen.

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# Topological methods in discrete geometry; new developments

Tuesday
11:45–12:30

## Rade T. Živaljević

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Some new applications of the *configurations space/test map scheme* can be found in Chapter 21 of the latest (third) edition of the Handbook of Discrete and Computational Geometry [2]. In this lecture we focus on some of the new developments which, due to the limitations of space, may have been included in the Chapter 21 only partially (or were not mentioned at all).

Among the promising new research directions is the 'algebraic topology of cooperative games'. Recall that in cooperative games the 'players' create coalitions in order to achieve some common goal (voting games, profit games, 'simple games' of von Neumann and Morgenstern, etc.).

One of the central new ideas is to study and compare configuration spaces (usually simplicial complexes) arising in cooperative game theory (threshold complexes, simple games) with complexes arising as obstructors for embedding (mapping) spaces into higher dimensional euclidean spaces without double (multiple) points (Kuratowski graphs, Tverberg–Van Kampen–Flores obstructions, $r$-unavoidable complexes (Gromov–Blagojević –Frick–Ziegler reduction), etc.)

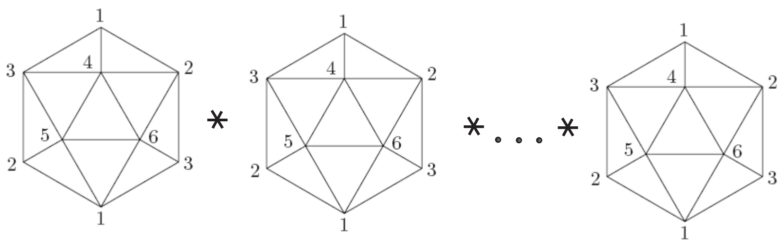

Figure 1: A join of $r$ hemi-icosahedra providing an example of an $r$-unavoidable complex which does not contain a threshold $r$-unavoidable complex.

As an illustration we outline a new proof (based on symmetrized deleted joins and discrete Morse theory) of the very general 'balanced Van Kampren–Flores theorem' ([2, Theorem 1.2]), which confirmed a conjecture of Blagojević, Frick, and Ziegler [1].

## References


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# Contributed lectures

# Topological data analysis on particulate materials

Friday
17:00–17:30

**Sergio Ardanza-Trevijano**

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We will show how persistent homology can be used to extract insight on certain particulate materials using only information about the position and size of the particles. We will study two types of particulate materials: Granular materials and colloids.

Granular materials are a paradigmatic example of a large set of particles with dissipative interactions, which can behave as a solid, liquid or gas. We focus on the evolution of packing of a vertical container under tapping and show how persistent homology can help to distinguish different static physical states with the same packing fraction.

Colloids are suspensions of particles in a liquid. We work with colloidal depositions where typically particles are much smaller than in granular materials and thus electrostatic interactions are more important while the deposit is being formed. In the spin coating technique there is a natural symmetry around the axis of rotation which impedes large compact domains (which will be square or triangular lattices). One way of improving this situation is breaking the symmetry by adding some obstacles. We show how Persistent homology can help to characterize the different structures arising.

# $A_\infty$-persistence

Monday
16:30–17:00

**Francisco Belchi-Guillamon**

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Persistent homology computes the (persistent) Betti numbers of a given filtration of topological spaces. $A_\infty$-persistence sharpens this tool by studying filtrations at the level of $A_\infty$-structures, which are algebraic constructions encoding information related to cup and Massey products.

In this talk I will explain the basics of the theory behind $A_\infty$-persistence and what we can gain by using $A_\infty$-persistence instead of ordinary persistence.

This is a joint work with Prof. Aniceto Murillo.

## Topological complexity and efficiency of motion planning algorithms

Thursday
16:30–17:00

**José Gabriel Carrasquel Vera**

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We introduce a variant of Farber's topological complexity, defined for smooth compact orientable Riemannian manifolds, which takes into account only motion planners with the lowest possible "average length" of the output paths. We prove that it never differs from topological complexity by more than 1, thus showing that the latter invariant addresses the problem of the existence of motion planners which are "efficient".

