AN ARCSINE LAW FOR MARKOV RANDOM WALKS

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The arcsine law for the number $N_n^> := n^{-1}\sum_{k=1}^n \mathbf{1}_{\{S_k>0\}}$ of positive terms, as $n\to\infty$, in an ordinary random walk $(S_n)_{n\ge 0}$ states that $n^{-1}N_n^>$ converges in distribution to a beta law with density

$$\frac{\sin(\pi\,\rho)}{\pi}\,\frac{1}{x^{1-\rho}\,(1-x)^{\rho}}\,\mathbf{1}_{(0,1)}(x),$$

also called generalized arcsine law with parameter $\rho\in[0,1]$ (the classic arcsine law if $\rho=1/2$) iff the Spitzer condition

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\mathbb{P}(S_n>0)\ =\ \rho$$

holds. The purpose of this talk will be to discuss an extension of this result to the case when the random walk is governed by a positive recurrent Markov chain $(M_n)_{n\ge 0}$ on a countable state space $\mathcal{S}$, that is, for a Markov random walk $(M_n,S_n)_{n\ge 0}$ with positive recurrent discrete driving chain. We will further show a result concerning the equivalence of the above Spitzer condition with the stronger variant

$$\lim_{n\to\infty}\mathbb{P}(S_n>0)\ =\ \rho$$

in the Markov-modulated context. For an ordinary random walk, this equivalence was shown by Doney [3] for $0<\rho<1$ and by Bertoin and Doney [2] for $\rho\in\{0,1\}$.

References

[1] G. Alsmeyer and F. Buckmann. An arcsine law for Markov random walks, 2017. Preprint available at `http://arxiv.org/abs/1703.00316`. To appear in *Stoch. Processes Appl.*

[2] J. Bertoin and R. A. Doney. Spitzer's condition for random walks and Lévy processes. *Ann. Inst. H. Poincaré Probab. Statist.*, 33(2):167–178, 1997.

[3] R. A. Doney. Spitzer's condition and ladder variables in random walks. *Probab. Theory Related Fields*, 101(4):577–580, 1995.