## Stochastic Dynamics on CW complexes

Monday
17:00–17:30

**Michael Catanzaro**

University of Florida
*catanzaro@ufl.edu*

In this talk, we will explore stochastic motion of subcomplexes inside CW complexes. This serves as a generalization of random walks on graphs, and a discretization of stochastic flows on smooth manifolds. We will define a notion of stochastic current, connect it to classical electric current, and show it satisfies quantization. Along the way, we will define the main combinatorial objects of study, namely spanning trees and spanning co-trees in higher dimensions, and relate these to the dynamics.

# Zariski topology on the spectrum of strongly prime submodules

Tuesday
17:30–18:00

**Ahmad Yousefian Darani**

University of Mohaghegh Ardabili
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Throughout this talk paper all rings are commutative with identity and all modules are unitary. Also we consider $R$ to be a ring and $M$ an $R$-module. For a submodule $N$ of $M$, let $(N :_R M)$ denote the set of all elements $r$ in $R$ such that $rM \subseteq N$. Let $N$ be a submodule of $M$ and let $x \in M$. We denote the ideal $(N + Rx :_R M)$ by $I_x^N$. Therefore, $I_x^N = \{r \in R \,|\, rM \subseteq N + Rx\}$.

A proper submodule $P$ of $M$ with $(P : M) = p$ is said to be *strongly prime* (or *strongly* $\mathfrak{p}$*-prime*) if $I_x^P y \subseteq P$, for $x, y \in M$, implies that either $x \in P$ or $y \in P$ (A. R. Naghipour, Comm. Algebra 37 (2009), 2193–2199). The collection of all strongly prime submodules of $M$ is called the *strongly spectrum* of $M$ and is denoted by $\text{S-Spec}_R(M)$. If $P$ is a strongly prime submodule of $M$ and $\mathfrak{p} = (P :_R M)$, we say that $P$ is a *strongly* $\mathfrak{p}$*-prime* submodule of $M$. The set of all strongly $\mathfrak{p}$-prime submodule of $M$ is denoted by $\text{S-Spec}_{\mathfrak{p}}(M)$. We will use $X$ to represent $\text{S-Spec}_R(M)$.

For any submodule $N$ of $M$, we define two different types of varieties:

$$\mathcal{V}(N) = \{P \in \text{S-Spec}_R(M) \,|\, (N : M) \subseteq (P : M)\},$$
$$\mathcal{V}^*(N) = \{P \in \text{S-Spec}_R(M) \,|\, N \subseteq P\}.$$

The Zariski topology on $X$ is the topology $\tau$ described by taking the set $\zeta = \{\mathcal{V}(N) \,|\, N \leqslant M\}$ as the set of closed sets of $X$. Moreover, the quasi-Zariski topology on $X$ is described as follows: put $\zeta^* = \{\mathcal{V}^*(N) \,|\, N \leqslant M\}$. Then there exists a topology $\tau^*$ on $X$ having $\zeta^*$ as the set of closed subsets of $X$ if and only if $\zeta^*$ is closed under the finite union. When this is the case $\tau^*$ is called the quasi-Zariski topology on $X$ and $M$ is called a top$^S$ top $R$-module.

In this talk we discuss some properties of Zariski topology and quasi-Zariski topology on $X$.

# Persistent Homology of Independence of Groups of Binary Variables

Friday
16:30–17:00

**Steven Ellis**

Columbia University

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"Concurrence topology" (S. Ellis, A. Klein, Homology, Homotopy, and Applications 16 (2014), 245–264) is a TDA method for binary data. The idea is to construct a filtration consisting of Dowker complexes then compute persistent homology. Persistent classes correspond to a form of negative statistical association among the variables. Suppose we have two groups of binary variables each displaying negative association, manifested in nontrivial concurrence homology in dimensions $p$ and in one group and $q$ in the other when the groups of variables are considered individually. Suppose, however, that the two groups of variables are statistically independent of each other. Now combine the two groups of variables and suppose the sample size is large. Then representative cycles, one from each group of variables, will combine to produce a cycle in dimension $p+q+1$. This is a chain level phenomenon, but we show it has a signature in homology. Looking for this signature can be used to study the dependence among groups of variables. We demonstrate it in simulated and real data.

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# An Approximate Nerve Theorem

Monday
17:00–17:30

**Dejan Govc**

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The Nerve Theorem [1], [2], [5] is an important result in algebraic topology. It states that the nerve $\mathcal{N}(\mathcal{U})$ of a good cover $\mathcal{U}$ of a paracompact space $X$ is homotopy equivalent to the space. There is also a homological version, where we assume that the cover is acyclic and conclude that $H_*\big(\mathcal{N}(\mathcal{U})\big) \cong H_*(X)$.

Recently, variations of this result have played an important role in applied topology, in particular in persistent homology. For instance, the Persistent Nerves Theorem of Sheehy [6], a consequence of the Persistent Nerve Lemma of Chazal and Oudot [3], states that given a good filtered cover of a filtered space, we have $\mathrm{Dgm}\big(\mathcal{N}(\mathcal{U})\big) = \mathrm{Dgm}(X)$, i.e. the persistence diagrams agree.

These theorems, however, do not account for the fact that in applications, measurements are often imprecise and verifying that a cover is good can only be done up to a certain precision. If the cover is good only in an approximate sense, we would still like to conclude that the nerve approximates the space well, so the need arises for an approximate version of the nerve theorem. In [4] we introduce the notion of an $\varepsilon$-acyclic cover, where each finite intersection of cover elements is either empty or has the persistent homology of a point. Using the Mayer–Vietoris spectral sequence, we show that such covers satisfy an Approximate Nerve Theorem, namely that $H_*\big(\mathcal{N}(\mathcal{U})\big) \overset{2(Q+1)\varepsilon}{\sim} H_*(X)$, that is the persistent homology of the nerve is $2(Q+1)\varepsilon$-interleaved with the persistent homology of the space, where $Q = \min\{\dim \mathcal{N}(\mathcal{U}), \dim X\}$.

This is join work with Primož Škraba.

**References**

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# Collapsing Čech to Delaunay complexes in persistence of sampled dynamical systems

Thursday
17:00–17:30

**Grzegorz Jabłonski**

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The goal of our research is to embed persistence in the computational analysis of dynamical systems. We narrow our research to the case of the homomorphism induced in homology by a continuous map on a topological space. It is characterized up to conjugacy by its Jordan form, therefore the inference of the eigenvectors from sampling of the map allows for partial reconstruction of the Jordan normal form.

Assume a continuous self-map $f\colon \mathbb{M} \to \mathbb{M}$, where $\mathbb{M} \subset \mathbb{R}^n$, is known only through an approximation map $g\colon X \to X$ and $X \subset \mathbb{M}$ is finite. The pair $(g, X)$ is called sampled dynamical system and was introduced in [1]. Let $L$ be the Lipschitz constant of $g$, and $Cech_r(X)$ denote Čech complex with radius $r$ and vertices $X$. Extending $g$ to a simplicial map $g\colon Cech_r(X) \to Cech_{Lr}(X)$ is straightforward but requires construction of a Čech complex. We use the Delaunay complexes, called also alpha complexes, in place of Čech complexes, as the size of the latter grows rapidly in comparison with the former. This significantly decreases the running time, but as a consequence there is number of challenging algorithmic problems that we solve. Using the collapsibility of Čech to Delaunay complexes [2] and tools from Discrete Morse theory we show how to construct the simplicial version of $g$ on Delaunay complexes without constructing Čech complexes explicitly. The collapse is computed with an iterative algorithm whose running time depends on geometric predicates. We focus on implementation of one geometric predicate that decides whether there exists a minimal sphere in $\mathbb{R}^d$ enclosing a given set of points inside, and excluding another given set outside. This predicate is critical in the construction of a collapse of a single edge and as a result also of a cycle. The final step is an implementation of the chain map induced by $g\colon Del_r(X) \to Del_{Lr}(X)$, where $Del_r(X)$ denotes Delaunay complex with radius $r$ and vertices $X$. After constructing the map on a Delaunay complex, we compute the homology over a finite field, and the eigenvalues along with corresponding eigenvectors. Growing the radius parameter $r$ in the Delaunay complex we obtain a sequence, called a tower, of simplicial complexes, maps induced in homology and corresponding eigenvectors. Finally, using analogous technique to the one proposed in [1] we extract persistenceof eigenvectors from the above constructed sequence.

This is a joint work with U. Bauer, H. Edelsbrunner and M. Mrozek.

**References**


[1] H. Edelsbrunner, G. Jabłoński, M. Mrozek, *The persistent homology of a self-map*, Found. Comput. Math. 15 (2015), 1213–1244.

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# Certifying numerical estimates of spectral gaps

Friday
17:00–17:30


## Marek Kaluba

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Estimation a lower bound on the spectral gap of the Laplace operator on a finitely presented groups (and thus a lower bound on the Kazhdan constant) is a fundamental problem in analytic group theory. In particular, much work was devoted only to establishing and then improving bounds for $\mathrm{SL}(n, \mathbb{Z})$, special linear groups. The obtained estimates are analytical in nature, often providing a homogeneous bound for all $n$. In this talk I will show how to use conic optimisation (semi-definite programming) to obtain significantly better numerical bounds for spectral gaps of Laplacians on low dimensional special linear groups ($n = 3, 4, 5$) as well as for special linear groups over certain finite fields. This provides a constructive (but computer assisted) proof that these groups have Kazhdan property (T).

This is joint work with Piotr W. Nowak.

# Counting thick embeddings

Friday
17:00–17:30


## Fedor Manin

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Geometric knot theory — the study of knots with “thickness” — has seen significant study as well as applications to topics such as protein folding. Thickness for embeddings in higher dimensions is considerably harder to define; moreover, embedding theory in higher dimensions is different depending on whether one is interested in piecewise linear or smooth embeddings. Several nonequivalent notions of thick PL embedding have been put forward in papers of Gromov–Guth and Freedman–Krushkal. I will describe possible notions of thick smooth embedding and the following main result:

*Let $M$ be an $m$-dimensional Riemannian manifold, and let $n \geqslant m + 3$. Then (for a suitable definition of thickness) the number of 1-thick embeddings of $M$ in an $R$-ball in $\mathbb{R}^n$ is polynomial in $R$.*

This is joint work with Shmuel Weinberger.

# Perfect Discrete Morse Functions on Connected Sums

Tuesday
17:00–17:30

**Mehmetcik Pamuk**

Middle East Technical University
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In this talk, we study perfect discrete Morse functions on closed oriented $n$-dimensional manifolds. First, we show how to compose such functions on connected sums of manifolds. Then we discuss how to decompose such functions, particularly in dimensions 2 and 3.

This is a joint work with Neza Mramor Kosta and Hanife Varli.

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# On discretized configuration spaces

Friday
16:30–17:00

**Leonid Plachta**

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The purpose of the present talk is to discuss a conception of the discretized configuration space. It can be considered as some combinatorial or simplicial approximation of classical configuration spaces of the square or more generally, a polyhedron. The topological structure of discretized configuration spaces of $n$-dimensional simplex has been described in [2].

It also appears with relation to topological description of the configuration space $F(k, r)$ of $k$ hard discs of the same radii $r$ in the unite square [3] or the unite disc [1]. We describe some algebraic-topological properties of discretized configuration spaces of the square. We also indicate applications of discretized configuration spaces in the problem of strong coloring of $k$-uniform hypergraphs. The configuration spaces of some graph puzzles [4] are also considered.

### References

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## Topological complexity of subgroups of the braid groups

Thursday
17:00–17:30

**David Recio-Mitter**

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Topological complexity (TC) was introduced in the early 2000s by Michael Farber in the context of topological robotics. It is a numerical homotopy invariant of a space which measures the instability of motion planning. Moreover, TC can also be defined for a (discrete) group $\pi$, as the TC of its Eilenberg–Mac Lane space $K(\pi, 1)$. In particular the TC of the full braid group $B_n$ is by definition equal to the TC of the unordered configuration space of $n$ points on the plane.

In this talk the TC of groups will be introduced and calculated for some subgroups of the full braid groups, for instance mixed (or coloured) braid groups and congruence subgroups. The methods used in the calculations are algebraic rather than topological.

This is joint work with Mark Grant.

## On the Bourgin–Yang theorem

Monday
17:00–17:30

**Edivaldo L. dos Santos**

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In 1933, S. Ulam posed and K. Borsuk showed that if $n > m$, then it is impossible to map $f\colon S^n \to S^m$ preserving symmetry in the sense that $f(-x) = -f(x)$. In 1954–55, C. T. Yang, and D. Bourgin, showed that if $f\colon S^n \to \mathbb{R}^{m+1}$ preserves this symmetry, then $\dim f^{-1}(0) \geqslant n - m - 1$.

We will present a survey of different versions of the latter for some other groups of symmetries, and also discuss the case $n = \infty$. Specifically, let $V$ and $W$ be orthogonal representations of a compact Lie group $G$ with $V^G = W^G = \{0\}$. Let $S(V)$ be the sphere of $V$ and $f\colon S(V) \to W$ be a $G$-equivariant mapping. We estimate the dimension of set $Z_f = f^{-1}(0)$ in terms of $\dim V$ and $\dim W$, where $G$ is one of the following groups: the torus $\mathbb{T}^k$, the $p$-torus $\mathbb{Z}_p^k$, or the cyclic group $\mathbb{Z}_{p^k}$, where $p$ is a prime. We also show that for any $p$-toral group $1 \to \mathbb{T}^k \to G \to P \to 1$, $P$ a finite

$p$-group, and a $G$-map $f\colon S(V) \to W$, with $\dim V = \infty$ and $\dim W < \infty$, then we have that $\dim Z_f = \infty$.

This is joint work with Wacław Marzantowicz and Denise de Mattos.

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## A Persistent Homological Analysis of Network Data Flow Malfunctions

Tuesday
16:30–17:00

**Nicholas Scoville**

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Persistent homology has recently emerged as a powerful technique in topological data analysis for analyzing the emergence and disappearance of topological features throughout a filtered space, shown via persistence diagrams. In this presentation, we develop an application of ideas from the theory of persistent homology and persistence diagrams to the study of data flow malfunctions in networks with a certain hierarchical structure. In particular, we formulate an algorithmic construction of persistence diagrams that parametrize network data flow errors, thus enabling novel applications of statistical methods that are traditionally used to assess the stability of persistence diagrams corresponding to homological data to the study of data flow malfunctions. We conclude with a discussion of an application to network packet delivery systems.

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## Hyperplane Equipartitions Plus Constraints

Monday
16:30–17:00

**Steven Simon**

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Although equivariant methods have yielded a number of fruitful applications in combinatorial geometry, their inability to settle the long-standing but now settled Topological Tverberg conjecture has made clear the need to move “beyond” the use of Borsuk–Ulam type theorems alone. Such concerns hold equally well for many hyperplane mass equipartition problems dating back to Grünbaum, for which the best known topological upper bounds nearly always exceed the conjectured values arising from simple dimension counting. By analogy with the “constraint” method of Blagojević, Frick, and Ziegler, we show how this gap can be removed by the imposition of further conditions — by including further masses with specified partition-types (e.g., cascades and/or those of a "Makeev" variety) along with prescribed arrangements of the hyperplanes themselves (e.g., orthogonality and/or affine containment) — thereby yielding a wide variety of optimal results still obtainable via classical group cohomological techniques.

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## Equivariant maps between representation spheres of compact Lie groups

Thursday
17:00–17:30

**Mahender Singh**

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A basic problem in the theory of transformation groups is to find necessary and sufficient conditions for the existence of a $G$-equivariant map between two $G$-spaces. The most celebrated result in the necessary direction is the classical Borsuk–Ulam theorem, which states that if $U$ and $\bar{U}$ are two orthogonal fixed-point free $\mathbb{Z}_2$-representations, then the existence of a $\mathbb{Z}_2$-equivariant map $S(U) \to S(\bar{U})$ implies that $\dim(U) \leqslant \dim(\bar{U})$. This result has numerous and far reaching generalizations. One of these being replacing $\mathbb{Z}_2$ by an arbitrary compact Lie group. In this talk, we give a brief survey of various results in this direction, and present some recent results for $G = (\mathbb{S}^1)^k \times (\mathbb{Z}_p)^l$ ($p$ a prime) obtained in a joint work with Zbigniew Błaszczyk and Wacław Marzantowicz.

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## Some applications of persistent cohomology

Tuesday
17:00–17:30

**Jan Spaliński**

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We discuss the possibility of lifting $\mathbb{F}_p$-cocycles to $\mathbb{Z}$-cocycles for filtered complexes coming from point cloud data in $\mathbb{R}^n$. This problem is motivated by a paper of V. de Silva, D. Morozov and M. Vejdemo-Johansson.

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## Topology in the Furnace — using topology to find failure modes in industrial models

Thursday
16:30–17:00

**Mikael Vejdemo-Johansson**
CUNY College of Staten Island
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Steel smelting is a high-volume, high-throughput industry, where the smallest performance gains translate into large dividends. Model construction to predict conditions inside the furnace is a centrally important part of process control. Machine learning and statistical methods have been shown to improve on purely metallurgical models, but in either case, the failure modes of the model are poorly understood, and tools for analyzing them not well developed.

We work in collaboration with Outukumppu Stainless with their electric-arc scrap furnace, to analyze and improve their temperature prediction models. Temperature prediction in particular is an important model to improve: reference measurements can be done by inserting probes, but these are costly and if measuring too early, more probes will be needed — measuring too late risks overheating the steel and spoiling the entire batch.

We are studying the use of the Mapper algorithm to construct intrinsic models of the fibres (preimages) of failed predictions. These models help classify different modes of failure for the models, and direct attention for improvement or for learning compensation transforms to improve precision of temperature detection.

In this talk, I will describe the approach we take for modeling and classifying failure modes, and give some examples from our ongoing study of the steel smelting data.

## Strong discrete Morse theory

Tuesday
16:30–17:00

**José Antonio Vilches**
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The goal of this talk is to establish a Lusternik–Schnirelmann theorem for discrete Morse functions and the recently introduced simplicial Lusternik–Schnirelmann category of a simplicial complex. To accomplish this, a new notion of critical object of a discrete Morse function is presented, generalizing the concept of critical simplex (in the sense of R. Forman). We show that the non-existence of such critical objects guarantees the strong homotopy equivalence (in the Barmak and Minian's sense) between the corresponding sublevel complexes. Finally, we establish that the number of critical objects of a discrete Morse function defined on $K$ is an upper bound for the non-normalized simplicial Lusternik–Schnirelmann category of $K$.

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## Multiple chessboard complexes and some theorems of Tverberg type

Thursday
16:30–17:00

**Siniša Vrećica**

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Chessboard complexes appear in many different areas of Mathematics. In Geometric combinatorics and Topology they appear as the configuration spaces in the proofs of theorems of van Kampen–Flores and Tverberg type. The generalized, multiple and symmetrized, versions of these complexes appear also in the natural way in the problems of similar type.

We determine some topological properties of these complexes, and using them establish some new theorems of van Kampen–Flores and Tverberg type. One of them confirms a conjecture of Blagojević, Frick and Ziegler about the existence of "balanced Tverberg partitions".

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## Instantaneous dimension of metric spaces via spread and magnitutude

Tuesday
16:30–17:00

**Simon Willerton**

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Some spaces seem to have different dimensions at different scales. A long thin strip might appear one-dimensional at a distance, then two-dimensional when zoomed in on, but when zoomed in on even closer it is seen to be made of a finite array of points, so at that scale it seems zero-dimensional. I will present a way of quantifying this phenomenon.

The main idea is to think of dimension as corresponding to growth rate of size: when you double distances, a line will double in size and a square will quadruple in size. You then just need some good notions of size of metric spaces. One such notion is 'magnitude' which was introduced by Leinster, using category theoretic ideas, but was found to have links to many other

areas of maths such as biodiversity and potential theory. There's a closely related, but computationally more tractable, family of notions of size called 'spreads' which I introduced following connections with biodiversity.

Meckes showed that the asymptotic growth rate of the magnitude of a metric space is the Minkowski dimension (ie. the usual dimension for squares and lines and the usual fractal dimension for things like Cantor sets). But this is zero for finite metric spaces. However, by considering growth rate nonasymptotically you get interesting looking results for finite metric spaces, such as the phenomenon described in the first parargraph.

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# A Random Bockstein Operator

Monday
16:30–17:00

**Matthew Zabka**

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As more of topology's tools become popular in analyzing high dimensional data sets, the goal of understanding the underlying probabilistic properties of these tools becomes even more important. While much attention has been given to understanding the probabilistic properties of methods that use homological groups in topological data analysis, the probabilistic properties of methods that employ cohomology operations remain largely unstudied. In this talk, we shall investigate a Bockstein operator with randomness in a strictly algebraic setting.

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# Directed paths on cubical complexes

Friday
16:30–17:00

**Krzysztof Ziemiański**

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Higher Dimensional Automaton is a cubical complex with two distinguished vertices: the initial and the final state and some labeling of vertices. Higher Dimensional Automata are used to model concurrent programs; possible states (respectively possible executions) of a given program correspond to points (respectively directed paths) of the geometric realization of the underlying cubical complex.

I will present a construction of a CW-complex which is homotopy equivalent to the space of directed paths on a cubical complex. This construction

satisfies certain minimality condition which makes it useful for direct calculations. Furthermore, such CW-complexes carry an interesting combinatorial structure — their cells can be identified with products of permutohedra and attaching maps are inclusions of faces. I will also discuss the relationship of this construction with other models of directed path spaces.

# Posters



## Inverse limits of $T_0$-Alexandroff spaces

### Paweł Bilski

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In this work we study $T_0$-Alexandroff spaces and their inverse limits which have applications in digital topology. We present some know results concerning approximations of compact Hausdorff spaces by finite spaces. We also present some generalizations showing that also non-compact polyhedra and locally compact, paracompact Hausdorff spaces can be approximated by $T_0$-Alexandroff spaces.

# Topological pattern analysis of atmospheric boundary layer turbulence

**Jose Licon**
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A current topic of research in atmospheric science is the effect of the land surface on turbulent mixing and transport processes in the planetary boundary layer. This affects the evolution of model variables, as well as parametrizations for large-scale climate models. We present a study of turbulent flow using homology to describe the structure of the boundary layer, and show how the Betti numbers of binarized plane domains for the vertical velocity and temperature fields reflect the underlying system dynamics.

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# On topological Tverberg type theorem

**Carlos Henrique Felicio Poncio**
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Helge Tverberg showed that any set of $(d+1)(q-1)+1$ points in $\mathbb{R}^d$ admits a partition into $q$ subsets such that the intersection of their convex hulls is non-empty. Bárány's topological Tverberg conjecture from 1976 states that any continuous map of an $N$-simplex $\Delta_N$ to $\mathbb{R}^d$, for $N \geqslant (d+1)(r-1)$, maps points from $r$ disjoint faces in $\Delta_N$ to the same point in $\mathbb{R}^d$. This conjecture is true for $r$ a prime power (Bárány-Shlosman-Szücs for $r$ prime, Özaydin and Volovikov for $r$ a prime power), and recent spectacular counter-examples in high dimensions for the case when $r$ is not a prime power were found in a series of papers by Frick, Blagojević–Frick–Ziegler and Mabillard–Wagner.

In this work, we will discuss some results in progress related to topological Tverberg type theorem for continuous map of an $N$-simplex $\Delta_N$ to $Y^k$, where $Y$ is a $k$-dimensional CW-complex.

This work is a part of my PhD's thesis under the supervision of professor Edivaldo L. dos Santos and it is supported by FAPESP 2016/06456-4.

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# Theory of interleavings on $[0, \infty)$-actegories

**Anastasios Stefanou**
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The interleaving distance was originally defined in the field of Topological Data Analysis (TDA) by Chazal et al. as a metric on the class of persistence modules parametrized over the real line. Bubenik et al. subsequently extended the definition to categories of functors on a poset, the objects in these categories being regarded as 'generalized persistence modules'. These metrics typically depend on the choice of a lax semigroup of endomorphisms of the poset. The purpose of the present paper is to develop a more general framework for the notion of interleaving distance using the theory of 'actegories'. Specifically, we extend the notion of interleaving distance to arbitrary categories equipped with a lax monoidal action by the monoid $[0, \infty)$, such categories being known as $[0, \infty)$-actegories. In this way, the class of objects in such a category acquires the structure of a Lawvere metric space. Functors that are colax $[0, \infty)$-equivariant yield maps that are 1-Lipschitz. This leads to concise proofs of various known stability results from TDA, by considering appropriate colax $[0, \infty)$-equivariant functors. Along the way, we show that several common metrics, including the Hausdorff distance and the $L_\infty$-norm, can be realized as interleaving distances in this general perspective.

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# Composing and decomposing perfect discrete Morse functions on a connected sum of surfaces

**Hanife Varlı**
Middle East Technical University
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We study perfect discrete Morse functions on closed, connected surfaces. We show how to compose and decompose such functions on a connected sum of closed surfaces.

This a joint work with Neza Mramor Kosta and Mehmetcik Pamuk.

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# Ordered Configuration Spaces in the collision-free motion planning problem

**Cesar Augusto Ipanaque Zapata**

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In robotics, in *the motion planning problem*, Michael Farber [1] proved that a continuous motion planning algorithm on space $X$ exists if and only if $X$ is contractible. In *the problem of simultaneous motion planning without collisions* for $k$ robots, we want to know if exists a continuous motion planning algorithm on the ordered configuration space $F(M,k)$. Thus, an interesting question is whether $F(M,k)$ is contractible.

It seems likely that the configuration space $F(M,k)$ is not contractible for certain topological manifolds $M$. Evidence for this statement is given in the work of F. Cohen and S. Gitler [2]. In this work I will show that for $M$ a finite dimensional topological manifold, the ordered configuration space $F(M,k)$ is not contractible, $\forall k \geqslant 2$. Thus its Lusternik-Schnirelmann category ($cat$) and its Topological complexity ($TC$) is at least 2. As an example, I will calculate this invariants for the ordered configuration space of 2-distinct points in Complex Projective $n-$space. On the other hand, I will show that $cat(\Omega F(M,k))$ and $TC(\Omega F(M,k))$ are infinite, $\forall k \geqslant 2$, where $M$ is a certain finite dimensional topological manifold.

This work is a part of my PhD's thesis under the supervision of professor Denise de Mattos and it is supported by FAPESP 2016/18714-8.

### References

[1] M. Farber, *Topological complexity of motion planning*, Disc. Comp. Geom. 29 (2003),211–221.

[2] F. Cohen, S. Gitler, *On loop spaces of configuration spaces*, Trans. Amer. Math. Soc. 354 (2002), 1705–1748.

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